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Analysis of Transient Modes of Peltier Thermoelectric Elements under Various Input Influences

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ABSTRACT:
stationary operation of a thermoelectric system is often not economical due to the impossibility of soft adjustment of the microclimate. The aspect makes relevant the research of the transient modes of Peltier elements and thermoelectric systems built on their basis. The study of the dynamic properties of thermoelectric devices and systems with high accuracy requires the use of complex high-order models, which makes it difficult to obtain analytical solutions. Using the spectral approach and piecewise linear approximation, analytical formulae of the transient modes of Peltier thermoelectric elements for various deviations of control currents are obtained. The calculated relations of piecewise linear approximating functions, spectral densities and time forms of input and output parameters of the model under study are presented. Time diagrams of the temperature response of a particular Peltier element in linear and nonlinear modes are constructed, calculated for the sinusoidal deviation of the control current from the stationary value.

KEYWORDS: Peltier Element, Transition Mode, Piecewise Linear Function, Spectral Method.

1. INTRODUCTION

Existing and prospective thermoelectric systems in various fields of application operate on the basis of standard Peltier thermoelectric modules. Information about the characteristics and parameters of the modules used is given in many works, for example, [1-6]. It is necessary to note that the stationary mode of operation of the thermoelectric system is often not economical due to the impossibility of soft adjustment of the microclimate. This aspect makes the study of the transient modes of Peltier elements and thermoelectric systems built on their basis. There are existing algorithms for identifying the dynamic properties of Peltier elements using parametric heat transfer models [7-11] and thermoelectric models [12], autoregression models, and adaptive recursive least squered algorithm (RLS) [13,14]. There are simple 1st order linear models as well [15]. It should be noted that the issue of analyzing the transients of the Peltier element under various input influences, carried out on the basis of the results of its identification, is promising and insufficiently investigated.

The aim of the work is to derive analytical formulae of the temperature response of the Peltier element based on the

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spectral approach and approximation by piecewise linear functions, as well as to calculate the characteristics of a particular device for various deviations of the control current.

2. NECESSARY EXPRESSIONS

A thermoelectric module based on the Peltier effect is an object under study shown in Fig. 1. General Peltier modules consist of two insulating dies with thermocouples connected in series and located between them (Fig. 1). The module sides contact either p-n or n-p junctions, which depends on the polarity of the applied voltage.

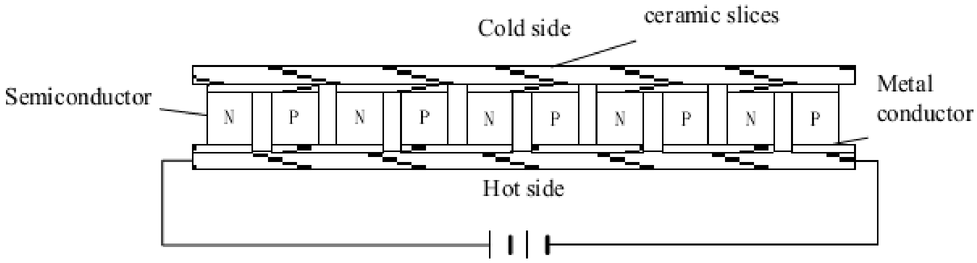

Fig. 1. The design of the thermoelectric module based on the Peltier effect

The simplified heat transfer formula in the Peltier element, without taking into account Seebeck, Thomson, Joule, and Fourier effects, has the following form [16]

$$E\frac{\partial \Delta T}{\partial t} + kA\Delta T = \Delta Q\,,$$

Where E is the temperature coefficient, ΔT is the temperature of the hot side minus the temperature of the cold side, k is the heat transfer coefficient, A is the contact surface between the Peltier module and the ambient environment, $\Delta Q=U^2/R$ is the cooling capability of the Peltier module, U is the applied voltage at the terminals of the module, R is the resistance.

Using the Lagrange transformation, the heat transfer formula can be written as

$$\frac{\Delta T}{\Delta U} = \frac{\Delta Q}{\Delta U kA}\frac{1}{1+\frac{Ep}{kA}}\,,$$

Where p is the Laplace differential operator. So, the transfer temperature response function of the Peltier module from the applied voltage is the 1st-order link:

$$G(p) = \frac{K_1}{1+\tau_1 p}\,,$$

Where $K_1 = \frac{\Delta Q}{\Delta U kA}$ is the gain, $\tau_1 = \frac{Ep}{kA}$ is the link time constant.

The operator transfer functions of the Peltier element $H(p)$ represent the ratio of the temperature response of the element to the change in the control current $X(p) = \Delta I_{control}(p)$ that caused this response and can have a high order and nonlinear properties.

$$H(p) = \Delta T(p)/X(p)$$

Substituting the differentiation operator p by the complex frequency $j\omega$, we obtain the complex transfer function of the simulated system $H(j\omega)$. The spectrum of deviation of the input parameter (control current) is denoted as $S_{in}(j\omega)$. Then the spectrum of deviation of the output parameter (temperature response)

$$S_{out}(j\omega) = S_{in}(j\omega)\cdot H(j\omega)\,. \quad (1)$$

Let us consider the problem of studying the transient process of the Peltier element when changing the control current of an arbitrary shape. In this case, the analytical expression of the spectrum of the control current is absent or has a bulky appearance. Numerical calculation of the spectrum based on the direct fast Fourier transform requires significant computational costs and does not make it possible to obtain generalized solutions. Piecewise linear approximation of the control current function from time (Fig. 2) allows us to obtain a generalized expression of its spectrum [17-19].

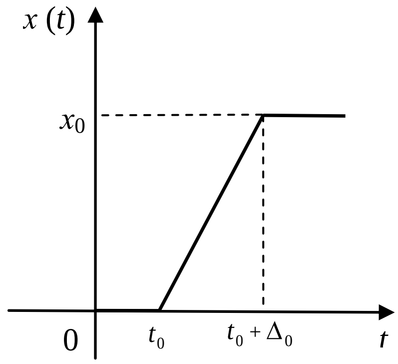

Fig. 2. Piecewise linear function approximating the transient of control current.

The deviation of the current controlling the Peltier element is approximated by the sum of several pulses

$$\tilde{o}(t) = \begin{cases} 0, \quad t < t_0, \\ \dfrac{\tilde{o}_0}{\Delta_0}(t - t_0), \quad t_0 \le t \le t_0 + \Delta_0, \\ \tilde{o}_0, \quad t > t_0 + \Delta_0, \end{cases}$$

Where x_0 is the amplitude of the steady-state pulse of the control current deviation,
t_0 is the time when the pulse appears, Δ_0 is the pulse setting time.

In general, the expression of the control current for the sum of pulses is the index of the current pulse, Δ_m is the setting time.

$$\tilde{o}(t) = \sum_{m=0}^{M-1} \tilde{o}_m(t) = \sum_{m=0}^{M-1} \begin{cases} 0, \quad t < t_m \\ \dfrac{\tilde{o}_m}{\Delta_m}(t - t_m), \quad t_m \le t \le t_m + \Delta_m, \\ \tilde{o}_m, \quad t > t_m + \Delta_m \end{cases} \qquad (2)$$

Piecewise linear approximation of $x(t)$ allows us to describe the control current deviation function without dividing it into definition intervals (3). The approximated function has the form

$$\tilde{o}(t) = \sum_{m=0}^{M-1} \tilde{o}_m(t) = \sum_{m=0}^{M-1} \frac{1}{2\Delta_{\grave{o}}} \left(|t - t_m| - |t - t_m - \Delta_{\grave{o}}| + \Delta_{\grave{o}} \right). \qquad (3)$$

The spectral density of the input signal parameter (control current deviation) is found in expression (3) using the direct Fourier transform

$$S_{in}(\omega) = \sum_{m=0}^{M-1} \frac{\tilde{o}_m}{\Delta_m \omega^2} \left[e^{-j\omega(t_m + \Delta_m)} - e^{-j\omega t_m} \right], \qquad (4)$$

To find the response of the system in time form $f(t) \leftarrow S(j\omega)$, we apply the inverse Fourier transform. However, this transformation is often associated with cumbersome calculations, since in general the Fourier integral cannot be solved analytically. Piecewise linear approximation of the spectral form of the control current allows to derive general formulae of the transient of the Peltier element.

In general, the spectral density of the temperature response of the Peltier element $S(j\omega)$ can be represented as real $S_1(\omega)$ and imaginary $S_2(\omega)$ components

$$S(j\omega) = S_1(\omega) - jS_2(\omega). \tag{5}$$

It is known [5] that it is enough to know only one component to find $f(t)$:

$$f_1(t) = \frac{2}{\pi}\int_{0+}^{\infty} S_1(\omega)\cos(\omega t)d\omega + \frac{1}{\pi}\int_{0-}^{0+} S_1(\omega)\cos(\omega t)d\omega, \tag{6}$$

$$f_2(t) = \frac{2}{\pi}\int_{0+}^{\infty} S_2(\omega)\sin(\omega t)d\omega + \frac{1}{\pi}\int_{0-}^{0+} S_2(\omega)\sin(\omega t)d\omega. \tag{7}$$

We consider (6) and omit the indices 1 for $f(t)$ and $S(\omega)$. We approximate $S(\omega)$ by the sum of piecewise linear functions, similar to the approximation of the control current, only in the frequency domain. Substitute the resulting expression in (6) and after the transformation, we obtain a generalized analytical formula of the transition mode of the Peltier element based on approximation by the sum of piecewise linear functions

$$f(t) = \frac{2}{\pi t}\sum_i \frac{c_{0i}}{\Delta_i}[Si(\omega_{i+1}t) - Si(\omega_i t)], \tag{8}$$

Where i s the number of the approximating function, $c_{0i} = \mathrm{S}(\omega_{i+1}) - S(\omega_i)$ is the height of the i-th function, Δ_i is the frequency step at node i.

To exclude the situation $S(\omega) \to \infty$ at $\omega \to 0$, we present the complex transfer function of the simulated system $H(j\omega)$ normalized to one in the form

$$\text{Í}\ (j\omega) = \text{Í}\ (0) + [\text{Í}\ (j\omega) - \text{Í}\ (0)]. \tag{9}$$

By converting (8) taking into account (9), we obtain

$$f(t) = S_{in}(0)\cdot \text{Í}\ (0) + \frac{2}{\pi t}\sum_i \frac{\tilde{n}^*_{0\mathrm{i}}}{\Delta_i}[Si(\omega_{\hat{e}\,i}t) - Si(\omega_{\acute{\imath}\,i}t)], \tag{10}$$

Where $c^*_{0i} = S^*(\omega_{нi}) - S^*(\omega_{кi})$, $\mathrm{S}^*(\omega) = \mathrm{Re}\{S_{\hat{a}\tilde{o}}(j\omega)\cdot[\text{Í}\ (j\omega) - \text{Í}\ (0)]\}$.

The height of the function is $\tilde{n}^*_{0\mathrm{i}} = \tilde{n}_{0\mathrm{i}}$ at $H(0) = 0$.

3. MODELING OF TRANSIENT MODES OF PELTIER ELEMENTS

The thermoelectric cooler is characterized by an inertial linear dynamic model. At the same time, we assume that the nonlinear component identifies thermal conditions on the cold side. We simulate the transient process of the Peltier element, represented by an experimentally verified model in the form of a sequential connection of a linear link $D(p)$ and a nonlinear link $F(p)$ [20,21]:

$$D(p) = \frac{p + 0.1323}{p^2 + 0.5964p + 0.00855}, \tag{11}$$

$$F(d(t)) = -0.4141\sinh(0.5d(t)). \tag{12}$$

As the control current of the Peltier element, we take a sinusoidal function with a frequency of 0.2 Hz, its spectral representation is $X(j\omega) = \dfrac{10\pi}{-25\omega^2 + 4\pi^2}$.

Then the spectrum of the temperature response of the Peltier element in the linear mode will take the form

$$T_{lin}(j\omega) = X(j\omega)\cdot D(j\omega) = \frac{10\pi(p+0.1323)}{(-25\omega^2+4\pi^2)(p^2+0.5964p+0.00855)},$$

and the temperature response in the nonlinear mode is $T_{nonlin}(t) = F(T_{lin}(t))$.

After approximation of $T_{lin}(j\omega)$ (the range of circular frequency $\omega = 10^{-3}...100$ c^{-1}, the number of approximating piecewise linear functions $N = 100$) and substitution of approximation coefficients in (10), we obtain an expression for calculating the transition process of the Peltier element. To increase accuracy and reduce computational costs, the nodes of the cyclic frequency approximation were arranged exponentially. Figure 3 shows the time diagrams of the control current, as well as the temperature response in linear and nonlinear modes. Temperature responses represent the temperature deviation in degrees Celsius from the stationary value before the start of the transition process t= 0 c. The calculation result for the linear mode based on piecewise linear approximation coincides with the transition process obtained analytically based on the Laplace table of originals and images.

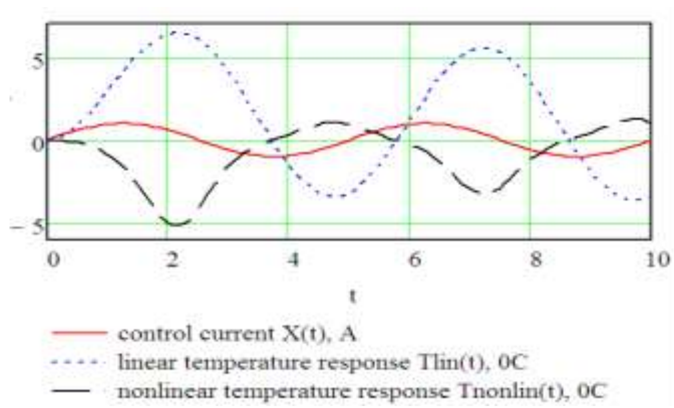

Fig. 3. Transients of the control current and temperature responses of the Peltier element in linear and nonlinear regime

4. CONCLUSION

The proposed approach for analyzing transient modes can be used to study the temperature response of the Peltier element based on an arbitrary order model and with arbitrary deterministic laws of control currents, which will not sufficiently increase the computational cost. The approach is a quite simple tool for obtaining the characteristics of Peltier elements and thermoelectric systems in general, it might be used for optimization and synthesis tasks, including multichannel systems with various transfer characteristics.

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Conflicts of interest. The authors declare no conflict of interest.

Ethics. The authors declare that the present research work has fulfilled all relevant ethical guidelines required by COPE.

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