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OPTICS
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# Monochromatic Optical Cherenkov Radiation of Moderately Relativistic Ions in Radiators with Frequency Dispersion

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Optical Cherenkov radiation of moderately relativistic ions in a CVD-diamond plate with frequency dispersion is considered. It has been shown that Cherenkov radiation extracted from the inclined diamond plate to vacuum at a fixed observation angle becomes monochromatic. The wavelength of the spectral line depends on the energy of an ion and on the geometry of an experiment (observation angle and plate inclination angle). An experiment has been proposed to study the monochromatization of Cherenkov radiation on the beam of the JINR Nuclotron for the purpose of its subsequent use in the diagnostics of ion beams. The method can be applied to monitor the NICA ion beams energy.

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Optical Cherenkov radiation (CR) [1] is widely used in various fields such as the development of elementary-particle detectors [2–5], diagnostics of modern accelerator beams [6–8], infrastructure of nuclear fusion devices [9], and measurements of dose fields of radiotherapy facilities [10].

In almost all cited works, CR was characterized by the directivity

$$\cos\theta_{Ch} = 1/(n(\lambda)\beta), \quad (1)$$

where $\theta_{Ch}$ is the emission angle of Cherenkov photons in a medium through which a charged particle passes at the velocity $v = \beta c$ ($c$ is the speed of light in vacuum), and $n(\lambda)$ is the refractive index of the medium at the wavelength $\lambda$.

It is clear that Eq. (1) also determines the "threshold" character of the Cherenkov effect appearing when the velocity of the particle exceeds the phase velocity of light in the medium:

$$\beta > 1/n(\lambda). \quad (2)$$

The spectral composition of CR in the medium is determined by the condition

$$n(\lambda) > 1; \quad (3)$$

i.e., the spectrum of CR for almost all insulators is continuous from the ultraviolet to infrared range.

Formula (1) is strictly speaking valid for an infinitely thick emitter. In the real case, radiation formed in the finite segment of the trajectory $L$ propagates in the form of a cone of Cherenkov photons with the finite "width" [11]

$$\Delta\theta_{ch} \sim \frac{2\lambda}{\pi L \sin\theta_{ch}}. \quad (4)$$

In the traditional geometry of generation of CR by the moderately relativistic charge in the dielectric plate (see Fig. 1a), the radiation cone is extracted to vacuum if the velocity of the charge satisfies the condition

$$\beta_{thr} = 1/n(\lambda) < \beta < \beta_{max} = 1/\sqrt{n^2(\lambda) - 1}. \quad (5)$$

In the far-field region, where the length of the trajectory in which radiation is generated can be neglected and radiation can be considered as that from a point source, inequality (5) corresponds to the emission angle to vacuum $\theta_{vac}$, which is measured from the momentum of the electron (see Fig. 1a):

$$0 < \theta_{vac} < \pi/2. \quad (6)$$

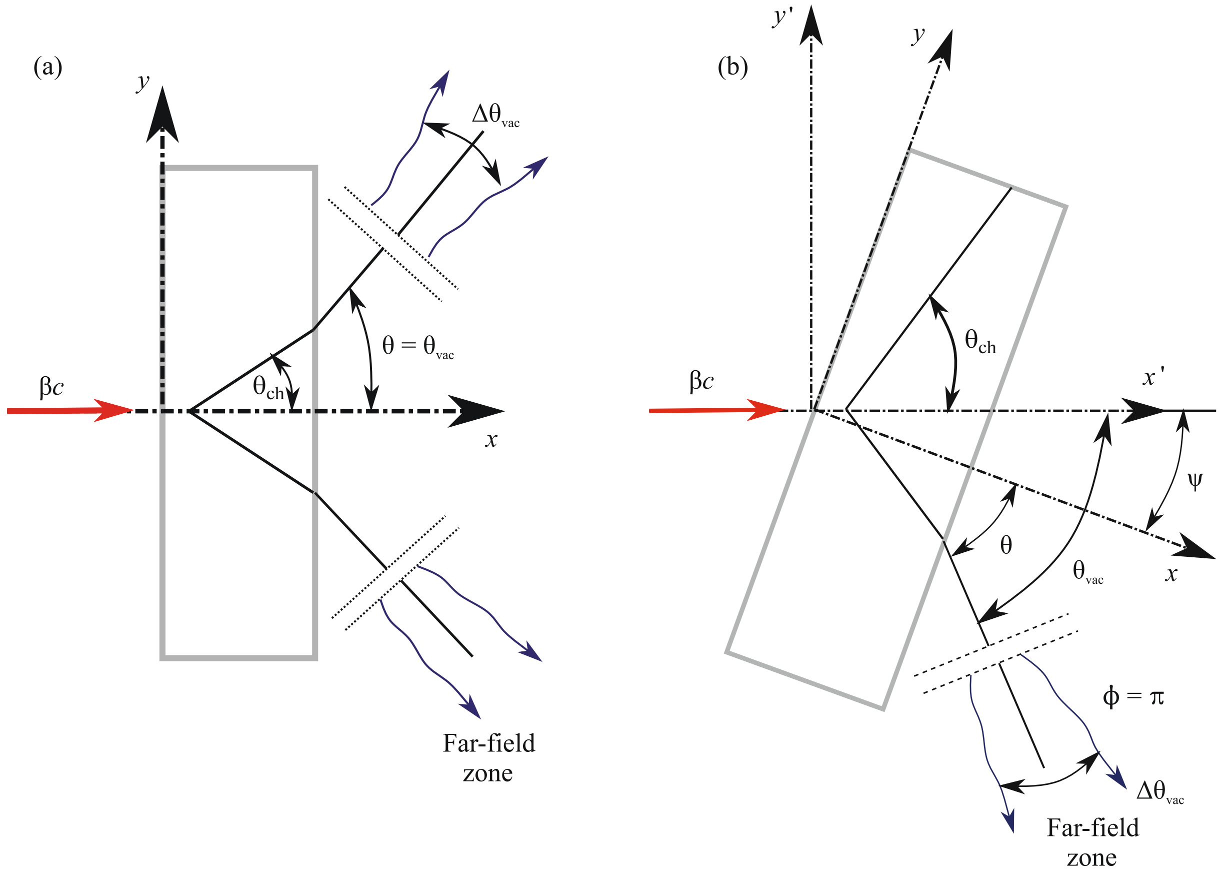


**Fig. 1.** (Color online) Schematic of the generation of Cherenkov radiation in the dielectric plate: (a) perpendicular flight of the charge and (b) flight through the inclined plate.

To extract the radiation of the charge with $\beta > \beta_{max}$ from the radiator to vacuum, its "output" surface should be conical [12]. An alternative simpler way is to use an inclined dielectric plate as the emitter [13] (see Fig. 1b). In this case, only a part of the Cherenkov cone is extracted to vacuum. If the medium has frequency dispersion, because of refraction on the output surface of the emitter at a fixed $\theta$ = const (measured from the normal, see Fig. 1b) in the plane specified by the momentum of the electron and the normal to the output surface (coplanar geometry), the emitted radiation is monochromatic with the wavelength $\lambda$ determined by the relation

$$\theta = \arcsin\left\{n(\lambda)\sin\left[\arccos\left(\frac{1}{n(\lambda)\beta}\right) - \psi\right]\right\}. \quad (7)$$

Here, $\psi$ is the angle between the plate and the momentum of the charged particle (see Fig. 1b).

This effect of monochromatization of CR was recently observed in the experiment with an 855-MeV electron beam and a 200-μm quartz plate used as the emitter [14]. The number of Cherenkov photons that are emitted at the angle $\theta_{vac}$ by the electron from such an inclined emitter and are observed in the far-field region was calculated in [14] within the polarized current model [15, 16]:

$$\frac{d^2N}{d\lambda d\Omega} = 4\alpha\frac{\cos^2\theta}{\left((1-\beta_y n_y)^2 - \beta_z^2\cos^2\theta\right)^2}\left|\frac{\varepsilon - 1}{\varepsilon}\right|^2$$
$$\times\frac{L^2}{\lambda^3}\mathrm{sinc}^2\left(\pi\frac{L}{\lambda}\frac{1-\beta_z Z - n_y\beta_y}{\beta_z}\right)$$
$$\times\left(\beta_y^2\beta_z^2\sin^2\varphi\times\left(|Z|^2+\sin^2\theta\right)\left|\frac{\sqrt{\varepsilon}}{\cos\theta + Z}\right|^2 \quad (8)$$
$$+\left|\frac{\varepsilon}{\varepsilon\cos\theta + Z}\right|^2$$
$$\times\left|(\beta_z^2 + n_y\beta_y + \beta_z Z - 1)\sin\theta - \beta_y\beta_z\cos\varphi Z\right|^2\right).$$

Here, $\alpha \approx 1/137$ is the fine structure constant, $L$ is the thickness of the emitter, $\theta$ is the polar emission angle of a Cherenkov photon to vacuum, $\mathrm{sinc}x = \sin x/x$, $Z = \sqrt{\varepsilon(\lambda) - \sin^2\theta}$, and $\lambda$ is the wavelength of CR. This expression, being simpler, is generally in good agreement with Pafomov's formula given by Eq. (1) in [17].

In the coordinate system $\{x, y, z\}$ associated with the emitter, the components of the velocity of the charge measured in the speed of light are given by the expressions (see Fig. 1b)

$$\beta_y = \beta\sin\psi, \quad \beta_z = \beta\cos\psi,$$

and the components of the unit vector in the direction of the wave vector have the form

$$n_x = \sin\theta\sin\varphi; \quad n_y = \sin\theta\cos\varphi; \quad n_z = \cos\theta.$$

Relation (7) directly follows from the condition that the function sinc$x$ in Eq. (8) is maximal, i.e., its argument is zero:

$$1 - \beta_z Z - n_y\beta_y = 0. \tag{9}$$

In the general case (for a noncoplanar geometry of the process), Eq. (9) specifies a two-dimensional region of angles $\{\theta, \varphi\}$ that describes the part of the Cherenkov cone emitted to vacuum:

$$\begin{aligned} 1 - \beta_z Z - n_y\beta_y = 1 - \beta\cos\psi\sqrt{\varepsilon - \sin^2\theta} \\ - \sin\theta\cos\varphi\beta\sin\psi = 0. \end{aligned} \tag{10}$$

The solution of Eq. (10) for $\sin\theta$ has the form

$$\begin{aligned} \sin\theta = (\sin\psi\cos\varphi + \cos\psi \\ \times \sqrt{\varepsilon\beta^2(1 - \sin^2\psi\sin^2\varphi) - 1})/\beta(1 - \sin^2\psi\sin^2\varphi). \end{aligned} \tag{11}$$

For the coplanar geometry with $\varphi = \pi$ (see Fig. 1), Eq. (7), which is a consequence of Snell's law, can easily be obtained from Eq. (11) by straightforward algebra.

The azimuth region limiting the part of the Cherenkov cone in vacuum

$$\pi + \Delta\varphi \le \varphi \le \pi - \Delta\varphi \tag{12}$$

is determined from Eq. (11) under the boundary condition

$$\sin\theta = 1.$$

Under the condition $\Delta\varphi < 1$, one can obtain the estimate

$$\begin{aligned} (\Delta\varphi)^2 \\ = 4\beta[1 - n(\lambda)\sin(\theta_{\rm ch} - \psi)]/\sin\psi\sin 2(\theta_{\rm ch} - \psi). \end{aligned} \tag{13}$$

In particular, for 165-MeV/nucleon ions and the 565-nm Cherenkov spectral line (see below), the exact solution of Eq. (10) gives $\Delta\varphi = 0.764 \approx 43.8°$, whereas Eq. (13) yields an overestimated value of $\Delta\varphi \approx 1.11 \approx 63.6°$.

We note that, when the velocity of ions increases, the angular range $\Delta\varphi$ is narrowed and Eq. (13) provides not too large an error.

The vacuum angles $\theta_{\rm vac}$ and $\varphi_{\rm vac}$ defined in a more convenient coordinate system $\{x', y', z'\}$ with the $z'$ axis directed along the velocity of the charge (see Fig. 1b) are related to the angles $\theta$ and $\varphi$ through the rotation by the angle $\psi$:

$$\cos\theta_{\rm vac} = \cos\theta\cos\psi + \sin\theta\cos\varphi\sin\psi, \tag{14}$$

$$\tan\varphi_{\rm vac} = \frac{\sin\theta\sin\varphi}{\sin\theta\cos\psi - \sin\theta\cos\varphi\sin\psi}. \tag{15}$$

For the coplanar geometry ($\varphi = \pi$), Eqs. (14) and (15) give

$$\theta_{\rm vac} = \theta + \psi, \quad \varphi_{\rm vac} = 0.$$

Because of frequency dispersion, CR with different wavelength is emitted to vacuum at different angles $\theta_{\rm vac}$. If $\Delta\theta \ll \Delta\theta_{\rm Ch}$, where $\Delta\theta$ is the aperture of the detector and $\Delta\theta_{\rm Ch}$ given by Eq. (4) is the "natural" angular width of the Cherenkov cone emitted by the charge from the finite trajectory $L$, the quasimonochromatic spectrum of radiation should be expected. This expectation was confirmed in the experiment reported in [14].

When the emitter is made from a material with a high refractive index $n(\lambda)$, e.g., diamond with $n > 2.4$, CR is generated by the charge moving at the velocity $v > c/2.4$, i.e., at $\beta > 0.417$. The Lorentz factor for the ion beam with such a velocity is $\gamma \ge 1/\sqrt{1 - \beta^2} \ge 1.09$.

Synthetic diamond (CVD diamond) has a frequency dispersion described by the Sellmeier formula [18, 19]

$$\varepsilon(\lambda) = n^2(\lambda) = 1 + 4.658\lambda^2/(\lambda^2 - 112.5^2), \tag{16}$$

where $\lambda$ is the wavelength in nanometers.

Using Eq. (8), one can calculate the spectrum of Cherenkov photons from the moderately relativistic ion beam passing through the diamond plate after the multiplication by $z^2$, where $z$ is the charge number of the ion.

Figure 2 presents the dependences of the yield of Cherenkov photons with various wavelengths on the observation angle in the geometry shown in Fig. 1a. The calculations were carried out by Eq. (8) for the velocity of ions $\beta = 0.4166$ ($\gamma = 1.1$) passing through the 100-μm diamond target. According to Fig. 2, $\Delta\theta_{\rm vac} \approx$ FWHM $\approx 1.5° = 0.026$ at the wavelength $\lambda = 0.4$ μm. The velocity at the wavelength $\lambda = 0.5$ μm is $\beta_{\max} = 0.45116$, corresponding to the Lorentz factor $\gamma_{\max} = 1.119$, which is only 1.7% larger than the threshold value $\gamma_{\rm thr} = 1/\sqrt{1 - \beta_{\rm thr}^2}$.

To extract CR from more strongly relativistic ions to vacuum, it is necessary to use the inclined geometry. Figure 3 shows the spectra of CR emitted at the angle $\theta_{\rm vac} = 79°$ from ions with energies less than 0.2 GeV/nucleon passing through the 100-μm diamond target inclined at an angle of 17°.

Figure 4 shows the spectra of CR for the same geometry obtained after the integration of Eq. (8) over

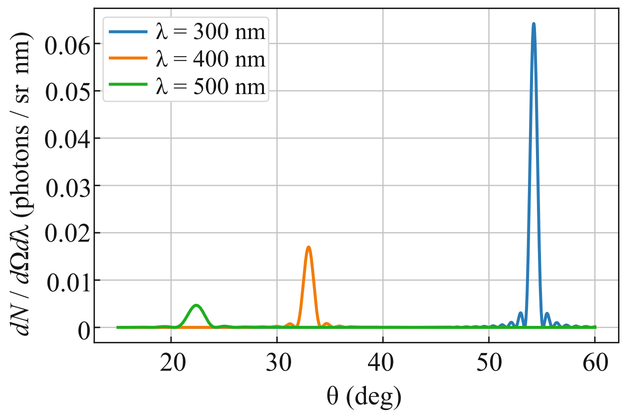


**Fig. 2.** (Color online) Angular dependence of the intensity of Cherenkov radiation of the charge passing at the velocity $\beta = 0.4166$ ($\gamma = 1.1$) through the 100-μm perpendicular diamond plate ($\psi = 0$).

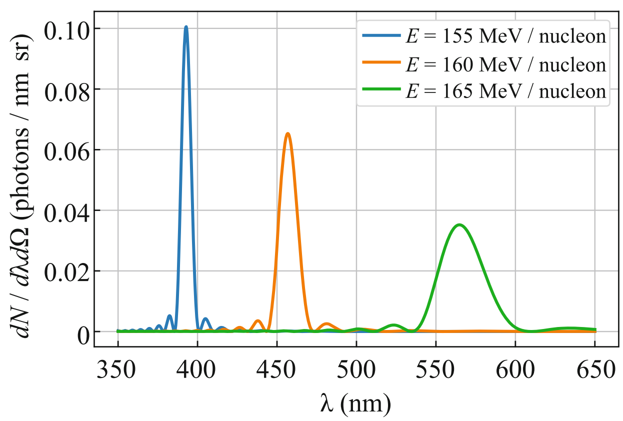


**Fig. 3.** (Color online) Spectrum of monochromatic Cherenkov radiation of moderately relativistic ions passing through the 100-μm inclined diamond plate ($\psi = 17°$) at the observation angle $\theta_{vac} = 79°$ ($\theta = 62°$).

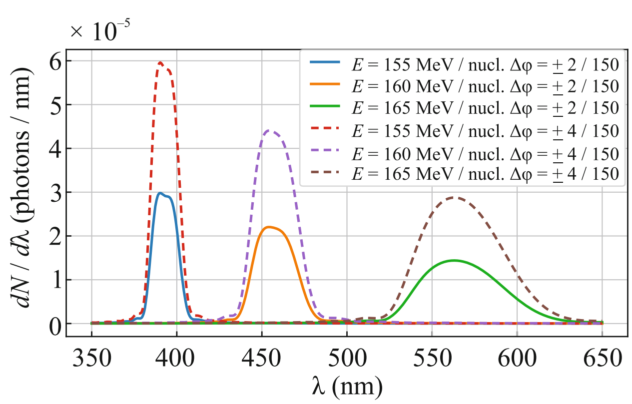


**Fig. 4.** (Color online) Spectrum of Cherenkov radiation of ions under the same condition as in Fig. 3 at detection by a spectrometer with finite apertures (solid lines) $\Delta\theta_{ap} = \Delta\varphi_{ap} = 0.76°$ and (dashed lines) $\Delta\theta_{ap} = 0.76°$ and $\Delta\varphi_{ap} = 1.52°$.

the solid angle ($\theta_{vac} \pm \Delta\theta_{ap} = 79° \pm 0.76°$, $\varphi \pm \Delta\varphi_{ap} = \pm 0.76°$), where ($\Delta\theta_{ap}$, $\Delta\varphi_{ap}$) is the angular aperture of the detector corresponding to the $4 \times 4$-mm collimator placed at a distance of 150 mm from the emitter, as well as for the aperture ($\Delta\theta_{ap} = 0.76°$, $\Delta\varphi_{ap} = 1.52°$).

The broadening of the line is due primarily to "capture" in the polar angle $\Delta\theta_{ap}$, whereas broadening caused by finite capture in the azimuth is much smaller; therefore, a collimator with an increased azimuth aperture can be used to increase the number of collected events.

The yield of Cherenkov photons from 165-MeV/nucleon ions into the aperture ($\Delta\theta_{ap} = 0.76°$, $\Delta\varphi_{ap} = 1.52°$) in the geometry under consideration is estimated as $\Delta N = 0.00174z^2$ photons/ion, where $z$ is the charge number of the ion.

To conclude, we note that monochromatic optical CR from a moderately relativistic ion beam propagating in the CVD-diamond emitter can be detected in the appropriately chosen geometry of the experiment. Since the spectrum of this monochromatic optical CR "carries" information on the energy of the ion beam, this technique can be used as a new approach to spectrometry.

The average energy of the ion beam was measured in [6] using the photometric method based on the directivity of optical CR, but the achieved accuracy was low. The proposed method based on spectral measurements is apparently more accurate.

The reported results can provide a foundation for a new method of measurement of the energy of weakly relativistic ions. The reported calculations will be verified at the Laboratory of High-Energy Physics, Joint Institute for Nuclear Research, where a system will be developed to measure the energy dispersion of ions from carbon to gold with energies up to 4 GeV/nucleon. The corresponding experiments are planned on the beams extracted from the NICA at the MARUSYA setup in the SPD test region, where the necessary infrastructure exists [20, 21].

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## CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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