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EFFECT OF MEDIUM ON THE THERMO-, PHOTO- AND DIFFUSIOPHORESIS OF SPHEROIDAL SOLID PARTICLE

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Abstract. Steady motion of spheroidal aerosol particle with inner nonuniformly distributed heat sources that are placed in some external temperature and concentration gradients is studied in the Stokes approximation. The mean temperature of the particle surface is assumed to be differed slightly from that of gaseous environment. The analytic expression of the force and the rate of thermo-, photo- and diffusiophoresis are found by solving gas-dynamic equations in view of environment evolution.

Keywords: spheroid; thermophoresis; photophoresis; diffusiophoresis.

1. Introduction. In modern techniques different types of multiphase mixtures are often used especially in such spheres as chemical technology, hydrometeorology, environmental protection etc. From this viewpoint, the study of dispersion mixtures consisted of two phases one of which is particles, and the other is viscous liquid spheres are of greatest interest. Gas (liquid) with weighted particles in it is called aerosol (hydrosol) and those particles is called aerosol (hydrosol). Hydro- and aerosol particles can influence sufficiently on physical and chemical processes of different types in dispersion system (e.g. in mass and heat changing processes). The particles' sizes of dispersion phase can vary from macroscopic (500 mkm) scales to molecular (10 nm) ones as well as the concentration of the particles can change from 1 particle to multiconcentrated systems ($> 10^{10}$ см $^{-3}$). In today's nanotechnology, the usage of ultra dispersion nanomaterials, Nan particles is important demand, e.g. in such spheres as Nan electronics, Nan mechanics, etc.

Particles of dispersion systems can be effected by many ways. It leads to regulated movement relative of their inertia centers. At this way, their sedimentation happens in gravity field. In gaseous spheres with heterogeneously spread temperature, there can happen regular particles' movement influenced by some forces of molecular origin. This process is the result of the transmission of uncompensated impulse to particles by gas molecules. In this case the movement of particles influenced by outward temperature and concentration gradient is called thermophoresis and diffusiophoresis, correspondingly [1]. If the movement is influenced by the inner heat source heterogeneously spreaded in particles volume, so this movement is called photophoretical [2, 3].

The average distance between aerosol particles in aerodispersial systems is much longer than the usual particle size. In such systems the stock of aerosol influence on physical process can be taken in account basing on dynamic laws, heat-and-mass changing of environment. Without appropriate knowledge of such a behavior, the mathematical simulation of aerosol systems evolution is impossible as well as the solution of such a problem with correct account of their influence on aerosols.

Some types of particles which are met in industrial arrangements and nature have the nonspherical form, e.g. spheroid (ellipsoid). Therefore, the question of movements of particles with different appropriate forms in gaseous (liquid) being both homogeneous and heterogeneous is very actual which lays theoretical and practical interest.

2. Problem formulation. We examine enormous solid particle of spheroid form weighted in binomial gaseous mixture with the temperature T_∞, the density ρ_g and the viscousness μ_g. The particle motion is caused by outside sources of the little temperature gradient ∇T and the concentration gradient ∇C_1 in this binary gaseous mixture. In this case $C_1 + C_2 = 1$, $C_1 = n_1/n_g$, $C_2 = n_2/n_g$, $n_g = n_1 + n_2$, $\rho_g = \rho_1 + \rho_2$, $\rho_1 = m_1 n_1$, $\rho_2 = m_2 n_2$, m_1, n_1 and m_2, n_2 are mass and concentration of first and second components, correspondingly, in binary gaseous mixture. In the works published till now under the theory of thermo-, photo- and diffusiophoresis of large solid aerosol particles having spheroidal form at small relative temperature drops [4] were not studied influence of environment evolution (i.e. convective terms in the heat conduction equation and the diffusion one) and the surface heating on thermo-, photo- and diffusiophoresis simultaneously, that represents theoretical and practical interest. In this work the estimate of this influence is resulted.

In theoretical description of thermo-, photo-, diffusiophoresis we suppose that the heat process and the mass movement is quasistatic in the system. It is due to smallness of heat and diffusion relaxation. The movement happens with small Peclet and Reynolds numbers and the temperature drop in particle neighborhood is assumed to be small, i.e. $(T_s - T_\infty)/T_\infty \ll 1$ where T_s is the mean temperature of spheroid surface and T_∞ is the gas temperature far away of particle. In this case, the thermal conductivity as well as the dynamic and kinematic viscosities can be considered as constants [11]. The problem is solved by hydrodynamic method. So, the hydrodynamic equations with appropriate bordering conditions should be solved. It is considered that there are no some phase transformations and particles are homogenous in its structure.

We suppose that the flat monochromium wave falls on the particle with the intensity I_0 during a time interval. The energy of electromagnetic radiation having been absorbed in particle volume transforms into heat energy. The heat spreads heterogeneously in volume, the local distribution of appeared heat sources can be described by the function q_p called the volume density of inner heat sources [10].

We describe the thermo-, photo- and diffusiophoresis of the particle in the spheroidal coordinate system $(\varepsilon, \eta, \varphi)$ with the origin at spheroid center; i.e. the origin of fixed coordinate system coincides with the instantaneous position of particle center. The curvilinear coordinates $\varepsilon, \eta, \varphi$ are related to the Cartesian coordinates by the relations [5]:

$$x = c\,\mathrm{ch}\,\varepsilon \sin\eta \cos\varphi\,, \;\; y = c\,\mathrm{ch}\,\varepsilon \sin\eta \sin\varphi\,, \;\; z = c\,\mathrm{sh}\,\varepsilon \cos\eta\,, \tag{1}$$

$$x = c\,\mathrm{sh}\,\varepsilon \sin\eta \cos\varphi\,, \;\; y = c\,\mathrm{sh}\,\varepsilon \sin\eta \sin\varphi\,, \;\; z = c\,\mathrm{ch}\,\varepsilon \cos\eta \tag{2}$$

where $c = \sqrt{a^2 - b^2}$ when the spheroid is oblate ($a > b$, Eq.(1)) or $c = \sqrt{b^2 - a^2}$ when it is prolate ($a < b$, Eq.(2)); a and b are spheroid semiaxis. The OZ axis of the Cartesian coordinate system coincides with the spheroid symmetry axes.

In the frame of formulated theory, spreadings of the velocity U_g, the pressure P_g, temperatures T_g, T_p and the concentration of first component C_1 in binary gaseous mixture are described in the following equation system [6]:

$$\nabla P_g = \mu_g \triangle \mathbf{U}_g, \quad \operatorname{div} \mathbf{U}_g = 0\,,$$

$$\rho_g c_{pg}(U_g \nabla) T_g = \lambda_g \Delta T_g, \quad \Delta T_p = -\frac{q_p}{\lambda_p}\,, \tag{3}$$

$$(U_g \nabla) C_1 = D_{12} \Delta C_1\,.$$

The equation system (3) is solved with boundary conditions [4]

$$\varepsilon = \varepsilon_0: \qquad U_\varepsilon = -\frac{cU \operatorname{ch} \varepsilon}{H_\varepsilon} \cos \eta,$$

$$U_\eta = \frac{cU \operatorname{sh} \varepsilon}{H_\varepsilon} \sin \eta - K_{TS} \frac{\nu_g}{T_g} (\nabla T_g \cdot \mathbf{e}_\eta) - K_{DS} D_{12} (\nabla C_1 \cdot \mathbf{e}_\eta)\,,$$

$$T_g = T_p, \quad \lambda_g \frac{\partial T_g}{\partial \varepsilon} = \lambda_p \frac{\partial T_p}{\partial \varepsilon}, \quad \frac{\partial C_1}{\partial \varepsilon} = 0\,. \tag{4}$$

$$\varepsilon \to \infty: \qquad T_g \to T_\infty + |\nabla T_g|_\infty c \operatorname{sh} \varepsilon \cos \eta,$$

$$C_1 \to C_{1\infty} + |\nabla C_1|_\infty c \operatorname{sh} \varepsilon \cos \eta\,,$$

$$P_g \to P_\infty; \; U_g \to 0, \tag{5}$$

$$\varepsilon \to 0: \qquad T_p \neq \infty\,. \tag{6}$$

Here, $\mathbf{e}_\eta$, $\mathbf{e}_\varepsilon$ are unit vectors of spheroidal coordinate system; U_ε, U_η are components of the mass velocity $\mathbf{U}_g$; $U = |\mathbf{U}|$; λ_g, λ_p are thermal conductivities of gas and particles, correspondingly, ν_g, μ_g are kinematical and dynamic viscosities; $H_\varepsilon = c\sqrt{\operatorname{ch}^2 \varepsilon - \sin^2 \eta}$ is the Lame coefficient; K_{TS}, K_{DS} are thermal and diffusion creep coefficients which are calculated from kinetic theory of gases, the gas kinetic coefficient $K_{TS} \approx 1,152$ when accommodation coefficients of tangential momentum and energy equal to unity (in the case of spheroid particle) [1,7]; $K_{DS} \approx 0,3$; $\varepsilon = \varepsilon_0$ is the coordinate surface corresponding to the particle surface.

3. Temperature and concentration distributions. Let us study temperature and concentration distributions inside and outside of the particle. We transform equations (3) and boundary conditions (4)-(6) into dimensionless form by introducing dimensionless values: $t_k = T_k/T_\infty, V_g = U_g/U, (k = g, p)$.

In this problem, besides Reynolds and Peclet dimensionless numbers, there are two controllable small parameters $\xi_1 = a|\nabla T_g|_\infty/T_\infty \ll 1$ and $\xi_2 = a|\nabla C_1|_\infty \ll 1$ characterizing relative overfall of temperature and concentration. Characteristic velocity $U \sim (\mu_g/\rho_g T_\infty)|\nabla T_g|_\infty$ on the size order for the purely thermophoresis and $U \sim D_{12}|\nabla C_1|_\infty$ for the purely diffusiophoresis. Therefore, we look for the solution of the boundary-value problem

(3)-(6) in the form of expansion corresponding physical sizes in powers of ξ_1. Small parameter ξ_2 is expressed through ξ_1 and the Reynolds number calculated on the pure thermophoresis characteristic velocity coincides with ξ_1.

$$V_g = V_{g0} + \xi_1 V_{g1} + \dots, \quad p_g = p_{g0} + \xi_1 p_{g1} + \dots,$$

$$t = t_0 + \xi_1 t_1 + \dots, \qquad C_1 = C_{10} + \xi_1 C_{11} + \dots. \tag{7}$$

We restrict out our consideration to the first order terms on ξ_1 when calculating the force acting on particle and the velocity of its thermo-, photo-, and diffusiophoretic motion. In order to find these quantities, one has to know the distributions of velocity, pressure, temperature and concentration both outside and inside the spheroid. Substituting (7) into (3), leaving terms ξ_1 and solving sets of equations found by the method of separation of variables, we finally find at zero and first approximations

$$t_{g0}(\lambda) = 1 + \gamma\lambda_0 \operatorname{arcctg}\lambda, \tag{8}$$

$$t_{p0}(\lambda) = 1 + (1-\delta)\gamma\lambda_0 \operatorname{arcctg}\lambda_0 + \delta\gamma\lambda_0 \operatorname{arcctg}\lambda + \int_{\lambda_0}^{\lambda} f_0 \operatorname{arcctg}\lambda d\lambda - \operatorname{arcctg}\lambda \int_{\lambda_0}^{\lambda} f_0 d\lambda, \tag{9}$$

$$C_{10} = C_{1\infty}; \tag{10}$$

$$t_{g1} = \cos\eta \Big\{ \frac{c\lambda}{a} + \Gamma(\lambda \operatorname{arcctg}\lambda - 1) + Pr_\infty \frac{\gamma\lambda_0}{ac} \Big\{ A_2 \Big(\operatorname{arcctg}\lambda - \frac{\lambda}{2} \operatorname{arcctg}^2\lambda \Big) + $$
$$+ \frac{A_1}{2} \Big(\operatorname{arcctg}\lambda - \lambda \operatorname{arcctg}^2\lambda \Big) \Big\} \Big\}, \tag{11}$$

$$t_{p1} = \cos\eta \Big\{ B\lambda + \frac{3(1-\lambda \operatorname{arcctg}\lambda)}{4\pi c^2 \lambda_p T_\infty} \int_V q_p z dV - \lambda \int_{\lambda_0}^{\lambda} f_1 (\lambda \operatorname{arcctg}\lambda - 1) d\lambda + $$
$$+ (\lambda \operatorname{arcctg}\lambda - 1) \int_{\lambda_0}^{\lambda} f_1 \lambda d\lambda \Big\}, \tag{12}$$

$$C_{11} = \cos\eta \Big\{ \frac{c\lambda}{a} - \frac{c(1+\lambda_0^2)}{((1+\lambda_0^2)\operatorname{arcctg}\lambda_0 - \lambda_0)a} \Big(\lambda \operatorname{arcctg}\lambda - 1 \Big) \Big\} \tag{13}$$

where $\lambda = \operatorname{sh}\varepsilon, \lambda_0 = \operatorname{sh}\varepsilon_0$ and $\delta = \lambda_g/\lambda_p, \gamma = t_s - 1$ is the dimensionless parameter characterizing the temperature on spheroid surface; $t_s = T_s/T_\infty$,

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi c \lambda_0 \lambda_g T_\infty} \int_V q_p dV. \tag{14}$$

In (14) the integral is taken over the total entire particle volume,

$$f_n = -\frac{2n+1}{2\lambda_p T_\infty} \int_{-1}^{1} c^2 q_p (\lambda^2 + x^2) P_n(x) dx,$$

$x = \cos\eta, P_n(x)$ — Legendre's polynomials.

Constants in expressions (11) and (12) of temperature fields inside and outside the particle are connected with corresponding boundary conditions on the spheroid surface. Since an expression for the coefficient Γ is of interest, we write it in the explicit form:

$$\Gamma = -\frac{c(1-\delta)}{a\Delta} + \frac{3}{4\pi c^2\lambda_p\lambda_0 T_\infty(1+\lambda_0^2)\Delta}\int_V q_p z dV +$$

$$+Pr_\infty\frac{\gamma\lambda_0}{ac}\Big\{A_2\Big(\operatorname{arcctg}\lambda_0 - \frac{1-\delta}{2\Delta}\operatorname{arcctg}^2\lambda_0 - \frac{\delta}{(1+\lambda_0^2)\Delta}\Big) + + \frac{A_1}{2}\Big(\operatorname{arcctg}\lambda_0 + \frac{\delta(\lambda_0\operatorname{arcctg}\lambda_0 - 1)}{(1+\lambda_0^2)\Delta}\Big)\Big\}, \tag{15}$$

$$\Delta = (1-\delta)\operatorname{arcctg}\lambda_0 + \frac{\delta\lambda_0}{1+\lambda_0^2} - \frac{1}{\lambda_0}.$$

4. Force and velocity of thermo-, photo- and diffusiophoresis. The general solution of the hydrodynamic equations in the spheroidal coordinate system has form [5]:

$$U_\varepsilon(\varepsilon,\eta) = \frac{U}{c\operatorname{ch}\varepsilon H_\varepsilon}\cos\eta\Big\{\lambda A_2 + \Big[\lambda - (1+\lambda^2)\operatorname{arcctg}\lambda\Big]A_1 + c^2(1+\lambda^2)\Big\},$$

$$U_\eta(\varepsilon,\eta) = -\frac{U}{cH_\varepsilon}\sin\eta\Big\{\frac{A_2}{\lambda} + \Big[1 - \lambda\operatorname{arcctg}\lambda\Big]A_1 + c^2\lambda\Big\}, \tag{16}$$

$$P_g(\varepsilon,\eta) = P_\infty + c\frac{\mu_g U}{H_\varepsilon^4}x(x^2+\lambda^2)A_2.$$

Integration constants A_1, A_2 are found from boundary conditions on the spheroid surface:

$$A_2 = -\frac{2c^2}{\lambda_0 + (1-\lambda_0^2)\operatorname{arcctg}\lambda_0} -$$

$$-2K_{TS}\frac{c^2\nu_g\delta}{Ut_s}\cdot\frac{|\nabla T_g|_\infty}{T_\infty(1+\lambda_0^2)\Delta}\cdot\frac{\lambda_0 - (1+\lambda_0^2)\operatorname{arcctg}\lambda_0}{\lambda_0 + (1-\lambda_0^2)\operatorname{arcctg}\lambda_0}\Big(1 + \frac{3a(1-\lambda_0\operatorname{arcctg}\lambda_0)}{4\pi c^3\lambda_0\lambda_g T_\infty}\int_V q_p z dV -$$

$$-\frac{Pr_\infty\gamma\lambda_0}{2(\lambda_0 + (1-\lambda_0^2)\operatorname{arcctg}\lambda_0)}\Big[2\operatorname{arcctg}^2\lambda_0 + 4(\lambda_0\operatorname{arcctg}\lambda_0 - 1) + (1-\lambda_0^2)(\lambda_0\operatorname{arcctg}\lambda_0 - 1)^2\Big]\Big) -$$

$$-2K_{DS}D_{12}\frac{c^2|\nabla C_1|_\infty}{U(\lambda_0 + (1-\lambda_0^2)\operatorname{arcctg}\lambda_0)}. \tag{17}$$

The resultant force acting on spheroidal particle is defined by integrating the stress tensor over the aerosol particle surface [5, 6] and has the form:

$$F_z = 4\pi\frac{\mu_g U}{c}A_2. \tag{18}$$

In view of the explicit form of coefficient A_2, we find the general expression of force acting on spheroidal solid aerosol particle. This force is the sum of the viscous force F_μ, the thermophoretic force F_{th}, the photophoretic force F_{ph} proportional to the dipole moment of heat sources nonuniformly distributed over particle volume, the force connected with the medium and the diffusiophoretic force F_{dh}

$$F_z = F_\mu + F_{th} + F_{ph} + F_{dh}\,, \tag{19}$$

$$F_\mu = -8\pi\mu_g U \frac{c}{\lambda_0 + (1-\lambda_0^2)\operatorname{arcctg}\lambda_0}\,,$$

$$F_{th} = -8\pi K_{TS}\frac{\mu_g\nu_g}{t_s}\cdot\frac{|\nabla T_g|_\infty}{T_\infty}\cdot\frac{\lambda_0-(1+\lambda_0^2)\operatorname{arcctg}\lambda_0}{\lambda_0+(1-\lambda_0^2)\operatorname{arcctg}\lambda_0}\cdot\frac{c\delta}{(1+\lambda_0^2)\Delta}\times$$

$$\times\left(1-\frac{Pr_\infty\gamma\lambda_0}{2(\lambda_0+(1-\lambda_0^2)\operatorname{arcctg}\lambda_0)}\left[\operatorname{arcctg}^2\lambda_0-\lambda_0^2(\lambda_0\operatorname{arcctg}\lambda_0-1)^2\right]\right)$$

$$F_{ph} = -8\pi K_{TS}\frac{\mu_g\nu_g}{t_s}\cdot\frac{|\nabla T_g|_\infty}{T_\infty}\cdot\frac{\lambda_0-(1+\lambda_0^2)\operatorname{arcctg}\lambda_0}{\lambda_0+(1-\lambda_0^2)\operatorname{arcctg}\lambda_0}\cdot\frac{c\delta}{(1+\lambda_0^2)\Delta}\times$$

$$\times\left(\frac{3a(1-\lambda_0\operatorname{arcctg}\lambda_0)}{4\pi c^3\lambda_0\lambda_g T_\infty}\int_V q_p z dV -\right.$$

$$\left.-\frac{Pr_\infty\gamma\lambda_0}{2(\lambda_0+(1-\lambda_0^2)\operatorname{arcctg}\lambda_0)}\left[\operatorname{arcctg}^2\lambda_0+4(\lambda_0\operatorname{arcctg}\lambda_0-1)+(\lambda_0\operatorname{arcctg}\lambda_0-1)^2\right]\right)\,,$$

$$F_{dh} = -8\pi K_{DS}D_{12}\mu_g\frac{c|\nabla C_1|_\infty}{\lambda_0+(1-\lambda_0^2)\operatorname{arcctg}\lambda_0}\,.$$

Equating the resultant force F to zero, we arrive to the general expression of the drift (thermo-, photo- and diffusiophoretic) velocity of solid oblate spheroidal particle in external temperature and concentration gradient fields:

$$U = -\frac{b}{a}K_{TS}\cdot\frac{\nu_g\delta}{t_s}\cdot\frac{|\nabla T_g|_\infty}{T_\infty}\frac{(1-(\lambda_0+\lambda_0{}^{-1})\operatorname{arcctg}\lambda_0)}{\sqrt{1+\lambda_0^2}\Delta}\left(1+\frac{3a(1-\lambda_0\operatorname{arcctg}\lambda_0)}{4\pi c^3\lambda_0\lambda_g T_\infty}\int_V q_p z dV-\right.$$

$$\left.-\frac{Pr_\infty\gamma\lambda_0}{2(\lambda_0+(1-\lambda_0^2)\operatorname{arcctg}\lambda_0)}\left[2\operatorname{arcctg}^2\lambda_0+4(\lambda_0\operatorname{arcctg}\lambda_0-1)+(1-\lambda_0^2)(\lambda_0\operatorname{arcctg}\lambda_0-1)^2\right]\right)$$

$$-K_{DS}D_{12}|\nabla C_1|_\infty\,. \tag{20}$$

In order to find the rate of thermo-, photo- and diffusiophoresis for the case of prolate spheroid, one has to substitute $i\lambda$ for λ and $-ic$ for c (i is the imaginary unit) in (19), (20).

Thus, formulas (19) and (20) have the most general form and make it possible to estimate the resultant force acting on a solid spheroidal aerosol particle and its drift velocity in the external temperature and concentration gradient fields for the case when heat sources (sinks) are nonuniformly distributed inside the particle. In this approach, the environment evolution is taken into account for small temperature differences in the vicinity of particle.

5. Results and discussion. From formulas (18), (19), it is visible that environment evolution does not bring the contribution in diffusiophoresis. Convective terms in the diffusion equation do not influence on movement of solid aerosol particle in the field of concentration gradient. It can be of great importance in practical appendices, for example, at the description of aerosol particle movement in diffusiophoresis fields. That convective terms influence on the diffusiophoresis, it is necessary to change boundary conditions of diffusion part. It can be made by different ways. For example, we can consider movement not solid particle taking into account the evaporating drop.

Movement of environment brings the contribution as in thermophoresis and in fotophoresis. These contributions are proportional to product of Prandtl's number on average temperature of particle surface. Prandtl's number for the most of gases is about of one. As the problem dared at small temperature differences, the movement contribution can make no more than 10-12 percents.

It is interesting to examine the special cases of movement of spheroidal particles. If one does not take into account the motion of environment and internal heat sources, (20) is reduced to the expression of the purely thermo- and diffusiophoresis velocity of spheroidal particle:

$$U = -\frac{b}{a} K_{TS} \frac{\nu_g}{t_s} \cdot \frac{|\nabla T_g|_\infty}{T_\infty} \cdot \frac{\delta(1-(\lambda_0+\lambda_0{}^{-1})\operatorname{arcctg}\lambda_0)}{\sqrt{1+\lambda_0^2\Delta}} - K_{DS} D_{12} |\nabla C_1|_\infty$$

which coincides with formula (9) in [4].

In the spherical case, (20) turns into the expression for the thermo-, photo- and diffusiophoretic velocity of solid spherical particle of radius R that includes the flow of environment and internal heat sources:

$$U_{a=b=R} = -K_{TS} \frac{\nu_g}{t_s} \cdot \frac{|\nabla T_g|_\infty}{T_\infty} \cdot \frac{2\delta}{1+2\delta} \left(1 + \frac{1}{4\pi R^2 \lambda_g T_\infty} \int_V q_p z dV - Pr_\infty \frac{5\gamma}{24}\right) - K_{DS} D_{12} |\nabla C_1|_\infty . \tag{21}$$

Disregarding the environment flow and internal heat sources yields the conventional formula for the thermophoretic velocity of large spherical particle [8, 9].

$$U|_{a=b=R} = -K_{TS} \frac{\nu_g}{t_s} \cdot \frac{|\nabla T_g|_\infty}{T_\infty} \cdot \frac{2\delta}{1+2\delta} . \tag{22}$$

In order to estimate how the environment motion effects on the thermo- and photophoretic velocity of spheroidal particle, one has to specify the nature of heat sources nonuniformly distributed over its volume. As an example, let us consider the simplest case when the particle absorbs radiation as the black body. In this case, the radiation is absorbed in the thin layer of depth $\delta\varepsilon \ll \varepsilon_0$ that is adjacent to the heated particle surface. The density of heat sources inside the layer of depth $\delta\varepsilon$ is equal to [10, 11]

$$q_p(\varepsilon,\eta)=\begin{cases}-\dfrac{\operatorname{ch}\varepsilon\cos\eta}{c(\operatorname{ch}^2\varepsilon-\sin^2\eta)\delta\varepsilon}I_0, & \frac{\pi}{2}\le\eta\le\pi,\quad \varepsilon_0-\delta\varepsilon\le\varepsilon\le\varepsilon_0\,;\\ 0, & 0\le\eta\le\frac{\pi}{2}\end{cases}\qquad(23)$$

where I_0 is the intensity of incident radiation.

Integrals $\int\limits_V q_p dV$ and $\int\limits_V q_p z dV$ appear in the expression for the thermo- and photophoretic velocity. Substituting into (23) these integrals in view of the fact that $\delta\varepsilon \ll \varepsilon_0$ and performing integration, we find

$$\int\limits_V q_p z dV=-\frac{2}{3}\pi c^3 I_0\lambda_0^3\Big(1+\lambda_0^{-2}\Big)\,,\qquad(24)$$

$$\int\limits_V q_p dV=\pi c^2 I_0\lambda_0^2\Big(1+\lambda_0^{-2}\Big)\,.\qquad(25)$$

In view of (24), (25) the expression (20) takes the form

$$U=-\frac{b}{a}K_{TS}\frac{\nu_g}{t_s}\cdot\frac{|\nabla T_g|_\infty}{T_\infty}\cdot\frac{\delta(1-(\lambda_0+\lambda_0{}^{-1})\operatorname{arcctg}\lambda_0)}{\sqrt{1+\lambda_0^2}\Delta}\left(1+\frac{aI_0(1+\lambda_0^2)}{2\lambda_g T_\infty}\right)\times$$

$$\left[\lambda_0\operatorname{arcctg}\lambda_0-1-Pr_\infty\frac{2\operatorname{arcctg}^2\lambda_0+4(\lambda_0\operatorname{arcctg}\lambda_0-1)+(1-\lambda_0^2)(\lambda_0\operatorname{arcctg}\lambda_0-1)^2}{4\sqrt{1+\lambda_0^2}(\lambda_0+(1-\lambda_0^2)\operatorname{arcctg}\lambda_0)}\right]$$

$$-K_{DS}D_{12}|\nabla C_1|_\infty\,.\qquad(26)$$

In the case of sphere, (26) is recast as

$$U(a=b=R)=-K_{TS}\frac{\nu_g}{t_s}\cdot\frac{|\nabla T_g|_\infty}{T_\infty}\cdot\frac{2\delta}{1+2\delta}\Big(1-\frac{I_0R}{6\lambda_g T_\infty}(1+\frac{5Pr_\infty}{16})\Big)-K_{DS}D_{12}|\nabla C_1|_\infty\,.$$

In order to illustrate contributions of the form-factor (ratio of spheroid semiaxes), the environment flow and the internal heat release (nonuniform distribution of heat sources over the particle volume) to the thermo- and photophoretic velocity (26), in drawing list curves corresponding to values $f=(f^*_{t.ph}/f^{**}_{t.ph})|_{T_\infty=300K}$ with intensity of incident radiation of borated graphite ($\lambda_p=55W/(mK)$) particles with spheroidal (the curve 1 taking into account movement of environment, the curve 2 is the same without movement) and spherical (the curve 3) forms of surfaces suspended in air at $T_\infty=300K$ and $P_g=10^5Pa$ for various relations of spheroid semiaxes.

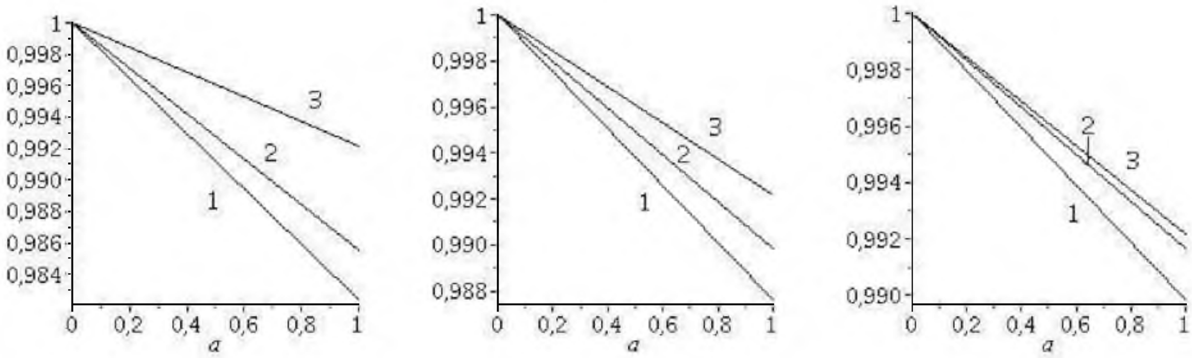

Fig. 1. Dependence of function f on the intensity of an incident radiation at the relation of semiaxes $b/a = 0,2$ — 1); $b/a = 0,5$ — 2); $b/a = 0,7$ — 3). $I_0 - W/sm^2$.

Numerical analysis showed that, at the given ratio between semiaxes, the relative contribution of other factors (flow of environment, internal heat release) leads to monotonous reduction of velocity thermo-, photo- and diffusiophoresis with increasing incident radiation intensity. This effect depends significantly on the equatorial radius a of spheroid.

Quantitative research of the discussed phenomen for firm hearted particles represents quite real experimental problem.

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