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DIFFRACTED TRANSITION RADIATION OF A BEAM OF RELATIVISTIC ELECTRONS IN A THIN SINGLE-CRYSTAL PLATE

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Abstract

In the present work, diffracted transition radiation (DTR) of the beam of relativistic electrons crossing a thin single-crystal plate has been considered. The expression for the DTR angular density has been derived for the case when the path of the electron in the target is considerably less than extinction length. For the first time the kinematic character of DTR of the beam of ultra-relativistic electrons crossing a thin single-crystal plate has been proved. The numerical calculation carried out shows the considerable influence of divergence of the beam on the angular density of DTR for high and super high energies of the electrons.

Keywords: relativistic electron, parametric X-radiation, diffracted transition radiation.

Introduction

The knowledge of spatial and angular distributions of particles in the incident beam is important for the experimental data interpretation in the physics of interaction of relativistic electrons with matter. That is why the working out of express methods of obtaining information about the characteristics of the beam used in the experiment is an actual problem. One of the approaches to this problem is the use of different types of radiation excited by relativistic charged particles in the matter. Recently the possibility of use of parametric X-radiation (PXR) for the diagnostics of relativistic electron beams was experimentally studied in [1, 2]. In [3] it was suggested getting the operative information on spatial distribution of the relativistic electron in the beam using the PXR generated in a thin crystal. The applicability of transition radiation (TR) in the range of vacuum ultraviolet for measuring of the electron beam cross dimensions was demonstrated in [4].

When a charged particle crosses the crystal plate surface, the transition radiation (TR) arises [5]. TR appearing on the border diffracts then on a system of parallel atomic planes of the crystal forming DTR in a narrow spectral range [6–9]. The DTR photons move near Bragg scattering direction.

The process of coherent X-ray radiation by a single relativistic electron in a crystal have been described in the framework of the dynamical theory of x-rays diffraction in [10–14]. In these papers, the dynamic theory of coherent X-ray radiation generated by a relativistic electron in a crystal was built for general case of asymmetric (relative to the crystal (target) surface) reflection of the electron Coulomb field. In this case, the system of the parallel reflecting atomic planes in the target can be located at arbitrarily given angle to the target surface. For divergent beam of relativistic electrons, the dynamic theory was developed in the resent works in Laue scattering geometry [13,14].

In the present work DTR of a relativistic electron crossing a monocrystalline target have been considered in Bragg scattering geometries. The expressions for angular density of the radiation are derived. The obtained expressions have been compared in the interesting cases when the path of the relativistic electron is much less or much more than the extinction length of x-ray waves in the monocrystalline target.

Geometries of the radiation processes

Let us consider a beam of relativistic electrons crossing a monocrystalline plate in Bragg (fig. 1) scattering geometries.

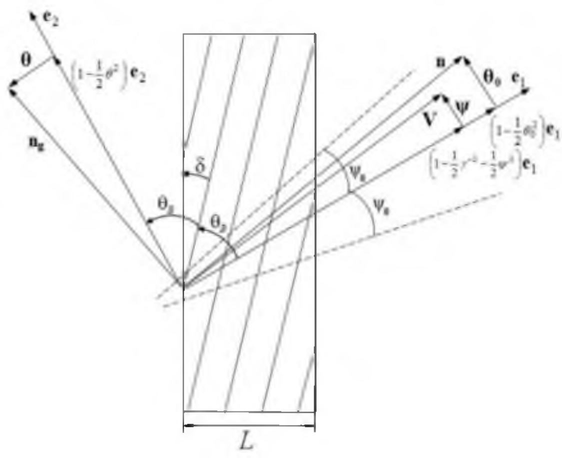

Fig. 1. Bragg scattering geometry

Let us introduce into consideration the angular variables $\boldsymbol{\psi}$, $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_0$ in accordance with the definition of relativistic electron velocity $\mathbf{V}$ and unit vectors $\mathbf{n}$ in direction of momentum of the photon radiated in the direction near electron velocity vector and unit vector $\mathbf{n}_g$ in the of Bragg scattering direction:

$$\mathbf{V}=\left(1-\frac{1}{2}\gamma^{-2}-\frac{1}{2}\psi^2\right)\mathbf{e}_1+\boldsymbol{\psi},\qquad \mathbf{e}_1\boldsymbol{\psi}=0,\ \mathbf{n}=\left(1-\frac{1}{2}\theta_0^2\right)\mathbf{e}_1+\boldsymbol{\theta}_0,$$

$$\mathbf{e}_1\mathbf{e}_2=\cos 2\theta_B,\qquad \mathbf{n}_g=\left(1-\frac{1}{2}\theta^2\right)\mathbf{e}_2+\boldsymbol{\theta},\ \mathbf{e}_2\boldsymbol{\theta}=0, \tag{1}$$

where $\boldsymbol{\theta}$ is the radiation angle, counted from direction of axis of radiation detector $\mathbf{e}_2$, $\boldsymbol{\psi}$ is the incidence angle of an electron in the beam counted from the electron beam axis $\mathbf{e}_1$, $\boldsymbol{\theta}_0$ is the angle between the movement direction of incident photon and axis $\mathbf{e}_1$, $\gamma=1/\sqrt{1-V^2}$ is Lorentz-factor of the particle. Let us decompose the angular variables into the components parallel and perpendicular to the figure plane: $\boldsymbol{\theta}=\boldsymbol{\theta}_{\parallel}+\boldsymbol{\theta}_{\perp}$, $\boldsymbol{\theta}_0=\boldsymbol{\theta}_{0\parallel}+\boldsymbol{\theta}_{0\perp}$, $\boldsymbol{\psi}=\boldsymbol{\psi}_{\parallel}+\boldsymbol{\psi}_{\perp}$.

The variable ψ_0 is the divergence parameter of the beam of the radiating electrons.

Let us consider the electromagnetic processes in crystalline medium, which are characterized by a complex permittivity

$$\varepsilon(\omega,\mathbf{r})=1+\chi(\omega,\mathbf{r}), \tag{2}$$

where $\chi(\omega,\mathbf{r})=\chi_0(\omega)+\sum_{\mathbf{g}}{}'\chi_{\mathbf{g}}(\omega)\exp(i\mathbf{gr})$, $\chi(\omega,\mathbf{r})$ is the dielectric susceptibility, $\chi_{\mathbf{g}}(\omega)=\chi'_{\mathbf{g}}(\omega)+i\chi''_{\mathbf{g}}(\omega)$ is the Fourier coefficient of the expansion of the dielectric susceptibility of the crystal into series over the reciprocal lattice vectors $\mathbf{g}$, and $\chi_0(\omega)$ is the average dielectric susceptibility.

In the present work the two-wave approach of dynamic diffraction theory are used, in which both the incident and diffracted waves are considered as equitable in process of self repumping one into another in crystalline target. While solving the problem, let us consider an equation for a Fourier image of an electromagnetic field $\mathbf{E}(\mathbf{k},\omega)=\int dt\, d^3r\, \mathbf{E}(\mathbf{r},t)\exp(i\omega t-i\mathbf{kr})$. The strengths of electromagnetic fields, excited by electron in the crystal are

$$\begin{aligned}\mathbf{E}(\mathbf{k},\omega)&=E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)}+E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)},\\ \mathbf{E}(\mathbf{k}+\mathbf{g},\omega)&=E_{\mathbf{g}}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)}+E_{\mathbf{g}}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)},\end{aligned}\tag{3}$$

$\mathbf{e}_0^{(1)}\perp\mathbf{k}$, $\mathbf{e}_0^{(2)}\perp\mathbf{k}$, $\mathbf{e}_1^{(1)}\perp\mathbf{k}_{\mathbf{g}}$ и $\mathbf{e}_1^{(2)}\perp\mathbf{k}_{\mathbf{g}}$, $\mathbf{k}_{\mathbf{g}}=\mathbf{k}+\mathbf{g}$. Vectors $\mathbf{e}_0^{(2)}$, $\mathbf{e}_1^{(2)}$ are situated on the plane of vectors $\mathbf{k}$ и $\mathbf{k}_{\mathbf{g}}$ (π-polarization) and $\mathbf{e}_0^{(1)}$, $\mathbf{e}_1^{(1)}$ are perpendicular to this plane (σ-polarization);

The system of equations for the Fourier transform images of the electromagnetic field in a two wave approximation of dynamic theory of diffraction has the form obtained in [15]. For the Bragg scattering geometry, the system of equations has the form

$$\begin{cases}(\omega^2(1+\chi_0)-k^2)E_0^{(s)}+\omega^2\chi_{-\mathbf{g}}C^{(s,\tau)}E_{\mathbf{g}}^{(s)}=8\pi^2 ie\omega\mathbf{e}_0^{(s)}\mathbf{V}\delta(\omega-\mathbf{kV}),\\ \omega^2\chi_{\mathbf{g}}C^{(s,\tau)}E_0^{(s)}+(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)}=0,\end{cases}\tag{4}$$

where $C^{(s,\tau)}=\mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)}=(-1)^{\tau}C^{(s)}$, $C^{(1)}=1$, $C^{(2)}=|\cos 2\theta_B|$.

The system of equations (4) for $s=1$ and $\tau=2$ describes the σ- polarized fields. For $s=2$, the system of equations (4) describes π- polarized fields. Let us note that if $2\theta_B<\frac{\pi}{2}$, then $\tau=2$, otherwise $\tau=1$. In the system of equations (4) following table of symbols is denoted: $P^{(1)}=\sin\varphi$, $P^{(2)}=\cos\varphi$, $\mathbf{e}_0^{(1)}\mathbf{V}=(\theta-\psi)P^{(1)}=\theta_\perp-\psi_\perp$, $\mathbf{e}_0^{(2)}\mathbf{V}=(\theta+\psi)P^{(2)}=\theta_{//}+\psi_{//}$,

$$\chi'_{\mathbf{g}}=\chi'_0\left(F(g)/Z\right)\left(S(\mathbf{g})/N_0\right)\exp\left(-g^2u_\tau^2/2\right),\ \chi''_{\mathbf{g}}=\chi''_0\exp\left(-\frac{1}{2}g^2u_\tau^2\right),\tag{5}$$

where $\chi_0=\chi'_0+i\chi''_0$ – is the average dielectric susceptibility, F(g) – is the form factor of atom containing Z electrons, $S(\mathbf{g})$ is the structural factor of a unit cell containing N_0 atoms, u_τ is the r.m.s. amplitude of thermal vibrations of crystal atoms. The work addresses the X-ray frequency range, where $\chi'_{\mathbf{g}}<0, \chi'_0<0$. We will consider a crystal with symmetry $(\chi_g=\chi_{-g})$, φ is the azimuthal radiation angle counted from the plane formed by the vectors $\mathbf{V}$ and $\mathbf{g}$. The value of the reciprocal lattice vector is determined by the expression $g=2\omega_B\sin\theta_B/V$, where ω_B is the Bragg frequency.

Spectral-angular density DTR in a thin crystal

Let us consider the relativistic electron coherent X-radiation in Bragg scattering geometry (see Fig.1). If we perform the analytical procedures similar to those used in [11], we will obtain the expressions for the spectral-angular density DTR for the propagation direction of the emitted photon $\mathbf{k}_{\mathbf{g}}=k_{\mathbf{g}}\mathbf{n}_{\mathbf{g}}$ (see Fig.1) taking into account the direction deviation of the electron velocity $\mathbf{V}$ relative to the electron beam axis $\mathbf{e}_1$:

$$\begin{aligned}&\omega\frac{d^2N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega}=\frac{e^2}{\pi^2}\Omega^{(s)2}\times\\ &\left(\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2}-\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi'_0}\right)^2\times\\ &\times\frac{\varepsilon^2}{\xi^{(s)^2}-\left(\xi^{(s)^2}-\varepsilon\right)\coth^2\left(\frac{b^{(s)}\sqrt{\varepsilon-\xi^{(s)^2}}}{\varepsilon}\right)},\end{aligned}\tag{6}$$

where the notations are used:

$$\Omega^{(1)}=\theta_{\perp}-\psi_{\perp},\quad \Omega^{(2)}=\theta_{//}+\psi_{//},\quad \varepsilon=\frac{\sin(\theta_B-\delta)}{\sin(\theta_B+\delta)},\ b^{(s)}=\frac{1}{2\sin(\theta_B+\delta)}\frac{L}{L_{ext}^{(s)}},$$

$$\xi^{(s)}(\omega)=\eta^{(s)}(\omega)+\frac{1+\varepsilon}{2\nu^{(s)}},\quad \eta^{(s)}(\omega)=\frac{2\sin^2\theta_B}{V^2|\chi_{\mathbf g}'|C^{(s)}}\left(\frac{\omega(1-\theta_{//}\cot\theta_B)}{\omega_B}-1\right),\quad \nu^{(s)}=\frac{\chi_{\mathbf g}'C^{(s)}}{\chi_0'},$$

$$L_{ext}^{(s)}=1/\omega|\chi_{\mathbf g}'|C^{(s)}. \tag{7}$$

The parameter ε is an important parameter in (6); it determines the degree of asymmetry of the reflection of the radiation field in a crystal plate with respect to the target surface. Under a fixed value of θ_B the parameter ε defines the orientation of crystal plate in relation to the system of diffracting atomic planes. When the angle of electron incidence on the target surface $\theta_B+\delta$ decreases the value of δ parameter can become negative and then will increase in magnitude (in extreme case $\delta\to-\theta_B$) that leads to increase of ε. On the contrary, when the angle of electron incidence decreases the value of ε decrease (in extreme case $\delta\to\theta_B$).

Parameter $b^{(s)}$ characterizing the thickness of the crystal plate is the ratio of half of the path of the electron in the target $L_e=L/(\theta_B+\delta)$ to the extinction length $L_{ext}^{(s)}$. Parameter $\nu^{(s)}$ can take the values in the interval $0\le\nu^{(s)}\le1$ and determines the degree of reflection of the radiation waves from the crystal, which is caused by the nature of the interference of the waves reflected from different planes (constructive ($\nu^{(s)}\approx1$) or destructive ($\nu^{(s)}\approx0$)). The spectral function $\eta^{(s)}(\omega)$ rapidly changes with the frequency of the radiation therefore this function is convenient for use as an argument in the diagrams demonstrating the spectra of PXR and DTR.

Angular density DTR in a thin crystal

To find the angular density of DTR let us integrate the expression (6) over the frequency function $\xi^{(s)}(\omega)$ using the relation $\frac{d\omega}{\omega}=-\frac{|\chi_{\mathbf g}'|C^{(s)}}{2\sin^2\theta_B}d\xi^{(s)}$. As the result we will obtain the expression describing the angular densities of DTR in Bragg scattering geometries:

$$\frac{dN_{\mathrm{DTR}}^{(s)}}{d\Omega}=\frac{e^2|\chi_{\mathbf g}'|C^{(s)}}{2\pi^2\sin^2\theta_B}\Omega^{(s)2}\times$$
$$\left(\frac{1}{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2}-\frac{1}{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi_0'}\right)^2\times \tag{8}$$
$$\int_{-\infty}^{\infty}\frac{\varepsilon^2}{\xi^{(s)2}-\left(\xi^{(s)2}-\varepsilon\right)\coth^2\left(\dfrac{b^{(s)}\sqrt{\varepsilon-\xi^{(s)2}}}{\varepsilon}\right)}d\xi^{(s)}(\omega),$$

Let us consider the extreme case when the electron path in the target expressed in extinction length is $b^{(s)}<<\sqrt{\varepsilon}$ or $b^{(s)}>>\sqrt{\varepsilon}$. Let us write these inequalities in view $\frac{L_e}{L_{ext}^{(s)}}<<2\sqrt{\varepsilon}$, $\frac{L_e}{L_{ext}^{(s)}}>>2\sqrt{\varepsilon}$. Because the parameter ε in the real experiments possesses the value $0.5<\varepsilon<3$

the pointed inequalities practically correspond to inequalities $L_e << L_{ext}^{(s)}$, $L_e >> L_{ext}^{(s)}$.

The extreme approximations of integral of DTR spectra for Bragg scattering geometry we will obtain in the following form

$$\int_{-\infty}^{\infty} \frac{\varepsilon^2}{\xi^{(s)^2} - \left(\xi^{(s)^2} - \varepsilon\right)\coth^2\left(\frac{b^{(s)}\sqrt{\varepsilon - \xi^{(s)^2}}}{\varepsilon}\right)} d\xi^{(s)}(\omega) = \\ = \varepsilon\sqrt{\varepsilon}\pi \cdot \tanh\left(\frac{b^{(s)}}{\sqrt{\varepsilon}}\right) \approx \begin{cases} \varepsilon\sqrt{\varepsilon}\pi, & b^{(s)} >> \sqrt{\varepsilon} \\ \pi\varepsilon b^{(s)}, & b^{(s)} << \sqrt{\varepsilon} \end{cases} \quad (9)$$

Using the obtained approximations, we will derive the angular densities of DTR in Bragg scattering geometries for conditions $b^{(s)} << \sqrt{\varepsilon}$:

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{b^{(s)}<<\sqrt{\varepsilon}} = \frac{e^2\omega_B \chi_{\mathbf{g}}'^2 C^{(s)^2}}{4\pi\sin^2\theta_B}\Omega^{(s)^2} \times \\ \left(\frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2} - \frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0'}\right)^2 \varepsilon \frac{L}{\sin(\theta_B + \delta)}. \quad (10)$$

In the case when $b^{(s)} >> \sqrt{\varepsilon}$ the expression (8) will take the form

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{b^{(s)}>>\sqrt{\varepsilon}} = \frac{e^2|\chi_{\mathbf{g}}'|C^{(s)}}{2\pi\sin^2\theta_B}\Omega^{(s)^2} \times \\ \left(\frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2} - \frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0'}\right)^2 \varepsilon\sqrt{\varepsilon}. \quad (11)$$

Let us note that the condition $b^{(s)} << \sqrt{\varepsilon}$ meaning that the length of electron path in the target is considerably less than the extinction length of x-ray waves in the crystal $L_e << L_{ext}^{(s)}$ completely excludes the repumping of the incident and diffracted waves in each other.

In case of enough high energy of electrons ($\gamma >> \frac{1}{\sqrt{|\chi_0'|}}$) for the beams where the magnitude of deviation angle of electron in the beam $\psi(\psi_\perp, \psi_{//})$ is less or close to characteristic value of angle of DTR angular density maximum γ^{-1}, i.e. when the condition $\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 << -\chi_0'$ is fulfilled, the expressions (10) and (11), describing the angular density of DTR go correspondently over

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{\substack{b^{(s)}<<\sqrt{\varepsilon} \\ \gamma>>\frac{1}{\sqrt{|\chi_0'|}}}} = \frac{e^2\omega_B \chi_{\mathbf{g}}'^2 C^{(s)^2}}{4\pi\sin^2\theta_B} \frac{\Omega^{(s)^2}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2\right)^2} \varepsilon \frac{L}{\sin(\theta_B + \delta)} \quad (12)$$

$$\left(\frac{dN_{DTR}^{(s)}}{d\Omega}\right)_{\substack{b^{(s)}>>\sqrt{\varepsilon} \\ \gamma>>\frac{1}{\sqrt{|\chi_0'|}}}} = \frac{e^2|\chi_{\mathbf{g}}'|C^{(s)}}{2\pi\sin^2\theta_B} \frac{\Omega^{(s)^2}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2\right)^2} \varepsilon\sqrt{\varepsilon}. \quad (13)$$

In fig. 2 the curves plotted according to (8) and (12) for a relativistic electron crossing a thin ($L = 0.2\ \mu m$) monocrystalline plate of silicon Si (111), under conditions $\gamma >> \frac{1}{\sqrt{|\chi_0'|}}$ and $b^{(s)} << \sqrt{\varepsilon}$, notably under $\gamma \approx 10000$ and $\frac{1}{\sqrt{|\chi_0'|}} \approx 258$ are shown. One can see that the angular densities of DTR calculated by formulas (8) and (11) practically coincide. The calculations were carried out for σ-polarized ($s = 1$) waves under condition $\theta_{//} = 0$. The angular density of the radiation was calculated for a separated electron moving along the axis of the electron $\mathbf{e}_1$ ($\psi = 0$) (see in fig. 1).

In fig. 3 the curves presented were plotted by formulas (8) and (13) for the same conditions as in fig. 2 but for more thick crystalline target ($L = 5\,\mu m$) i.e. for the condition $b^{(s)} >> \sqrt{\varepsilon}$. One can see that in this case the curves also coincide with a high accuracy.

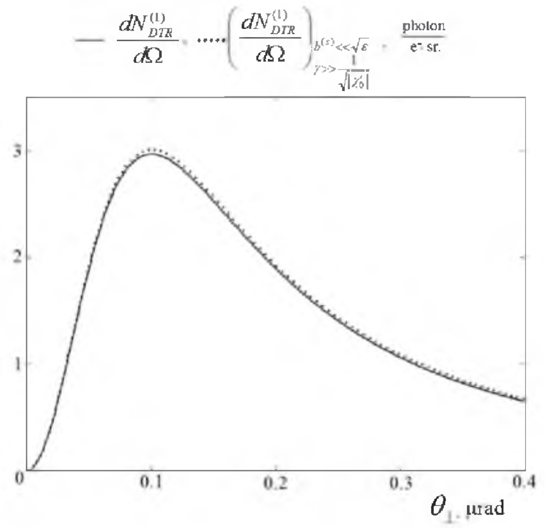

Fig. 2. The angular density of the DTR generated by a relativistic electron in the beam incident in direction of the beam axis

The curves are plotted by dynamical formula (8) (solid) and by its asymptotic kinematic formula (12) (dashed). $Si(111)$, $L = 0.2\,\mu m$, $\theta_B = 14.5^o$, $\delta = -7^o$, $\varepsilon = 3$ $\omega_B = 8$ *кэВ*, $E_e = 5$ *ГэВ*, $\psi = 0$, $\theta_{//} = 0$; $b^{(s)} \approx 0.264$, $\frac{1}{\sqrt{|\chi_0'|}} \approx 258$.

Let us compare the expression (12) describing the DTR angular density for the case of small path of electron in target ($b^{(s)} << \sqrt{\varepsilon}$), with good known kinematic expression for angular density of parametric X-ray radiation (PXR) written by us in following convenient form

$$\frac{dN_{PXR}^{(s)kin}}{d\Omega} = \frac{e^2\omega_B \chi_{\mathbf{g}}'^2 C^{(s)2}}{4\pi \sin^2\theta_B} \frac{\Omega^{(s)2}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0'\right)^2} \frac{L}{\sin(\theta_B + \delta)}. \quad (14)$$

One can see that in the case of symmetric reflection ($\varepsilon = 1$) the expressions (12) and (14) differ only by presence of real part of the average dielectric susceptibility χ_0' in the denominator of expression (14) that points on the fact that PXR is generated when the relativistic electron crosses the monocrystalline target. The quantity χ_0' determines the Fermi`s density effect, manifesting in the PXR yield saturation under the condition $\gamma > \frac{1}{\sqrt{|\chi_0'|}} \approx \frac{\omega}{\omega_0}$, where ω_0 is plasma frequency Therefore, we can conclude that DTR angular density of the ultra-relativistic electron can be described by the kinematic formula (14). The angular density is directly proportional to target thickness L. The condition $b^{(s)} << \sqrt{\varepsilon}$ indicate that the length of electron path in the monocrystalline target is considerable less than extinction length of the X-ray waves in the crystal, which completely excludes the repumping of the incident and diffracted waves in each other. So, DTR generation under high energy of the electron in a thin target have kinematic character.

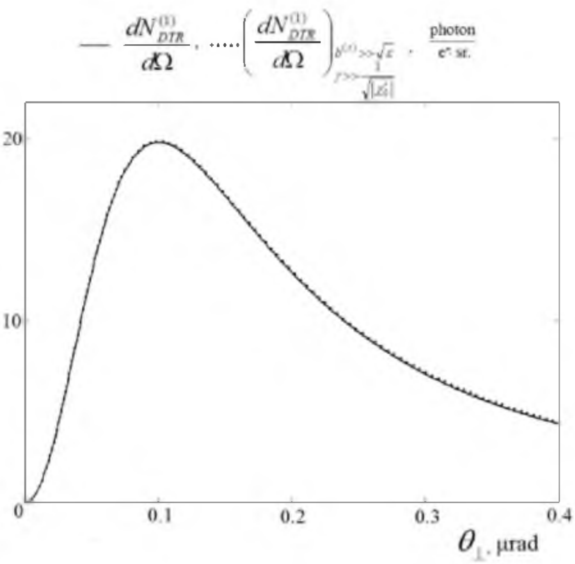

Fig. 3. The angular density of the DTR generated by a relativistic electron in the beam incident in direction of the beam axis. The curves are plotted by dynamical formula (8) (solid) and by its asymptotic kinematic formula (13) (dashed). $Si(111)$, $L = 5\,\mu m$, $\theta_B = 14.5^o$, $\delta = -7^o$, $\varepsilon = 3$ $\omega_B = 8$ кэВ, $E_e = 5$ ГэВ, $\psi = 0$, $\theta_{//} = 0$; $b^{(s)} \approx 6.59$, $\frac{1}{\sqrt{|\chi'_0|}} \approx 258$

Influence of divergence of electron beam on DTR angular density

Let us consider the influence of the divergence of the beam of relativistic electrons crossing a thin monocrystalline target on the angular density of DTR. For this purpose, we will average the expressions for angular density of the radiation generated by one of electrons over all its possible straight trajectories in the beam. As an example, let us carry out the averaging of the DTR density (12) using the Gauss distribution function

$$f(\psi) = \frac{1}{\pi\psi_0^2} e^{-\frac{\psi^2}{\psi_0^2}}, \tag{15}$$

where ψ_0 parameter would mean a divergence of the radiating electron beam (see in fig. 1). The angle ψ_0 defines the cone limiting the part of the electron beam outside of which the density is reduced by more than three times in comparison with the density on the beam axis.

For this distribution function the expression for averaged angular density of DTR generated by the electron beam normalized per number of electrons in the beam will take the form

$$\left\langle \frac{dN_{DTR}^{(s)}}{d\Omega} \right\rangle_{\substack{b^{(s)} << \sqrt{\varepsilon} \\ \gamma >> \frac{1}{\sqrt{|\chi'_0|}}}} = \frac{e^2 \omega_B \chi_g'^2 C^{(s)2}}{4\pi \sin^2\theta_B} \frac{\varepsilon L}{\sin(\theta_B + \delta)} \frac{1}{\pi\psi_0^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\Omega^{(s)2} e^{-\frac{\psi^2}{\psi_0^2}}}{\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2\right)^2} d\psi_\perp d\psi_{//}. \tag{16}$$

In fig. 4 the curves plotted by formula (16) demonstrate the angular density of DTR generated by relativistic electron beam in a thin crystalline plate for different values of beam divergence under other parameters the same as in Fig. 2. In fig. 5 the curves which describe the DTR angular density under the same condition as in fig. 2 and fig. 4, but for considerably more high electron energies are shown. The curves presented in fig.4 and in fig. 5 demonstrate epy significant dependence of DTR angular density on the electron beam divergence. As it follows from fig. 4 the angular density of DTR for electron energy $E_e = 5\ ГэВ$ becomes significantly sensitive to beam divergence under $\psi_0 \geq 0.08\, мрад$, but for electron energy $E_e = 100\ ГэВ$ (see in fig. 6.) under $\psi_0 \geq 0.003\, мрад$. One can see that under more high values of electron energy the angular distribution of DTR became narrower and more sensitive to the electron beam divergence.

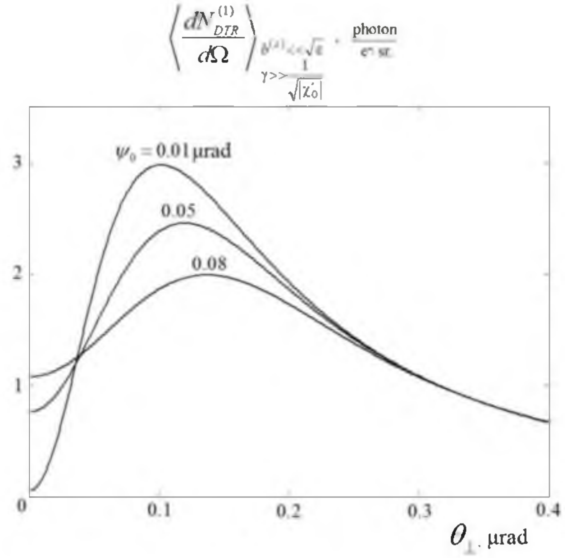

Fig. 4. Influence of the DTR angular density on the relativistic electron beam divergence (all the parameters are the same as in fig. 2)

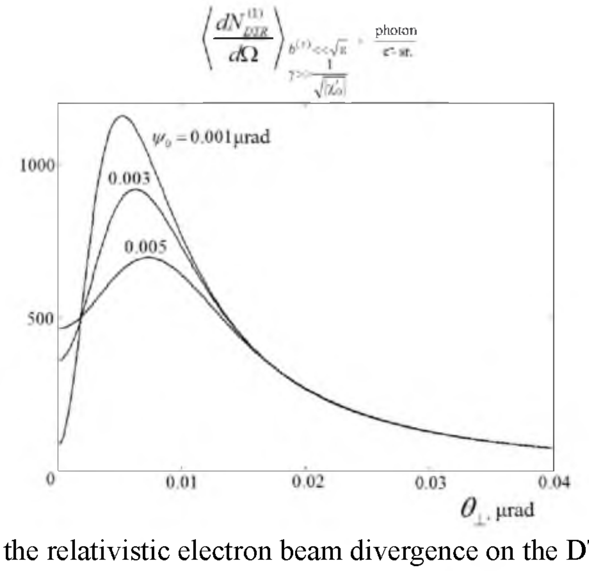

Fig. 5. Influence of the relativistic electron beam divergence on the DTR angular density (all the parameters are the same as in fig. 2, with the exception of the electron energy $E_e = 100\ ГэВ$)

Conclusion

In the present work the diffracted transition radiation (DTR) of the beam of super relativistic electrons crossing a thin single-crystal plate in scattering geometry of Bragg has been considered. In the first time the expression describing the DTR angular density was derived for case when the path of relativistic electron in the target is considerably less than length of X-ray extinction in crystal. The kinematic character of DTR by the beam of ultra-relativistic electrons crossing a thin single-crystal target has been proved. The numerical calculation carried out shown the effect of considerable influence of the beam divergence on the DTR angular density. The derived expression can be effectively used under working out the new methods of the beam divergence measurements on the new electron colliders on super high energy.

Acknowledgements

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