

Solving a Boundary Value Problem for a System of Equations of Gas Dynamics Describing the Motion of an Evaporating Droplet in an Electromagnetic Radiation Field

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Abstract—The article solves a boundary value problem for a stationary system of equations of gas dynamics (a velocity-linearized system of Navier–Stokes equations, convective equations of heat and mass transfer), which allows us to study the motion of an evaporating spherical droplet in the field of electromagnetic radiation. Convective heat and mass transfer equations are solved by the method of spliced asymptotic expansions. The solution of the problem allows us to evaluate the effects of the reactive, thermodiffusion and Stephan effects on the rate of photophoresis of an evaporating droplet.

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1. INTRODUCTION

The phenomenon of photophoresis in a gas consists in the movement of particles in the field of electromagnetic radiation. A particle located in the field of directional radiation absorbs electromagnetic energy, which is converted into heat and causes inhomogeneous heating of its surface. The gas, interacting with an inhomogeneously heated surface, begins to move along the surface in the direction of increasing temperature. This phenomenon is called thermal slip and it causes the appearance of a photophoretic force. Under the influence of this force, the drop begins to move. When the magnitude of the photophoretic force becomes equal in magnitude to the viscous drag force of the medium, the drop begins to move uniformly. The rate of uniform motion of the droplet is called the photophoretic velocity.

Despite the fact that this phenomenon was discovered at the beginning of the 20th century [1], interest in it does not weaken, but only grows. New fields of application of photophoresis are opening up, for example, this is a global problem of combating air pollution with aerosols of both artificial and natural origin (particle capture and levitation); numerous technological applications require information about the behavior of dispersed media in various conditions. Such problems are relevant for industries using particle deposition processes in channels; during fine cleaning of small volumes of gas; aerosol sampling; application of a given thickness of special coatings, etc.

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2. MATHEMATICAL FORMULATION OF THE PROBLEM

From a mathematical point of view, photophoresis is a complex task consisting of an electrodynamic problem (calculation of the characteristics of the absorbed electromagnetic field in the volume of a particle), a thermal problem (calculation of temperature fields in the volume and on the surface of a particle) and a gas kinetic problem (description of heat and mass transfer in the gas phase, calculation of fields of forces, pressures, velocities, etc.).

Consider a spherical large ($Kn \leq 0.01$, Kn is Knudsen number [2]) evaporating drop of radius R suspended in a temperature-homogeneous binary gas mixture with temperature T_e , density ρ_e , thermal conductivity λ_e , diffusion D_{12} , and viscosity μ_e .

A binary gas mixture is described by the relative numerical concentrations of the components C_1 and C_2 . Here $C_1 = n_1/n_e$, $C_2 = n_2/n_e$, $n_e = n_1 + n_2$ is the total number of molecules per unit volume of the mixture, $\rho_e = \rho_1 + \rho_2$, $\rho_1 = n_1 m_1$, $\rho_2 = n_2 m_2$, m_1, n_1 , and m_2, n_2 are mass and the numerical concentration of molecules of the first and second components of the mixture. The first component C_1 coincides in its physics–chemical composition with the substance of a liquid drop. The second component is considered the main one (carrier). Here and further, the indices “ e ” and “ i ” we will refer to the gas and the drop, the index “ s ” denotes the values of physical quantities taken at the average temperature of the drop surface T_s , and the index “ ∞ ” denotes the average values of physical quantities, characterizing the binary gas environment away from the droplet.

The diffusion mode of droplet evaporation is assumed ($C_1 \ll 1$). The main influence on the process of heat and mass transfer in the vicinity of a drop is determined by molecular diffusion. The radius of the droplet is considered unchanged (the time of noticeable change in the radius of the droplet is much longer than the relaxation time of diffusion and thermal inhomogeneities near it). When moving, the drop retains its spherical shape, i.e., the surface tension forces are much greater than the viscous drag force. Due to the short time of thermal and diffusive relaxation, the process of heat and mass transfer in the particle–gas system proceeds quasi-stationary, and free convection is neglected (the Grashof number is small). The problem is solved by the hydrodynamic method, i.e., the equations of hydrodynamics and heat and mass transfer with appropriate boundary conditions are solved.

We will describe the motion of the drop in a spherical coordinate system ($y = r/R, \theta, \varphi$), the beginning of which coincides with the center of mass of the drop. With this choice of reference frame, the task is reduced to analyzing the flow of an evaporating droplet by an infinite plane-parallel flow in the positive direction of the Oz axis, the velocity of which $\mathbf{U}_\infty$ is to be determined ($\mathbf{U}_\infty \parallel Oz$, $\mathbf{U}_\infty = -\mathbf{U}_{ph}$, $\mathbf{U}_{ph}$ is the photophoretic velocity).

The distributions of velocities, pressures, temperatures, and relative concentrations of the first component of the binary gas mixture (due to the symmetry of the problem) depend only on the radial coordinate y and the polar angle θ .

The problem is considered with small relative temperature differences in the vicinity of the droplet, the molecular transfer coefficients (viscosity, thermal conductivity, and diffusion) and the density of the binary gas mixture are considered constant.

The article solved the following stationary system of equations of gas dynamics [3–5]

$$\mu_e \Delta \mathbf{U}_e = \nabla P_e, \quad \text{div}(\rho_e \mathbf{U}_e) = 0, \quad \mu_e \Delta \mathbf{U}_i = \nabla P_i, \quad \text{div}(\rho_i \mathbf{U}_i) = 0, \quad (1)$$

$$\rho_e c_p (U_e \nabla) T_e = \lambda_e \Delta T_e, \quad (U_e \nabla) C_{1e} = D_{12} \Delta C_1, \quad (2)$$

$$\Delta T_i = -\frac{q_i}{\lambda_i}. \quad (3)$$

Here $\mathbf{U}_e(\mathbf{r})$ and $\mathbf{U}_i(\mathbf{r})$, $P_e(\mathbf{r})$ and $P_i(\mathbf{r})$ are the mass velocity and pressure outside and inside the evaporating droplet, $C_1(\mathbf{r})$ is the numerical concentration of the first component of the binary gas mixture and $T_e(\mathbf{r})$, $T_i(\mathbf{r})$ are the temperature fields outside and inside the evaporating droplet.

The stationary system of equations of gas dynamics (1)–(3) was solved with the following boundary conditions:

$$\begin{aligned} \lim_{y \rightarrow \infty} U_r^{(e)}(y, \theta) &= U_\infty \cos \theta, \quad \lim_{y \rightarrow \infty} U_\theta^{(e)}(y, \theta) = -U_\infty \sin \theta, \quad \lim_{y \rightarrow \infty} T_e(y, \theta) = T_\infty, \\ \lim_{y \rightarrow \infty} C_1(y, \theta) &= C_0, \quad \lim_{y \rightarrow \infty} P_e(y, \theta) = P_\infty, \quad U_\infty = |\mathbf{U}_\infty|, \end{aligned} \quad (4)$$

$$\lim_{y \rightarrow 0} P_i(y, \theta) = P_0, \quad \lim_{y \rightarrow 0} |\mathbf{U}_i(y, \theta)| \neq \infty, \quad \lim_{y \rightarrow 0} T_i(y, \theta) \neq \infty, \quad (5)$$

$$\lim_{y=1} \left(n_1 U_r^{(e)} - D_{12} \frac{n_e^2 m_2}{R \rho_e} \left[\frac{\partial C_1}{\partial y} + \frac{K_{TD}}{T_e} \frac{\partial T_e}{\partial y} \right] \right) = \lim_{y=1} \left(n_{1i} U_r^{(i)} \right), \quad (6)$$

$$\lim_{y=1} \left(n_2 U_r^{(e)} + D_{12} \frac{n_e^2 m_1}{R \rho_e} \left[\frac{\partial C_1}{\partial y} + \frac{K_{TD}}{T_e} \frac{\partial T_e}{\partial y} \right] \right) = 0, \quad C_{1s}^{(H)} = \frac{n_{1s}^{(H)}}{n_e} \Big|_{T_i=T_s}, \quad (7)$$

$$\lim_{y=1} U_\theta^{(e)} = \lim_{y=1} \left(K_{TS}^{(0)} \frac{\nu_e}{RT_e} \frac{\partial T_e}{\partial \theta} + K_{DS}^{(0)} \frac{D_{12}}{R} \frac{\partial C_1}{\partial \theta} \right), \quad \lim_{y=1} T_e = \lim_{y=1} T_i, \quad (8)$$

$$\lim_{y=1} \left(-\lambda_e \frac{\partial T_e}{\partial y} + \lambda_i \frac{\partial T_i}{\partial y} \right) = \lim_{y=1} \left(L \frac{n_e^2 m_1 m_2}{\rho_e} D_{12} \left[\frac{\partial C_1}{\partial y} + \frac{K_{TD}}{T_e} \frac{\partial T_e}{\partial y} \right] - \sigma_0 \sigma_1 R (T_i^4 - T_\infty^4) \right), \quad (9)$$

$$\lim_{y=1} \left(\mu_e \left[\frac{\partial U_\theta^{(e)}}{\partial y} + \frac{1}{y} \frac{\partial U_r^{(e)}}{\partial \theta} - \frac{U_\theta^{(e)}}{y} \right] + \frac{\partial \sigma}{\partial T_i} \frac{\partial T_i}{\partial \theta} \right) = \lim_{y=1} \left(\mu_i \left[\frac{\partial U_\theta^{(i)}}{\partial y} + \frac{1}{y} \frac{\partial U_r^{(i)}}{\partial \theta} - \frac{U_\theta^{(i)}}{y} \right] \right), \quad (10)$$

$$\lim_{y=1} C_1 = \lim_{y=1} \left(C_{1s}^{(H)} + C_{1s}^* \delta T_i \right), \quad C_{1s}^* = \frac{1}{n_e} \frac{\partial n_{1s}^{(H)}}{\partial T_i} \Big|_{T_i=T_s}. \quad (11)$$

At infinity at $y \rightarrow \infty$ and the finiteness of the physical quantities characterizing the drop at $y \rightarrow 0$ are taken into account in the boundary conditions (4), (5), and on the boundary surface, i.e., at $y = 1$, the boundary conditions (6)–(11). $U_r^{(e)}$ and $U_\theta^{(e)}$ are components of the mass velocity $\mathbf{U}_e$ in a spherical coordinate system and similarly for $\mathbf{U}_i$, σ is the coefficient of surface tension of a drop, L is the specific heat of evaporation of a liquid, σ_0 is the Stefan–Boltzmann constant, σ_1 is the integral degree of blackness, ν_e is the kinematic viscosity, $n_{1s}^{(H)}$ is the saturated concentration of molecules of the first component of the binary gas mixture, depending on the average temperature of the droplet surface T_i , K_{TD} is the coefficient of thermal diffusion [3], n_{1i} is the numerical concentration of molecules of the drop substance $K_{TS}^{(0)}$, and $K_{DS}^{(0)}$ is the coefficients of thermal and diffusion slip. Note that the gas kinetic slip coefficients included in the boundary conditions are determined from the solution of the kinetic Boltzmann equation [6] and with energy and tangential momentum accommodation coefficients equal to one $K_{TS}^{(0)} = 1.161$ and $K_{DS}^{(0)} = 0.3$ [6]. Undisturbed parameters (T_∞ and P_∞) observed at the location of the geometric center of the evaporating droplet in its absence (the value of C_0 is determined by the numerical concentrations of n_1 and n_2 gas molecules), $q_i = \frac{4\pi n_k a_k}{n_s \lambda_0} I_0 B_k$ is the volumetric density of internal heat sources inhomogeneously distributed inside the droplet, $m_k = n_k + i a_k$ is the complex refractive index of the droplet, n_s is the refractive index of the medium, I_0, λ_0 is the wavelength and intensity of incident radiation, B_k is the coordinate function calculated according to the theory of Mi [7], and δT_i is determined by from the boundary conditions on the surface of the evaporating droplet.

In boundary conditions on the surface of the evaporating droplet, the following are taken into account: the continuity of the radial flow for the first component of the binary gas mixture and the impermeability of the droplet surface for the second component are taken into account in (6) and (7). The left parts represent the sums of convective, diffusive and thermodiffusion flows; the following boundary condition (8) reflects the well-known phenomenon of thermal and diffusive sliding, proportional respectively to the coefficients $K_{TS}^{(0)}$ and $K_{DS}^{(0)}$ and the equality of temperatures on drop surface; condition (9) reflects the continuity of the radial heat flow, taking into account the heat going to the phase transition of the liquid droplet into steam, taking into account thermal diffusion and radiation, in the boundary condition (10), the continuity of the tangential components of the stress tensor is taken into account, taking into account the dependence of the surface tension coefficient of the droplet on temperature, the numerical relative concentration of molecules of the first (testing phase transition) of the component of the external mixture $C_1 = C_{1s}(T_s)$ at the surface of the droplet in a linear approximation by temperature perturbation δT_i must satisfy the condition (11).

Let's move on to the mathematical formulation of the problem.

Problem: A viscous binary gas mixture occupies an unlimited area $\Omega_e = \mathbb{R}^3 \setminus \Omega_i$ ($x \in \Omega_e$), where Ω_i is a spherical region centered at zero in Euclidean space R^3 . It is required to find the photophoretic velocity $\mathbf{U}_{ph}(x)$, which satisfies the system of differential equations (1)–(3) and boundary conditions (4)–(11).

3. SOLUTIONS OF EQUATIONS OF HYDRODYNAMICS AND CONVECTIVE EQUATIONS OF HEAT AND MASS TRANSFERS

To solve the problem formulated above, it is necessary to know the distributions of velocity, pressure and temperature fields in the vicinity of the evaporating droplet.

Based on the type of boundary conditions for the components of the mass velocity away from the evaporating droplet, the solution of the equations of hydrodynamics (1) for an axisymmetric Stokes flow in spherical coordinates is sought in the form of a series expansion by Legendre polynomials of the first kind and Gegenbauer polynomials of the first kind [8]. Due to the orthogonality conditions of these polynomials, only the first terms of these positions contribute to the total force acting on the drop. Therefore, we can write

$$U_r^{(e)}(y, \theta) = U_\infty G(y) \cos \theta, \quad U_\theta^{(e)}(y, \theta) = -U_\infty g(y) \sin \theta, \quad (12)$$

where the functions $G(y)$ and $g(y)$ are defined by setting (12) into the equations (1).

Solving these equations in a spherical coordinate system by separating variables, we find the functions $G(y)$ and $g(y)$:

$$G(y) = 1 + \frac{A_1}{y^3} + \frac{A_2}{y}, \quad g(y) = 1 - \frac{A_1}{2y^3} + \frac{A_2}{2y}.$$

Thus, we have the following expressions for the mass velocity and pressure components that satisfy the boundary conditions away from the drop ($y \rightarrow \infty$)

$$\begin{aligned} U_r^{(e)}(y, \theta) &= U_\infty \left(1 + \frac{A_1}{y^3} + \frac{A_2}{y} \right) \cos \theta, \quad U_\theta^{(e)}(y, \theta) = -U_\infty \left(1 - \frac{A_1}{2y^3} + \frac{A_2}{2y} \right) \sin \theta, \\ P_e(y, \theta) &= P_\infty + \cos \theta \frac{\mu_e}{Ry^2} A_2. \end{aligned} \quad (13)$$

The general solution of the equation of the system of equations describing the velocity and pressure field inside the evaporating droplet is determined in a similar way and has the form

$$\begin{aligned} U_r^{(i)}(y, \theta) &= U_\infty (A_3 + A_4 y^2) \cos \theta, \quad U_\theta^{(i)}(y, \theta) = -U_\infty (A_3 + 2A_4 y^2) \sin \theta, \\ P_i(y, \theta) &= P_0 + 10 \cos \theta \frac{\mu_i}{R} A_4 y^2. \end{aligned} \quad (14)$$

Here the constant integrations are A_1 , A_2 , A_3 , and A_4 included in equations (13) and (14) are uniquely determined from the boundary conditions on the surface of the drop $y = 1$.

In the mathematical problem formulated above, there is a small parameter, the role of which is played by the Reynolds number $\varepsilon = Re = U_\infty R \rho_e / \mu_e \ll 1$. Proudman and Pearson [9] for the hydrodynamic problem showed that, far from the sphere, the inertial terms become of the same order as the terms of molecular transport. Therefore, the usual method of finding a solution to the Navier–Stokes equations in the form of a small parameter expansion gives a known error, since already in the second approximation it does not strictly satisfy the boundary conditions at infinity and obtain an accurate uniform solution that is uniformly valid for the entire flow region.

Taking into account the above, solutions to the convective equations of thermal conductivity and diffusion (2) will be sought by the method of spliced asymptotic expansions [10, 11].

We show the application of the method of spliced asymptotic expansions by the example of solving the convective heat equation. To do this, in the equation (2), let's move on to dimensionless variables by entering the dimensionless temperature $t_e = T_e / T_\infty$ and velocity $\mathbf{V}_e = \mathbf{U}_e / U_\infty$ and then in the spherical coordinate system it takes the following form

$$\varepsilon Pr \left(V_r^{(e)}(y, \theta) \frac{\partial t_e(y, \theta)}{\partial y} + \frac{V_\theta^{(e)}(y, \theta)}{y} \frac{\partial t_e(y, \theta)}{\partial \theta} \right)$$

$$= \frac{1}{y^2} \frac{\partial}{\partial y} \left(y^2 \frac{\partial t_e(y, \theta)}{\partial y} \right) + \frac{1}{y^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t_e(y, \theta)}{\partial \theta} \right). \quad (15)$$

Here $Pr = \mu_e c_p / \lambda_e$ is the thermal Prandtl number [3].

The internal and external asymptotic expansions of a dimensionless temperature are represented as

$$t_e(y, \theta) = \sum_{n=0}^{\infty} f_n(\varepsilon) t_{en}(y, \theta), \quad f_0(\varepsilon) = 1, \quad (16)$$

$$t_e^*(\xi, \theta) = \sum_{n=0}^{\infty} f_n^*(\varepsilon) t_{en}^*(\xi, \theta), \quad (17)$$

where $\xi = \varepsilon y$ is the “compressed” radial coordinate [10, 11]. In this case, it is required that

$$\lim_{\varepsilon \rightarrow 0} \frac{f_{n+1}}{f_n} \rightarrow 0, \quad \lim_{\varepsilon \rightarrow 0} \frac{f_{n+1}^*}{f_n^*} \rightarrow 0.$$

The missing boundary conditions for internal and external decompositions follow from the condition of identity of asymptotic extensions of both into some intermediate domain

$$t_e(y \rightarrow \infty, \theta) = t_e^*(\xi \rightarrow 0, \theta). \quad (18)$$

Taking into account the compressed radial coordinate, we have the following equation for finding the temperature t_e^*

$$\begin{aligned} & \varepsilon Pr \left(V_r^*(\xi, \theta) \frac{\partial t_e^*(\xi, \theta)}{\partial \xi} + \frac{V_\theta^*(\xi, \theta)}{\xi} \frac{\partial t_e^*(\xi, \theta)}{\partial \theta} \right) \\ &= \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left(\xi^2 \frac{\partial t_e^*(\xi, \theta)}{\partial \xi} \right) + \frac{1}{\xi^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t_e^*(\xi, \theta)}{\partial \theta} \right), \end{aligned} \quad (19)$$

with boundary conditions

$$\begin{aligned} & \lim_{\xi \rightarrow \infty} t_e^*(\xi, \theta) \rightarrow 1, \\ & \mathbf{V}_e^*(\xi, \theta) = \mathbf{n}_z + \varepsilon \mathbf{V}_e^{(1)*}(\xi, \theta) + \dots \end{aligned}$$

Here $\mathbf{n}_z$ is a unit vector in the direction of the Oz axis.

The construction of the solution begins with the definition of the zero term of the external decomposition (17). In this case, obviously, the mathematical problem is satisfied by the solution

$$t_{e0}^*(\xi, \theta) = 1.$$

Let's find the zero term of the internal decomposition of (16). For $\varepsilon = 0$ we have the equation

$$\Delta t_{e0}(y, \theta) = 0. \quad (20)$$

The general solution of the equation (20) has the form [5]

$$t_{e0}(y, \theta) = D_0 + \frac{\Gamma_0}{y} + \sum_{n=1}^{\infty} \left(\frac{\Gamma_n}{y^{n+1}} + D_n y^n \right) P_n(\cos \theta). \quad (21)$$

Here D_0, D_n, Γ_0 , and Γ_n are constant integrations, $P_n(\cos \theta)$ are Legendre polynomials.

The constant integrations D_0, D_n, Γ_0 , and Γ_n must be determined from the splice condition (18). The external solution should be laid out in a row by ξ . Then, the value of the constants is set from the requirement of matching the behavior of the members of the resulting series at $\xi \rightarrow 0$ and the members of the decomposition (21) at $y \rightarrow \infty$. For zero approximations, splicing is trivial. We get $D_0 = 1, D_n = \Gamma_n = 0 (n = 1, 2, \dots)$. Therefore,

$$t_{e0}(y, \theta) = 1 + \frac{\Gamma_0}{y}.$$

Let's find the first approximation for the external decomposition of (17), which has the following form

$$t_e^*(\xi, \theta) = 1 + f_1^*(\varepsilon)t_{e1}^*(\xi, \theta). \quad (22)$$

It can be seen from (22) that in order to find the first approximation of the external decomposition, it is first necessary to determine the explicit form of the coefficient $f_1^*(\varepsilon)$. To do this, in the solution (21), let's move on to the external variable. Thus, $f_1^*(\varepsilon) = \varepsilon$ and

$$t_e^*(\xi, \theta) = 1 + \varepsilon(\varepsilon)t_{e1}^*(\xi, \theta). \quad (23)$$

Substituting (23) into the equation (19) and holding the terms of the order ε , we get

$$\begin{aligned} & Pr \left(x \frac{\partial t_{e1}^*(\xi, x)}{\partial \xi} + \frac{1-x^2}{\xi} \frac{\partial t_{e1}^*(\xi, x)}{\partial x} \right) \\ &= \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left(\xi^2 \frac{\partial t_{e1}^*(\xi, x)}{\partial \xi} \right) + \frac{1}{\xi^2} \frac{\partial}{\partial x} \left[(1-x^2) \frac{\partial t_{e1}^*(\xi, x)}{\partial x} \right], \quad x = \cos \theta, \end{aligned} \quad (24)$$

with a boundary condition

$$\lim_{\xi \rightarrow \infty} t_{e1}^* \rightarrow 0.$$

Go to (24) to the new variable $z = Pr\xi$. We are looking for a solution to the resulting equation in the form

$$t_{e1}^*(z, x) = G(z, x) \exp \left\{ \frac{1}{2}zx \right\},$$

and as a result of transformations, it reduces to the Helmholtz equation [5, 8]

$$\Delta G(z, x) - \frac{1}{4}G(z, x) = 0. \quad (25)$$

We are looking for the solution of the equation (25) in the form

$$G(z, x) = \sum_{n=0}^{\infty} \Phi(z) P_n(\cos \theta). \quad (26)$$

Substituting (26) into (25), we obtain the equation for modified spherical Bessel functions of half-integer order [8]: $\sqrt{\frac{\pi}{z}} I_{n+\frac{1}{2}} \left(\frac{z}{2} \right)$ and $\sqrt{\frac{\pi}{z}} K_{n+\frac{1}{2}} \left(\frac{z}{2} \right)$.

Considering the mathematical properties of the functions $\sqrt{\frac{\pi}{z}} I_{n+\frac{1}{2}} \left(\frac{z}{2} \right)$, $\sqrt{\frac{\pi}{z}} K_{n+\frac{1}{2}} \left(\frac{z}{2} \right)$ and the boundary condition for the function t_{e1}^* , we get

$$t_{e1}^*(\xi, \theta) = \exp \left\{ \frac{1}{2}Pr\xi x \right\} \sum_{n=0}^{\infty} L_n \sqrt{\frac{\pi}{Pr\xi}} K_{n+\frac{1}{2}} \left(\frac{Pr\xi}{2} \right) P_n(x).$$

Arbitrary constants L_n must be determined as a result of splicing, which in this case consists in comparing the behavior of the function $t_e^*(\xi, x)$ for $\xi \rightarrow 0$ and the functions $t_e(y, x)$ for $y \rightarrow \infty$. It is not difficult to establish that $L_0 = \Gamma_0 Pr/\pi$, $L_n = 0$ ($n = 1, 2, \dots$). Therefore,

$$t_{e1}^*(\xi, \theta) = \frac{\Gamma_0}{\xi} \exp \left\{ \frac{1}{2}Pr\xi(x-1) \right\}. \quad (27)$$

Let's find the first approximation for the internal decomposition. From (27) we see that $f_1(\varepsilon) = \varepsilon$. Thus, we have the following binomial internal decomposition

$$t_{e1}(y, \theta) = t_{e0}(y) + \varepsilon t_{e1}(y, \theta). \quad (28)$$

Let's define the behavior of the function $t_{e1}(y \rightarrow \infty, \theta)$. To do this, we combine the binomial internal and external decompositions, we get

$$t_{e1}(y \rightarrow \infty, \theta) = \frac{\omega}{2} (\cos \theta - 1), \quad \omega = Pr\Gamma_0. \quad (29)$$

Substituting (28) into (15), we obtain the equation for finding the function t_{e1} :

$$PrV_r^{(e)}(y, \theta) \frac{\partial t_{e0}(y)}{\partial y} = \Delta t_{e1}(y, \theta), \quad (30)$$

with boundary condition (29). Here $t_{e0} = 1 + \frac{\Gamma_0}{y}$, $V_r^{(e)}(y, \theta) = \left(1 + \frac{A_1}{y^3} + \frac{A_2}{y}\right)$.

We are looking for the solution of the equation (30) in the form

$$t_{e1}(y, \theta) = \tau(y) \cos \theta.$$

It can be seen that the variables are separated, we obtain an inhomogeneous ordinary equation for finding the function τ . Thus, we have the following expression for the function t_{e1}

$$t_{e1}(y, \theta) = \frac{\omega_0}{2y} (N_1 - y) + \cos \theta \left[\frac{\Gamma_1}{y^2} + \frac{\omega_0}{2} \left(1 + \frac{A_2}{y} - \frac{A_1}{2y^3} \right) \right].$$

The convective diffusion equation is solved similarly. We have

$$\begin{aligned} C_1(y, \theta) &= C_{10}(y) + \varepsilon C_{11}(y, \theta), \quad C_1^*(\xi, \theta) = C_{10}^*(\xi) + \varepsilon C_{11}^*(\xi, \theta), \\ C_{10}(y) &= C_0 + \frac{M_0}{y}, \quad C_{11}(y, \theta) = \frac{\omega_{0d}}{2y} (N_2 - y) + \cos \theta \left[\frac{M_1}{y^2} + \frac{\omega_{0d}}{2} \left(1 + \frac{A_2}{y} - \frac{A_1}{2y^3} \right) \right], \\ C_{10}^* &= C_0, \quad C_{11}^*(\xi, \theta) = \frac{M_0}{\xi} \exp \left\{ \frac{1}{2} Pr^{(D)} \xi (\cos \theta - 1) \right\}, \quad \omega_{0d} = Pr_d M_0, \quad Pr_d = \mu_e / (D_{12} \rho_e). \end{aligned}$$

Here Pr_d is the Prandtl diffusion number [3].

The asymptotic decomposition of the solution inside the droplet, as shown by the boundary conditions on the surface of the particle, should be sought in a form similar to (16). Thus, we have

$$\begin{aligned} t_i(y, \theta) &= t_{i0}(y) + \varepsilon t_{i1}(y, \theta), \\ t_{i0}(y) &= B_0 + \frac{H_0}{y} - \frac{1}{y} \int_1^y \psi_0 dy + \int_1^y \frac{\psi_0}{y} dy, \quad \psi_0(y) = -\frac{R^2}{2\lambda_i T_\infty} y^2 \int_{-1}^{+1} q_i dx, \quad H_0 = \frac{R^2 J_0}{3\lambda_i T_\infty}, \\ t_{i1}(y) &= N_3 + \cos \theta \left\{ B_1 y + \frac{H_1}{y^2} + \frac{1}{3} \left[y \int_1^y \frac{\psi_1}{y^2} dy - \frac{1}{y^2} \int_1^y y \psi_1 dy \right] \right\}, \quad J_0 = \frac{1}{V} \int_V q_i dV, \\ \psi_1(y) &= -\frac{3}{2} \frac{R^2}{\lambda_i T_\infty} y^2 \int_{-1}^{+1} q_i x dx, \quad H_1 = \frac{R J_1}{3\lambda_i T_\infty}, \quad V = \frac{4}{3} \pi R^3, \quad J_1 = \frac{1}{V} \int_V q_i z dV, \quad z = r \cos \theta, \end{aligned}$$

where $\int_V q_i z dV$ is the dipole moment of the density of thermal sources [11, 12]. Integration is carried out over the entire volume of the evaporating droplet.

The average value of the drop surface temperature $T_{iS} = t_{iS} T_\infty$ is determined from solving the following system of equations

$$\begin{cases} t_{eS} = t_{iS}, & t_{eS} = 1 + \Gamma_0, & M_0 = C_{1S}^{(H)} - C_0, \\ (t_{eS} - 1) \left[1 + L \frac{n_e^2 m_1 m_2}{\lambda_e \rho_e T_\infty} D_{12} \frac{K_{TD}}{t_{eS}} \right] = \frac{R^2 J_0}{3\lambda_e T_\infty} - L D_{12} \frac{n_e^2 m_1 m_2}{T_\infty \lambda_e \rho_e} D_{12} M_0 - \sigma_0 \sigma_1 \frac{R T_\infty^4}{\lambda_e} (t_{iS}^4 - 1), \end{cases}$$

which is obtained by setting the expressions $t_{e0}(y)$, $t_{i0}(y)$, and $C_{10}(y)$ to boundary conditions on the surface of an evaporating droplet. Here $t_{eS} = t_{e0}(y = 1)$, $t_{iS} = t_{i0}(y = 1)$.

It should be noted that the constant integrations Γ_0 , Γ_1 , N_1 , A_1 , A_2 , etc. are uniquely determined from the boundary conditions on the surface of the evaporating droplet ($y = 1$).

4. PHOTOPHORETIC VELOCITY. ANALYSIS OF THE RESULTS OBTAINED

After calculating the integration constants Γ_0 , Γ_1 , N_1 , A_1 , A_2 , etc., the total force acting on the drop is determined by integrating the stress tensor over the surface of the particle [3]. Since the droplet moves uniformly, equating this force to zero, we obtain an expression for the photophoretic velocity of an evaporating droplet in a binary gas mixture in the field of electromagnetic radiation

$$\begin{aligned}
 U_{ph} = & -\frac{2}{3} \frac{1}{R\delta(1+\mu_e/\mu_i)} \left\{ \frac{3}{4\pi R^2 \lambda_i T_\infty} \int_V q_i z dV \left[\Omega_0 + D_{12} \frac{n_e^2}{n_2} \left(C_{1S}^* T_\infty + \frac{K_{TD}(t_{eS} + 1)}{2t_{eS}^2} \right) \Omega_1 \right] \right. \\
 & + \frac{3\omega_D}{16} \Omega_2 \left[2L \frac{n_e^2 m_1 m_2}{\lambda_i \rho_e T_\infty} D_{12} \Omega_0 - D_{12} \frac{n_e^2}{n_2} \left(a_0 + 2 \frac{\lambda_e}{\lambda_i} \right) \Omega_1 \right] \\
 & + \frac{3\omega_T}{16} \Omega_2 \left[\left(2 \frac{\lambda_e}{\lambda_i} + 2L \frac{n_e^2 m_1 m_2}{\lambda_i \rho_e T_\infty} D_{12} \frac{K_{TD}}{t_{eS}} \right) \Omega_0 \right. \\
 & \left. \left. + \frac{n_e^2}{n_2} \left(2 \frac{\lambda_e}{\lambda_i} C_{1S}^* T_\infty - \frac{K_{TD}}{t_{eS}} \left(a_0 + \frac{\lambda_e}{\lambda_i} \frac{t_{eS} - 1}{t_{eS}} \right) \right) \Omega_1 \right] \right\}, \\
 \Omega_0 = & K_{TS}^{(0)} \frac{\nu_e}{t_{eS}} + K_{DS} D_{12} + \frac{RT_\infty}{3\mu_i} \frac{\partial \sigma}{\partial T_i}, \quad \Omega_1 = \frac{2}{n_{1i}} + \frac{m_1}{\rho_e} \left(1 + 2 \frac{\mu_e}{\mu_i} \right), \quad \Omega_2 = 2 - \frac{1 + \frac{2\mu_e}{3\mu_i}}{1 + \frac{\mu_e}{\mu_i}}, \\
 a_0 = & 1 + 4\sigma_0 \sigma_1 \frac{RT_\infty^3}{\lambda_i} t_{iS}^3, \quad \omega = \Gamma_0 Pr, \quad \omega_{0d} = M_0 Pr d, \quad z = r \cos \theta, \\
 \delta = & a_0 + 2 \frac{\lambda_e}{\lambda_i} + 2L \frac{n_e^2 m_1 m_2}{\lambda_i \rho_e} D_{12} C_{1S}^* + 2L \frac{n_e^2 m_1 m_2}{\lambda_i \rho_e T_\infty} D_{12} \frac{K_{TD}}{t_{eS}} \left(1 - \frac{t_{eS} - 1}{2t_{eS}} \right). \quad (31)
 \end{aligned}$$

Thus, as a result of the conducted research, the following theorem is proved.

Theorem. *A general expression of the photophoretic velocity of an evaporating droplet in an electromagnetic radiation field satisfying a system of differential equations (1)–(3) and boundary conditions (4)–(11) has the form (31).*

Consider the **consequences** of the theorem formulated above. It can be seen from the expression (31) that the photophoretic velocity of an evaporating droplet is the sum of three velocities: “purely,” photophoretic velocity, which is proportional to the dipole moment of the density of thermal sources $\int_V q_i z dV$; photophoretic velocity due to convective heat exchange, proportional to the coefficient ω_0 and photophoretic velocity due to convective mass transfer and proportional to the coefficient ω_{0d} . Each speed, in turn, consists of the sum of four terms: the first term, which is proportional to the coefficient of thermal slip $K_{TS}^{(0)}$ and, due to which the evaporating drop tends to move towards a drop in temperature along its unevenly heated surface, i.e., from an area with a higher temperature to an area with a lower temperature; due to the second term (diffusion sliding, which is proportional to the coefficient $K_{DS}^{(0)}$), the drop can move both in the direction of growth and in the direction of temperature drop, depending on the masses of the components of the binary gas mixture. If the mass of the molecules of the component of the external mixture undergoing a phase transition on the surface of the droplet is $m_1 < m_2$, then $K_{DS}^{(0)} > 0$. Otherwise, $K_{DS}^{(0)} < 0$. The third term is due to the variable interphase surface tension on the surface of the droplet. For most liquids, the surface tension decreases with increasing temperature $\frac{\partial \sigma}{\partial T_i} < 0$, therefore, the third term contributes to the velocity directed towards temperature increase and the last term is due to the influence of reactive (evaporation-related), thermodiffusion effects and internal currents.

If in the formula (31) we go to the limit at $\mu_i \rightarrow \infty$, $\sigma \rightarrow 0$ and $n_{1i} \rightarrow 0$, then, using the formula (31), we can estimate the photophoresis rate of a highly viscous droplet in the field of electromagnetic radiation and at $C_{1S}^* \rightarrow 0$, the formula (31) turns into formulas known in the literature for the photophoresis rate of a spherical solid particle, for example, [13].

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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