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Research article

Fuzzy-Logical model for analysis of sustainable development of fuel and energy complex enterprises

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Abstract: The purpose of this article is to build a mathematical model for analyzing the sustainability of the development of an enterprise in the fuel and energy complex, integrated into an information management system. It is noted that one of the strategic dominants in achieving the national goal of accelerating the technological development of any country is to ensure the effective functioning of enterprises in the fuel and energy complex. It is substantiated that these enterprises represent the basis of the material life of society, thus, ensuring their sustainable development is a significant factor for the formation of the structure of sectoral and inter-sectoral industrial complexes. In order to analyze the sustainable development of enterprises, an integral indicator is proposed, the components of which are the vectors of production, organizational, economic, environmental and social characteristics. Due to the weak structure of some characteristics, to solve the problem of their synthesis with quantitatively defined indicators, it is proposed to use the mathematical apparatus of fuzzy logic. Weakly structured indicators are formally described by linguistic variables. To establish the dependence of the integral indicator of sustainable development on production, organizational, economic, environmental and social indicators, a fuzzy-logical model has been built, which makes it possible to use the knowledge of experts by constructing rules of fuzzy inference. The fuzzy logic model is implemented using MATLAB tools. On the constructed model, experiments were carried out to assess the impact of each of the local indicators of sustainable development of an enterprise on the integral indicator. The advantage of the constructed model is its adaptability to changes in the operating conditions of enterprises.

Keywords: fuel technology; energy enterprise; sustainable development; mathematical model; fuzzy logic

1. Introduction

The fundamental component of the economy of any state is the enterprises of the fuel and energy complex, which form their basic potential. Thanks to their functioning, jobs are created, conditions are formed for the development of innovative processes, etc. Due to the fact that these enterprises make a significant contribution to ensuring the stability of the country, ensuring the conditions for their sustainable development is a key problem in the formation of the national economy. Sustainable development is understood as development in which the needs of the present should not be satisfied at the expense of future generations. We believe that the concept of sustainable development should be the dominant feature of the management of an enterprise in the fuel and energy complex that consists of resolving contradictions between three sets of characteristics: product quality indicators, as one of the leading components of the enterprise's competitiveness, as well as production and economic indicators, including ecological and social parameters. Solving this problem requires analytical approaches related to the development of mathematical models and their integration into digital technologies for managing the sustainable development of mining enterprises. The use of traditional methods of mathematical modeling is associated with a number of difficulties due to the convergence of poorly formalized social and environmental indicators of sustainable development with quantitatively de-fined production indicators. Our purpose is to use a fuzzy-logical approach to develop a mathematical model for assessing the sustainable development of enterprises in the fuel and energy complex, integrating quantitatively and qualitatively expressed economic, environmental and social indicators. Accounting for uncertainty conditions required the use of intelligent modeling methods based on the mathematical apparatus of fuzzy logic [1].

2. Literature review

Within the framework of the scientific discourse on the management of sustainable development of enterprises in the fuel and energy complex, it is appropriate to consider a number of works by modern authors. In this regard, it should be noted the studies that outline an integrated approach to intelligent mining management systems [2–7]. Various aspects in the search for conditions for sustainable development of organizations are in articles [8–17]. Works [18–20] are devoted to the study of geoecological factors influencing the sustainable development of mining enterprises. The issues of the need to study the effectiveness of sustainable development based on the application of mathematical methods are reflected in the articles [21,22]. The turbulence of the external environment entails the need to develop new principles and business models that allow not only maintaining your business potential, but also implementing sustainable development. In this aspect, a special place is occupied by mathematical models. The use of these models makes it possible to evaluate both the achieved level of development and the effectiveness of its management. The results of the analysis of scientific papers devoted to the application of mathematical methods to the study of sustainable development of organizations are summarized in Table 1. A lot of papers published on this topic are decomposed into subsets according to the criterion of the methods used. The subsets included studies based on econometric methods, peer review methods, decision tree construction and hierarchy analysis, Lyapunov function methods, the mathematical apparatus of neural networks and soft computing. The results of the analysis have demonstrated the increased attention of modern researchers to the application of econometric methods in solving the problems of sustainable development of organizations. In this case, the methods of regression analysis, linear modeling, and programming [24–26], statistical data processing [27] and econometric modeling [28–30,31,33,34] are used.

Table 1. Analysis of literary sources on sustainable development of enterprises.

Mathematical methods	Literary sources
Econometric methods, game theory	[24–34]
Methods of expert assessments, decision tree construction, hierarchy analysis	[35,36]
Methods of Lyapunov functions	[37–42]
Neural network	[43–46]
Soft computing	[47–53]

The methods of expert assessments [35] and the construction of a decision tree [36] have become effective ways to solve problem situations that arise in the management of sustainable development of enterprises. Applied developments on the application of Lyapunov functions in the study of the stability of economic systems have been used in articles [37–42]. At present, in connection with the development and use of computer systems in all areas of activity, a promising direction is the intellectualization of mathematical modeling tools. At present, there is a tendency to use the methods of intellectual modeling in solving the issues of sustainable development of enterprises. Among the works devoted to this issue, in the studies of many authors, are the mathematical apparatus of artificial neural networks [35,43,44,46], soft computing [47–55]. We propose an intellectualized approach to the synthesis of economic, environmental and social indicators in the analysis of the sustainable development of mining enterprises and its implementation through the construction of a fuzzy-logical model.

3. Materials and methods

It is indisputable that in ensuring the sustainable development of an enterprise in the fuel and energy complex, the determining factor is the quality of its products, which is the foundation for ensuring its competitiveness. However, improving the quality of products causes an increase in the cost of its manufacture (the use of high-quality materials, efficient technologies, compliance with the provisions of social and environmental policy, etc.) and, as a result, a decrease in profitability. Therefore, when analyzing the sustainable development of an enterprise, a model toolkit is needed that can resolve the contradictions between these factors. The initial data of the developed mathematical model for determining the level of sustainability of the development of enterprises in the fuel and energy complex is the integral indicator proposed in the article, which are the sets of production-economic, environmental and social characteristics, respectively. The components are vectors of the elements of which are listed in Table 2. The operating enterprises of the fuel and energy complex are very diverse in terms of the system of characterizing indicators and their quantitative expressions. Therefore, the creation of a mathematical model is subject to the requirements of flexibility in adapting indicators and to the conditions of functioning of a particular enterprise.

Table 2. Mining sustainability indicators.

Indicator group sustainability $\Omega_i \in \Omega$	Identification indicator	Verbal description of indicators $\Omega_{ij} \in \Omega_j$
$\Omega_1 = (\Omega_{11}, \Omega_{12}, \Omega_{13})$ − production and economic indicators	$N(\Omega_{11})$	Cost-effectiveness
	$N(\Omega_{12})$	The level of organizational sustainability of production
	$N(\Omega_{13})$	Product quality
$\Omega_2 = (\Omega_{21}, \Omega_{22}, \Omega_{23}, \Omega_{24})$ − environmental performance	$N(\Omega_{21})$	Presence in the environmental policy of the environmental management system for compliance with international standards ISO 14001
	$N(\Omega_{22})$	Availability of a system of preliminary assessment of the impact of the enterprise's activities on the environment
	$N(\Omega_{23})$	Presence of requirements for efficient use of resources
	$N(\Omega_{24})$	Availability of a response system for emergency and other emergency situations
$\Omega_3 = (\Omega_{31}, \Omega_{32}, \Omega_{33}, \Omega_{34})$ – social indicators	$N(\Omega_{31})$	Presence in the social policy of the procedure for hiring the local population
	$N(\Omega_{32})$	Availability of a system for providing employees with an insurance policy
	$N(\Omega_{33})$	Availability of regular medical check-ups
	$N(\Omega_{34})$	Availability of a system for regular monitoring of working conditions

In particular, the characteristics $\Omega_{13} \in \Omega_1$ of the quality of products of enterprises differ depending on the compliance with the main profile of the enterprise and reflect a set of properties that meet the needs of consumers. For example, if the model is designed to assess the sustainable development of a coal industry enterprise, then the major indicators of the consumer quality of coal include the characteristics of humidity, heat of combustion, ash content, volatile matter, etc. Depending on the combinations of product properties, the evaluation of the integral quality indicator Ω_{13} in the mathematical model is carried out according to a set of generalized verbal characteristics: low, medium and, high quality level. The task is to build an economic and mathematical model F that allows to evaluate the value of the integral indicator Ω of sustainable development of an enterprise according to the system of characteristics $\Omega_1, \Omega_2, \Omega_3$, i.e., model F must implement mapping $F: \{\Omega_1, \Omega_2, \Omega_3\} \to \Omega$. Thus, the multicriteria task of assessing the sustainability of an enterprise's development consists in the synthesis of quantitatively and qualitatively expressed characteristics. Table 2 shows that the components of the vectors $\Omega_2 = (\Omega_{21}, \Omega_{22}, \Omega_{23}, \Omega_{24})$, $\Omega_3 = (\Omega_{31}, \Omega_{32}, \Omega_{33}, \Omega_{34})$ are qualitatively defined, poorly formalized characteristics. Therefore, the problem of constructing a mathematical model $F: \{\Omega_1, \Omega_2, \Omega_3\} \to \Omega$ belongs to the class of weakly structured problems. To solve this problem, we propose the use of an intelligent approach to modeling based on the mathematical apparatus of fuzzy logic. The solution of the problem of convergence of quantitatively and qualitatively expressed characteristics of sustainable development in the construction of a mathematical model $F:\{\Omega_1, \Omega_2, \Omega_3\} \rightarrow \Omega$is carried out by means of a formal description of characteristics $\Omega_{ij} \in \Omega_i$ by linguistic variables. Linguistic variables are specified by tuples $\Omega_{ij} =< N(\Omega_{ij}), T(\Omega_{ij}), U_{ij}, \mu_{ij} >$, where $N(\Omega_{ij})$ is the name of the linguistic variable; $T(\Omega_{ij})$ − its term-set; U_{ij} − universe; μ_{ij} − set of membership functions of fuzzy sets identified by elements of the set $T(\Omega_{ij})$. In the formal description of linguistic variables $\Omega_{ij} =< N(\Omega_{ij}), T(\Omega_{ij}), U_{ij}, \mu_{ij} >$ their names$N(\Omega_{ij})$ are given in Table1. Indicators $\Omega_{1i} \in \Omega`_1$ vector $\Omega_1 = (\Omega_{11}, \Omega_{12}, \Omega_{13})$ take values from the set $(\Omega_{1j}) = \{Low,\ Middle,\ High\}, j = \overline{1,3}$ ,the elements of which are weakly formalized, verbal expressions: "low", "medium", "high". Indicators $\Omega_{2i} \in \Omega`_2$, $\Omega_{3i} \in \Omega`_3$ take values from the set $T(\Omega_{ij}) = \{Yes,\ Partially,\ No\}$. The elements of these sets reflect the assessment of the presence in the environmental or social management of an enterprise of various systems, procedures, requirements through verbal expressions: "yes", "partially", "no". Due to the diversity of both the scale of enterprises and the conditions for their functioning, the use of a point system is proposed to assess the level of their sustainable development. Furthermore, the areas of definition of indicators of sustainable development $\Omega_{ij} \in \Omega$ are set by the universe U_{ij} in the form of segments [0,5] that establish the range of score variation for each linguistic variable. Relationships between language expressions included in the sets of terms $T(\Omega_{ij}) = \{Yes,\ Partially,\ No\}$ and $T(\Omega_{1j}) = \{Low,\ Middle,\ High\}$ and the universe are described by a set of membership functions $\mu_{1j} = \{\mu_{1j}^{Low}, \mu_{1j}^{Middle}, \mu_{1j}^{High}\}$, $\mu_{ij} = \{\mu_{ij}^{Yes}, \mu_{ij}^{Partially}, \mu_{1j}^{No}\}$, given in the form of systems of equations that describe the semantics of fuzzy sets in an explicit form. To set analytical expressions for membership functions of terms of linguistic variables $\mu_{1j} = \{\mu_{1j}^{Low}, \mu_{1j}^{Middle}, \mu_{1j}^{High}\}$, $\mu_{ij} = \{\mu_{ij}^{Yes}, \mu_{ij}^{Partially}, \mu_{1j}^{No}\}$, a method of expert assessments is proposed, according to which a group of experts is invited to fill in the validity matrix $\lambda = \|\lambda_{ij}\|$. The rows of the matrix $\lambda = \|\lambda_{ij}\|$correspond to the numbers of experts, and the columns correspond to the areas of definition of sustainable development indicators $\Omega_{ij} \in \Omega$, specified by the universe U_{ij} in the form of segments [0,5]. Each expert with the number assigns the value $\lambda_{ij} = 1$ (or $\lambda_{ij} = 0$) to the matrix element, if from his position the verbal expression of the term can (or cannot) be estimated by the value $\alpha \in U_{ij}$. According to the results of the survey of experts, the degree of belonging of the value $\alpha \in U_{ij}$ to the fuzzy set is determined by the formula $\mu = \frac{1}{k}\sum_{i=1}^{k}\lambda_{ij}$, where k is the number of experts participating in the study. This article presents the results of an expert evaluation of membership functions of terms $T(\Omega_{1j}) = \{Low,\ Middle,\ High\}$ for identifiers $\Omega_1 = (\Omega_{11}, \Omega_{12}, \Omega_{13})$. Tables 3–5 present the results of expert surveys. Figures 1–3 show the results of processing the contents of tables according to the formula $\mu = \frac{1}{k}\sum_{i=1}^{k}\lambda_{ij}$ in the form of graphs of membership functions. The constructed graphs made it possible to put forward a hypothesis about the triangular nature of changes in the membership functions μ^{Low}, μ^{Middle} , μ^{High} of fuzzy sets $Low,\ Middle,\ High$:

$$\mu^{Low} = \begin{cases} 0, & x < 0; \\ \frac{5-x}{5}, & 0 < x < 5; \\ 0, & x > 5. \end{cases} \tag{1}$$

$$\mu^{Middle} = \begin{cases} 0, & x < 0; \\ \frac{x-0}{3}, & 0 \le x < 3; \\ \frac{5-x}{2}, & 3 \le x \le 5; \\ 0, & x > 5. \end{cases} \quad (2)$$

$$\mu^{High} = \begin{cases} 0, & x < 0; \\ \frac{x}{5}, & 0 \le x \le 5; \\ 0, & x > 5. \end{cases} \quad (3)$$

Table 3. The results of the examination when choosing the membership function of the term low.

Experts	Term 0	1	2	3	4	5
1	1	1	1	0	0	1
2	1	0	1	1	0	1
3	1	1	0	0	1	1
4	1	1	0	0	1	1
5	1	0	1	1	0	1
6	1	0	1	0	0	1

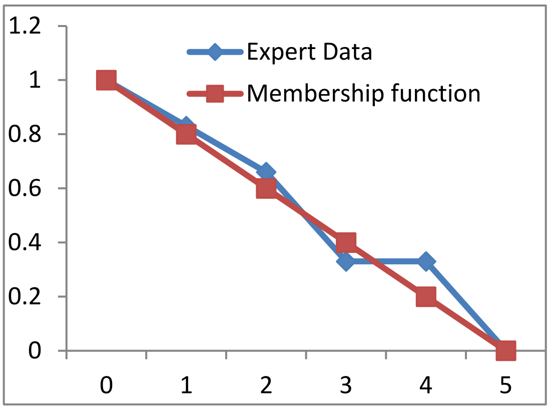

Figure 1. Membership function of the term “low”.

Table 4. The results of the examination when choosing the membership function of the middle term.

Experts	Term 0	1	2	3	4	5
1	0	0	1	1	0	1
2	0	1	1	1	0	0
3	0	1	1	1	1	1
4	0	0	0	1	1	0
5	0	0	1	1	0	1
6	0	0	1	1	0	1

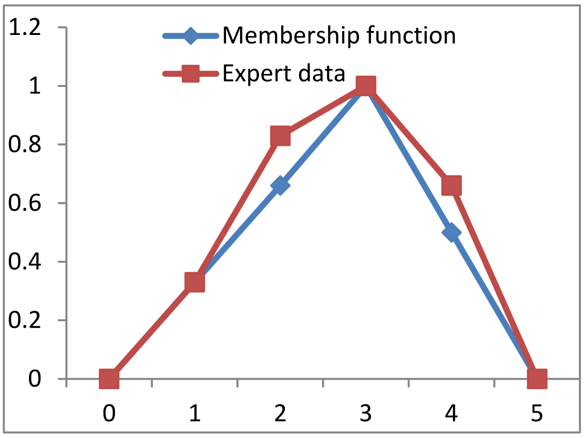

Figure 2. Membership function of the term "Middle".

Table 5. The results of the examination when choosing the membership function of the term High

Experts	Term					
	0	1	2	3	4	5
1	0	0	1	1	1	1
2	0	1	0	1	1	1
3	0	0	0	0	0	1
4	0	1	0	0	1	1
5	0	0	1	1	1	1
6	0	0	0	1	1	1

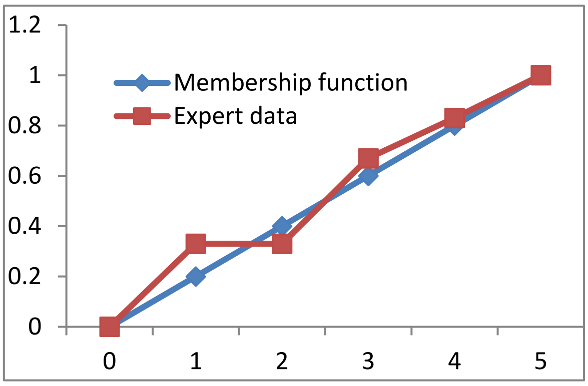

Figure 3. Membership function of the term "High".

Analytical expressions of membership functions of terms $T(\Omega_{ij}) = \{Yes, \ Partially, \ No\}$ of linguistic variables Ω_{21}, Ω_{22}, Ω_{23}, Ω_{24}, Ω_{31}, Ω_{32}, Ω_{33}, and Ω_{34} were constructed in a similar way. The results of expert surveys allowed us to draw conclusions about the trapezoidal nature of their functions:

$$\mu^{No} = \begin{cases} 0, & x < 0; \\ 1, & 0 \le x < 1; \\ \frac{5-x}{4}, & 1 \le x \le 5; \\ 0, & x > 5. \end{cases} \quad (4)$$

$$\mu^{Partially} = \begin{cases} 0, & x < 0; \\ x, & 0 \le x < 1; \\ 1, & 1 \le x < 4; \\ 5-x, & 4 \le x \le 5; \\ 0, & x > 5. \end{cases} \quad (5)$$

$$\mu^{Yes} = \begin{cases} 0, & x < 0; \\ \frac{x}{4}, & 0 \le x < 4; \\ 1, & 4 \le x \le 5; \\ 0, & x > 5. \end{cases} \quad (6)$$

In the fuzzy-logical model for analyzing the sustainable development of an enterprise, the central place is occupied by a fuzzy inference system that performs the function of formulating fuzzy conclusions based on qualitatively expressed factors about the current state of the enterprise. To implement fuzzy inference, there are currently many schemes, the most popular of which are the algorithms of Mamdani (Mamdani), Larsen (Larsen), Sagano (Sagano), Tsukamoto (Tsukamoto) and simplified inference. The use of each of the algorithms is carried out based on the target orientation of the modeling, the method of identifying the used fuzzy variables, interpreting the conclusions obtained, etc. The use of the Tsukamoto algorithm for this model is not possible due to the need for strict compliance in this system with the requirements of the monotonicity of membership functions of antecedents and consequents as fuzzy sets from which implicative statements in production rules are formed. Sagano's algorithm is also unacceptable for solving the stated modeling problem due to the peculiarity of building a knowledge base, which implies the representation of the right parts of the inference rules as linear functions of input variables X, Y, Z, described by fuzzy terms A_1, A_2: $< p_\beta >: if\ X\ \ is\ A_1\ and\ Y\ is\ A_2\ \ then\ Z\ is\ a_0 + a_1X + a_2Y$.

Although of Sagano type schemes, provide greater accuracy of the obtained simulation results, a meaningful interpretation of the construction of inference rules in the knowledge base and the interpretation of inference in connection with the requirements presented entails certain difficulties. Regarding the application of the simplified fuzzy inference algorithm, there are also problems associated with the representation of consequents of implicative statements in the knowledge base.

Conclusions in fuzzy production rules are specified in this algorithm discretely in the form of clear numerical values b: $< p_\beta >: if\ X\ \ is\ A_1\ and\ Y\ is\ A_2\ \ then\ Z\ =\ b$. The choice remains between the class schemes of Mamdani and Larsen. These schemes are similar to each other in terms of the type of knowledge base, but there is a fundamental difference in the approaches to the formation of the membership function of fuzzy statements A_1, A_2 and the method of determining the fuzzy implication $A_1 \to A_2$, as a way to specify a fuzzy relation $R \subseteq X \times Y$. In the Mamdani system, the mathematical representation of a fuzzy implication is given on the basis of the T - norm operation, and the semantics $\mu_{A_1 \to A_2}$ of the implication $A_1 \to A_2$ is modeled as $\mu_{A_1 \to A_2} = min(\mu_{A_1}, \mu_{A_2})$. In Larsen's algorithm, the fuzzy implication is modeled using the multiplication operation $\mu_{A_1 \to A_2} = \mu_{A_1} \cdot \mu_{A_2}$. Due to the intuitive intelligibility and proximity of the Mamdani system inference rules to the logical thinking of natural intelligence, as well as the possibility of their adjustment necessary in the process of the enterprise functioning, the Mamdani cash register system was used as an adaptive algorithm of the mathematical model for analyzing the sustainability of the enterprise development. In the model, dependence $F:\{\Omega_1,\Omega_2,\Omega_3\} \rightarrow \Omega$ is built by setting fuzzy production rules of the fuzzy inference system, reflecting the knowledge of specialist experts about the state of the enterprise in the process of managing its sustainable development. Elements of the system of fuzzy inference rules agreed with respect to the introduced linguistic variables, compiled by experts, are formally described by the logical expression: $< p_\alpha >: if\ X\ is\ Partially\ then\ Y\ is\ Middle$, where p_α is the identification of the fuzzy production rule, X and Y are its antecedent and consequent, respectively. The values of the variables X and Y are verbal expressions characterizing the current state of sustainability of the development of a mining enterprise. In this case, the values of the antecedent X and the consequent Y can be specified not only in the form of atomic terms defined by the sets $T(\Omega_{ij}) = \{Yes,\ Partially,\ No\}$ and $(\Omega_{1j}) = \{Low,\ Middle,\ High\}$. Variables X and Y can be represented by structured linguistic variables connecting atomic terms with logical links "AND", "OR", "NOT": $< p_\beta >: if\ X\ is\ High\ and\ Y\ is\ Middle\ then\ Z\ is\ Middle$.

The system of production rules being compiled is a knowledge base of an intellectual model for analyzing the level of sustainable development of an enterprise. Furthermore, it is assumed that the fuzzification of all indicators of sustainable development should be carried out by experts who have knowledge of the processes taking place in the mining enterprise.

4. Results and discussion

Suppose that after identifying the problem of assessing the sustainable development of an enterprise and extracting expert knowledge, they are structured in the form of a system of fuzzy inference rules $P =< p_1, p_2, \ldots, p_k >$ (only a subset of the constructed rules P is given in the article):

$$< p_1 >: if\ \Omega_{11}\ is\ Low\ and\ \Omega_{12}\ is\ Low\ and\ \Omega_{13}\ is\ Low\ end\ \Omega_{21}\ is\ No\ and$$

$$and\ \Omega_{22}\ is\ No\ and\ \Omega_{23}\ is\ Partially\ and\ \Omega_{24}\ is\ No\ and$$

$$and\ \Omega_{31}\ is\ No\ endand\ \Omega_{32}\ is\ Yes\ and\ \Omega_{33}\ is\ Partially\ and$$

$$and\ \Omega_{34}\ is\ Partially\ then\ \Omega\ is\ Low\ ;$$

$$< p_2 >: if\ \Omega_{11}\ is\ Middle\ and\ \Omega_{12}\ is\ Middle\ and\ \Omega_{13}\ is\ High\ end$$

$$\Omega_{21}\ is\ No\ and\ \Omega_{22}\ is\ Partially\ and\ \Omega_{23}\ is\ Partially\ and$$

$$and\ \Omega_{24}\ is\ No\ and\ \Omega_{31}\ is\ No\ end\ \Omega_{32}\ is\ Yes\ and$$

$$\Omega_{33}\ is\ Partially\ and\ \Omega_{34}\ is\ Partially\ then\ \Omega\ is\ Low;$$

$$< p_k >: if\ \Omega_{11}\ is\ High\ and\ \Omega_{12}\ is\ High\ and\ \Omega_{13}\ is\ High\ end$$

$$\Omega_{21}\ is\ Yes\ and\ \Omega_{22}\ is\ Yes\ and\ \Omega_{23}\ is\ Yes\ and$$

$$and\ \Omega_{24}\ is\ Yes\ and\ \Omega_{31}\ is\ Yes\ end\ \Omega_{32}\ is\ Yes\ and$$

$$\Omega_{33}\ is\ Yes\ and\ \Omega_{34}\ is\ Yes\ then\ \Omega\ is\ High\ .$$

In the rules drawn up by experts, all possible combinations of combinations of verbal characteristics of sustainable development indicators Ω_{ij} are considered. These rules perform the function of teaching the fuzzy-logical model the procedure for assessing the level of sustainable development of an enterprise based on the use of expert knowledge.

The constructed economic-mathematical model for analyzing the level of sustainable development of an enterprise makes it possible to study changes in the values of the integral indicator Ω, as a functional feature, when varying the values of factor variables $\Omega_{ij} \in \Omega$. Appendix A presents the results of studies of the influence of factor traits Ω_{ij} on the functional trait Ω. Improving the functioning of enterprise is a complex process that must be considered from the standpoint of multi-criteria.

The efficiency of the enterprises of the fuel and energy complex is influenced by a number of factors that can also act as a brake on its development. Consequently, the process of improving the efficiency of an enterprise consists in the continuous streamlining of dynamically changing requirements for various aspects of its activities, which necessitates the improvement of the system of indicators, setting priorities between them. In connection with the changing market conditions, the issues of adaptation of enterprises of the fuel and energy complex to changes in the impact of the external environment are of paramount importance in their activities.

These circumstances entail the need for continuous improvement of the system of methods, tools and tools for sustainability analysis based on a wide variety of technical, economic, environmental and social characteristics.

At present, in the context of research into the sphere of analysis of the effectiveness of mining operations, trends have been identified for the inclusion of various indicators of a quantitative and qualitative nature, which are part of the integral indicators for assessing sustainability. Along with traditional methods, the evaluation procedure includes an expert analysis of the activities of organizations, based on the information collected.

The variety of characteristics that make up integral indicators has given rise to many methodological schemes and algorithms for complex assessments. Currently, there is no single approach to assessing the level of sustainability and, therefore, there is no general methodology for performing this procedure. Therefore, the comparison and diagnosis of various methods remains an unusually difficult task, the solution of which is not unambiguous. Among the many approaches, the closest to the methods proposed in this article is the scheme developed by Prokofieva E. [23].

In [23], an expert-analytical model for conducting a technical and economic audit of mining enterprises based on production, environmental and social factors is proposed. Based on an expert assessment in [23], the measures of influence of each of the listed factors on the integral indicator were evaluated (Table 6).

Table 6. Measures of influence of factors on the integral indicator of technical and economic audit.

Factors	Production factor	Environmental factor	Social factor
Assessment of the influence of indicators P, E, S on the integral indicator	0,23	0,2	0,17
Assessment of the influence of indicators Ω_{11}, Ω_{21}, Ω_{34} on the integral indicator	0,95	0,818	0,63

To compare the methods, indicators were selected that are close in content to the indicators P, E, S: cost-effectiveness Ω_{11}, the presence in the environmental policy of an environmental management system for compliance with international standards ISO 14001 Ω_{21} and the presence of a system for regular monitoring of working conditions Ω_{34}. The comparison was carried out on the basis of determining the coefficients of pair correlations of the influence of characteristics Ω_{11}, Ω_{21}, Ω_{34} on the integral indicator Ω:$r_{\Omega_{ij}/\Omega} = \frac{\sum_{k=1}^{n}(\Omega_{ij}(k)-\overline{\Omega_{ij}})(\Omega(k)-\overline{\Omega})}{\delta_{\Omega_{ij}}\delta_{\Omega}}$, where $\overline{\Omega_{ij}}, \overline{\Omega}$ are arithmetic means; $\delta_{\Omega_{ij}}$, δ_{Ω}—standard deviations of indicators Ω_{ij}, Ω, k—number of experience.

The result of comparing the methods indicates that the knowledge base of the constructed fuzzy-logical model is trained on the basis of the production rules of experts in such a way as to give preference to the economic factor, as in the algorithm [23].

If it is necessary to strengthen the influence of certain factor characteristics on the integral indicator Ω, the mathematical model provides the opportunity to make changes both to the composition of the characteristics and to the system of fuzzy inference rules $P = < p_1, p_2, \ldots, p_k >$.

5. Conclusions

Ensuring the sustainable development of enterprises in the fuel and energy complex is a key element in the development of the national industrial production system, which leads to increased attention to solving the problems of their functioning. The level of sustainable development of enterprises is significantly influenced by many conflicting factors. In the aspect of sustainable development, in addition to production components, environmental and social components are distinguished, which play a translational role in the functioning of the enterprise, but are gaining increasing importance. Production and environmental factors, as a rule, are poorly formalized and their convergence with quantitatively expressed indicators cause a problem in creating a system for analyzing and assessing the level of sustainable development. The research results presented in this article are devoted to the development of a mathematical model for assessing the level of sustainability of the development of an enterprise in the fuel and energy complex based on the use of the mathematical apparatus of fuzzy logic. The description in the form of linguistic variables of qualitatively and quantitatively expressed indicators makes it possible to synthesize them in an integral indicator. The proposed fuzzy-logical model is universal and can be used to analyze the sustainable development of enterprises of any orientation. The advantage of the model is its ability to adapt to changing conditions for the functioning of enterprises by varying both the membership functions represented by fuzzy sets of qualitatively expressed indicators and the production rules created by experts to calculate the values of the integral indicator.

The research results allow us to draw the following conclusions:

-Assessment of the sustainable development of enterprises in the fuel and energy complex is a multi-criteria task that uses quantitatively and qualitatively defined economic, environmental and social characteristics as criteria;

-The proposed intellectualized approach based on the use of the mathematical apparatus of fuzzy logic makes it possible to implement the synthesis of formalized and weakly formalized indicators in the analysis of sustainable development of enterprises;

-An adaptive fuzzy-logical model has been built, in which the analysis of sustainable development of enterprises is carried out on the basis of expert knowledge of specialists formalized in the form of fuzzy inference rules.

Use of AI tools declaration

The authors declare that the research was conducted and presented in this article have not used AI tools at all stages of the research process.

Conflict of interest

The authors declare no conflict of interest.

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Appendix A

Table A1. Basic designations.

Variable	Designation	
$\Omega_1 = (\Omega_{11}, \Omega_{12}, \Omega_{13})$ – production and economic indicators of sustainability	Ω_{11}	Cost return
	Ω_{12}	Level of organizational sustainability of production
	Ω_{13}	Product quality
$\Omega_2 = (\Omega_{21}, \Omega_{22}, \Omega_{23}, \Omega_{24})$ – environmental indicators	Ω_{21}	The presence in the environmental policy of an environmental management system for compliance with international standards ISO 14001
	Ω_{22}	Availability of a system for preliminary assessment of the impact of the enterprise's activities on the environment
	Ω_{23}	Availability of requirements for efficient use of resources
	Ω_{24}	Availability of a response system to emergency and other emergency situations
$\Omega_3 = (\Omega_{31}, \Omega_{32}, \Omega_{33}, \Omega_{34})$ – social indicators	Ω_{31}	Availability of a procedure for hiring local people in social policy
	Ω_{32}	Availability of a system for providing employees with an insurance policy
	Ω_{33}	Availability of a procedure for regular medical examination
	Ω_{34}	Availability of a regular monitoring system for regular monitoring of working conditions
$\mu_{1j} = \{\mu_{1j}^{Low}, \mu_{1j}^{Middle}, \mu_{1j}^{High}\}$ – set of indicator membership functions $\Omega_1 = (\Omega_{11}, \Omega_{12}, \Omega_{13})$	μ_{ij}^{Low}	Term membership function «LOW»
	μ_{ij}^{Middle}	Term membership function «Mddle»
	μ_{ij}^{High}	Term membership function «High»
$\mu_{ij} = \{\mu_{ij}^{Yes}, \mu_{ij}^{Partially}, \mu_{1j}^{No}\}$ – set of indicator membership functions $\Omega_{2i} \in \Omega`_2$, $\Omega_{3i} \in \Omega`_3$, $i \in \{2,3\}$	μ_{ij}^{Yes}	Term membership function «YES»
	$\mu_{ij}^{Partially}$	Term membership function «Partially»
	μ_{1j}^{No}	Term membership function «No»
$\lambda = \lVert\lambda_{ij}\rVert$	$\lambda_{ij} \in \{0,1\}$	Validity matrix values filled in by experts

Table A2. The results of experiments on a fuzzy-logical model.

№	Production and economic indicators $\Omega_1 = (\Omega_{11}, \Omega_{12}, \Omega_{13})$			Environmental indicators $\Omega_2 = (\Omega_{21}, \Omega_{22}, \Omega_{23}, \Omega_{24})$				Social indicators $\Omega_3 = (\Omega_{31}, \Omega_{32}, \Omega_{33}, \Omega_{34})$				Integral indicator
	Ω_{11}	Ω_{12}	Ω_{13}	Ω_{21}	Ω_{22}	Ω_{23}	Ω_{24}	Ω_{31}	Ω_{32}	Ω_{33}	Ω_{34}	Ω
1	0,216	0,357	0,388	0,3	0,3	0,12	0,474	0,388	0,47	0,38	0,4	1,68
2	1,13	0,357	0,32	0,45	0,38	0,12	0,48	0,388	0,5	0,38	0,42	1,75
3	3,54	0,357	0,39	1,1	0,4	0,12	0,55	0,388	0,6	0,38	0,5	2,2
4	5	0,357	0,4	2	0,47	0,12	0,6	0,388	0,7	0,38	0,63	2,53
5	0,214	1,13	0,388	0,44	0,4	0,12	0,5	0,388	0,38	0,38	0,4	1,74
6	3,6	3,54	0,388	1,3	0,5	0,12	0,48	0,388	0,4	0,38	0,51	2,17
7	0,316	5	0,388	2,1	0,55	0,12	0,6	0,388	0,7	0,38	0,59	2,53
8	4,5	0,357	1,13	0,29	0,31	0,12	0,45	0,388	0,48	0,38	0,41	1,67
9	0,216	0,357	3,54	2,21	0,43	0,12	0,49	0,388	0,49	0,38	0,39	2,18
10	3	0,357	5	2,4	0,51	0,12	0,65	0,388	0,48	0,38	0,47	2,49
11	2,5	0,357	0,388	2,41	0,53	0,12	0,58	0,388	0,48	0,38	0,48	2,49
12	3,5	0,357	0,388	1,9	0,5	0,12	0,47	0,388	0,47	0,38	0,31	2,49