On the Flow of a Viscous Liquid around a Heated Spheroidal Particle

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Abstract—An expression is derived for the viscous drag of a spheroidal particle whose temperature differs from that of the carrier liquid. Calculations are performed for the case where the temperature dependence of the liquid viscosity may be represented by an exponential power series.

1. PROBLEM STATEMENT

The flow of viscous liquid and gaseous media around heated particles was considered in a number of studies [1–5]. A heated particle is a particle whose average surface temperature significantly exceeds the temperature of its environment. Heating of the particle surface may be due, for example, to a chemical reaction in the bulk, radioactive decay of the particle substance, or external radiation. If a flux of electromagnetic radiation (with the wavelength $\bar{\lambda}$ and intensity I_0) is incident on a particle, the energy absorbed by this particle is equal to $\pi R^2 I_0 K_n$, where R is the major semiaxis of the spheroid and K_n is the absorption factor [6]. If the heat conductance of the particle substance is much higher than the that of the ambient liquid (which is true for most liquids) and $\bar{\lambda} \gg R$, the absorbed energy is uniformly distributed over the particle surface; that is, the particle may be regarded as uniformly heated. A heated particle may exert a significant influence on the thermophysical characteristics of its environment and thus affect the distribution of the velocity and pressure fields in it.

Many particles that may be encountered in nature and industry have a nonspherical surface shape, for example, spheroidal. In this work, we derived (in the Stokes approximation) an analytical expression for the hydrodynamic force acting on a uniformly heated spheroidal particle with allowance for the temperature dependence of viscosity represented in the form of an exponential power series. The expression is valid for an arbitrary temperature drop between the particle surface and the liquid far away from the particle.

Let us consider a liquid flow around a heated oblate spheroid parallel to its symmetry axis at the velocity $\mathbf{U}_\infty(\mathbf{U}_\infty \parallel OZ)$. Let us assume that the densities, heat conductivities, and heat capacities of the liquid and particle are constant, and the heat conductivity coefficient of the particle is much greater than that of the ambient liquid. We also assume that the liquid flows around the spheroid slowly enough (the Reynolds and Peclet numbers are small).

Among all transport parameters of a liquid, only the dynamic viscosity factor significantly depends on the temperature [7]. To make allowance for the temperature dependence of viscosity, let us use expression (1.1), which makes it possible to describe a change in the viscosity within a wide temperature range with any desired accuracy (at $F_n = 0$, this formula may be reduced to the well-known Reynolds equation [7]):

$$\mu_{\text{liq}} = \mu_\infty \left[1 + \sum_{n=1}^{\infty} F_n \left(\frac{T_{\text{liq}}}{T_\infty} - 1\right)^n\right] \times \exp\left\{-A\left(\frac{T_{\text{liq}}}{T_\infty} - 1\right)\right\}. \tag{1.1}$$

Here, A is a constant, $\mu_\infty = \mu_{\text{liq}}(T_\infty)$, and T_∞ is the temperature of the liquid far away from the particle. The subscripts liq and p hereafter refer to the ambient liquid and the particle, respectively.

The viscosity of a liquid is known to exponentially decrease with temperature [7]. Analyzing the known semiempirical formulas, we showed that expression (1.1) provides the best description of viscosity changes within a wide temperature range with any desired accuracy. For example, let us consider water in the temperature range from 0 to 90°C; at the parameter values $A = 5.779$, $F_1 = -2.318$, and $F_2 = 9.118$ ($T_\infty = 273$ K), this formula reproduces the viscosity values with the relative error not exceeding 3%.

The flow of a liquid around a spheroid will be described in a spherical coordinate frame (ε, η, φ) with the origin at the center of the particle. The curvilinear coordinates ε, η, and φ are related to the Cartesian coordinates as follows [8]:

$$x = c\sinh\varepsilon \sin\eta \cos\varphi, \quad y = c\sinh\varepsilon in\eta \sin\varphi, \quad z = c\cosh\varepsilon \cos\eta, \tag{1.2}$$

$$x = c\cosh\varepsilon \sin\eta \cos\varphi, \quad y = c\cosh\varepsilon \sin\eta \sin\varphi, \quad z = c\sinh\varepsilon \cos\eta, \tag{1.3}$$

where $c = \sqrt{b_0^2 - a_0^2}$ in the case of a prolate spheroid [$a_0 < b_0$, formula (1.2)], and $c = \sqrt{a_0^2 - b_0^2}$ in the case of an oblate spheroid [$a_0 > b_0$, formula (1.3)]; a_0 and b_0 are the semiaxes of the spheroid. Here, the z-axis of the Cartesian coordinate system coincides with the symmetry axis of the spheroid.

At small Reynolds numbers, the distributions of the velocity $\mathbf{U}_{\text{liq}}$, pressure P_{liq}, and temperature T_{liq} are determined by the following set of equations [9]:

$$\frac{\partial P_{\text{liq}}}{\partial \varepsilon} = \frac{\partial \sigma_{\varepsilon\varepsilon}}{\partial \varepsilon} + \frac{\partial \sigma_{\varepsilon\eta}}{\partial \eta} + \frac{1}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \varepsilon}(\sigma_{\varepsilon\varepsilon} - \sigma_{\varepsilon\eta})$$
$$+ \frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \varepsilon}(\sigma_{\varepsilon\varepsilon} - \sigma_{\varphi\varphi}) + \left(\frac{2}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \eta} + \frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \eta}\right)\sigma_{\varepsilon\eta}, \tag{1.4a}$$

$$\frac{\partial P_{\text{liq}}}{\partial \eta} = \frac{\partial \sigma_{\varepsilon\eta}}{\partial \varepsilon} + \frac{\partial \sigma_{\eta\eta}}{\partial \eta} - \frac{1}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \eta}(\sigma_{\varepsilon\varepsilon} - \sigma_{\eta\eta})$$
$$+ \frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \eta}(\sigma_{\eta\eta} - \sigma_{\varphi\varphi}) + \left(\frac{2}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \varepsilon} + \frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \varepsilon}\right)\sigma_{\varepsilon\eta}, \tag{1.4b}$$

$$\frac{\partial U_\varepsilon}{\partial \varepsilon} + \left(\frac{1}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \varepsilon} + \frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \varepsilon}\right)U_\varepsilon$$
$$+ \frac{\partial U_\eta}{\partial \eta} + \left(\frac{1}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \eta} + \frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \eta}\right)U_\eta = 0, \tag{1.4c}$$

$$\Delta T_{\text{liq}} = 0, \tag{1.5}$$

where $H_\varepsilon = c\sqrt{\cosh^2\varepsilon - \sin^2\eta}$ and $H_\varphi = c\cosh\varepsilon\sin\eta$ are the Lamé coefficients; and $\sigma_{\varepsilon\varepsilon}$, $\sigma_{\eta\eta}$, $\sigma_{\varphi\varphi}$, and $\sigma_{\varepsilon\eta}$ are components of the stress tensor [8], which assume the following form in the spheroidal coordinate system:

$$\sigma_{\varepsilon\varepsilon} = \frac{2}{H_\varepsilon}\mu_{\text{liq}}\left(\frac{\partial U_\varepsilon}{\partial \varepsilon} + \frac{1}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \eta}U_\eta\right),$$

$$\sigma_{\eta\eta} = \frac{2}{H_\varepsilon}\mu_{\text{liq}}\left(\frac{\partial U_\eta}{\partial \eta} + \frac{1}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \varepsilon}U_\varepsilon\right),$$

$$\sigma_{\varphi\varphi} = \frac{2}{H_\varepsilon}\mu_{\text{liq}}\left(\frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \varepsilon}U_\varepsilon + \frac{1}{H_\varphi}\frac{\partial H_\varphi}{\partial \eta}U_\eta\right),$$

$$\sigma_{\varepsilon\eta} = \frac{1}{H_\varepsilon}\mu_{\text{liq}}\left(\frac{\partial U_\eta}{\partial \varepsilon} + \frac{\partial U_\varepsilon}{\partial \eta} - \frac{1}{H_\varepsilon}\frac{\partial H_\varepsilon}{\partial \varepsilon}U_\eta\right).$$

Solving the set of equations (1.4) and (1.5), let us use the following boundary conditions:

$$\mathbf{U}_{\text{liq}} = 0, \quad T_{\text{liq}} = T_s \quad \text{at} \quad \varepsilon = \varepsilon_0, \tag{1.6}$$

$$\mathbf{U}_{\text{liq}} \longrightarrow U_\infty\cos\eta\,\mathbf{e}_\varepsilon - U_\infty\sin\eta\,\mathbf{e}_\eta,$$
$$T_{\text{liq}} \longrightarrow T_\infty, \quad P_{\text{liq}} \longrightarrow P_\infty \quad \text{at} \quad \varepsilon \longrightarrow \infty. \tag{1.7}$$

Here, T_s is the average temperature at the particle surface, $\mathbf{e}_\varepsilon$ and $\mathbf{e}_\eta$ are the unit vectors of the spheroidal coordinate system, and $U_\infty = |\mathbf{U}_\infty|$.

The boundary conditions (1.6) at the particle surface involve the stick condition for the velocity and the constancy of the particle surface temperature. The particle surface corresponds to the ε value equal to ε_0.

The force exerted by the liquid on the particle is determined according to the formula

$$F_z = \int_S\left(-P_{\text{liq}}\cos\eta + \sigma_{\varepsilon\varepsilon}\cos\eta - \frac{\sinh\varepsilon}{\cosh\varepsilon}\sigma_{\varepsilon\eta}\sin\eta\right)dS, \tag{1.8}$$

where $dS = c^2\cosh^2\varepsilon\,\sin\eta\,d\eta\,d\varphi$ is a differential element of the surface.

Considering the form of the boundary condition (1.7), one should search for the expressions for the normal (U_ε) and tangential (U_η) components of the velocity $\mathbf{U}_{\text{liq}}$ in the form

$$U_\varepsilon(\varepsilon, \eta) = \frac{U_\infty}{c\cosh\varepsilon H_\varepsilon}G(\varepsilon)\cos\eta,$$
$$U_\eta(\varepsilon, \eta) = -\frac{U_\infty}{cH_\varepsilon}g(\varepsilon)\sin\eta, \tag{1.9}$$

where $G(\varepsilon)$ and $g(\varepsilon)$ are the arbitrary functions of the normal coordinate ε.

2. THE VELOCITY FIELD, TEMPERATURE DISTRIBUTION, AND VISCOUS DRAG

To find the force exerted by the liquid on a heated solid spheroidal particle, one should know the temperature field in its vicinity. Integrating Eq. (1.5) with relevant boundary conditions, we obtain

$$t_{\text{liq}} = 1 + \frac{\gamma}{c}a_0\operatorname{arccot}\lambda, \tag{2.1}$$

where $t_{\text{liq}} = T_{\text{liq}}/T_\infty$, $\lambda = \sinh\varepsilon$, $\gamma = \dfrac{1}{\sqrt{1+\lambda_0^2}}\dfrac{t_s - 1}{\operatorname{arccot}\lambda_0}$ is a dimensionless parameter characterizing the heating of the particle surface, $t_s = T_s/T_\infty$, and $\lambda_0 = \left[\left(\dfrac{a_0}{b_0}\right)^2 - 1\right]^{-1/2}$.

With allowance for Eq. (2.1), expression (1.1) assumes the form

$$\mu_{\text{liq}} = \mu_\infty\left[1 + \sum_{n=1}^{\infty}F_n\gamma_0^n(\operatorname{arccot}\lambda)^n\right]$$
$$\times\exp\{-\gamma_0\operatorname{arccot}\lambda\}, \tag{2.2}$$

where $\gamma_0 = \dfrac{A\gamma}{c}a_0$.

Substituting Eq. (1.9) into the continuity equation (1.4c), we find the relationship between the functions $g(\varepsilon)$ and $G(\varepsilon)$:

$$g(\varepsilon) = \frac{1}{2}\frac{dG}{d\lambda}. \tag{2.3}$$

Since the viscosity depends only on the radial coordinate λ, we will search for the solution to the set of hydrodynamic equations (1.4a), (1.4b) with allowance for expressions (1.9) and (2.3) using the method of separation of variable. In particular, we obtain the following expressions for components of the velocity U satisfying the boundary conditions (1.7) at infinity:

$$U_\varepsilon(\varepsilon, \eta) = \frac{U_\infty}{cH_\varepsilon}\cos\eta[c^2 + A_1G_1 + A_2G_2], \tag{2.4}$$

$$U_\eta(\varepsilon, \eta) = -\frac{U_\infty}{cH_\varepsilon}\sin\eta[c^2 + A_1G_3 + A_2G_4], \tag{2.5}$$

where

$$G_1 = -\frac{1}{\lambda^3}\sum_{n=0}^{\infty}\frac{\theta_n^{(1)}}{(n+3)\lambda^n},$$

$$G_2 = -\frac{1}{\lambda}\sum_{n=0}^{\infty}\frac{\theta_n^{(2)}}{(n+1)\lambda^n} - \frac{\beta}{\lambda^3}\sum_{n=0}^{\infty}\frac{\theta_n^{(1)}}{(n+3)\lambda^n}$$

$$\times\left[(n+3)\ln\frac{\lambda_0}{\lambda} - 1\right],$$

$$G_3 = G_1 + \frac{1+\lambda^2}{2\lambda}G_1^I, \quad G_4 = G_2 + \frac{1+\lambda^2}{2\lambda}G_2^I,$$

$$\theta_n^{(1)} = -\frac{1}{n(n+5)}\sum_{k=1}^{n}[(n+4-k)$$

$$\times\{(n+1-k)\alpha_k^{(1)} + \alpha_k^{(2)}\} + \alpha_k^{(3)}]\theta_{n-k}^{(1)} \quad (n \geq 1),$$

$$\theta_n^{(2)} = -\frac{1}{(n-2)(n+3)}\left[\sum_{k=1}^{n}\{(n+2-k)\right.$$

$$\times[(n+1-k)\alpha_k^{(1)} + \alpha_k^{(2)}] + \alpha_k^{(3)}\}\theta_{n-k}^{(2)}$$

$$+\beta\sum_{k=0}^{n}[(2n-2k+3)\alpha_k^{(1)} + \alpha_k^{(1)}]$$

$$\left.\times\theta_{n-k-2}^{(1)} - 6\alpha_n^{(4)}\right] \quad (n \geq 3),$$

$$H_\varepsilon = c\sqrt{\cosh^2\varepsilon - \sin^2\eta},$$

$$\theta_1^{(2)} = -\frac{1}{4}[2(\alpha_1^{(1)} + \alpha_1^{(2)}) + \alpha_1^{(3)} + 6\alpha_1^{(4)}],$$

$$\theta_2^{(2)} = 1, \quad \theta_0^{(1)} = -1, \quad \theta_0^{(2)} = -1,$$

$$\beta = -\frac{1}{5}[\{3(2\alpha_1^{(1)} + \alpha_1^{(2)}) + \alpha_1^{(3)}\}\theta_1^{(2)}$$

$$-2(\alpha_2^{(1)} + \alpha_2^{(2)}) - \alpha_2^{(3)} - 6\alpha_2^{(4)}], \tag{2.6}$$

$$\alpha_n^{(1)} = C_n + 12\sum_{k=0}^{\left[\frac{n-2}{2}\right]}(-1)^k$$

$$\times\frac{C_{n-2k-2}}{(2k+1)(2k+3)(2k+5)}, \quad \Delta_0 = 1,$$

$$\alpha_n^{(2)} = (n-2)C_n - \gamma_0 C_{n-1}$$

$$+12\sum_{k=0}^{\left[\frac{n-2}{2}\right]}(-1)^k\frac{(4k+5)C_{n-2k-2}}{(2k+1)(2k+3)(2k+5)}$$

$$-3\sum_{k=0}^{\left[\frac{n-3}{2}\right]}(-1)^k\frac{1}{(2k+3)(2k+5)}[(n-2k-2)C_{n-2k-2}$$

$$-\gamma_0 C_{n-2k-3} + (n-2k-4)C_{n-2k-4}] \quad (n \geq 1),$$

$$\alpha_n^{(4)} = \Delta_n/n!,$$

$$\alpha_n^{(3)} = -2(n+2)C_n + 2\gamma_0 C_{n-1} - 2(n-2)C_{n-2}$$

$$+12\sum_{k=0}^{\left[\frac{n-2}{2}\right]}(-1)^k\frac{C_{n-2k-2}}{(2k+5)}$$

$$+6\sum_{k=0}^{\left[\frac{n-3}{2}\right]}(-1)^k\frac{(k+2)(4k+5)}{(2k+3)(2k+5)}[(n-2k-2)C_{n-2k-2}$$

$$-\gamma_0 C_{n-2k-3} + (n-2k-4)C_{n-2k-4}] \quad (n \geq 1),$$

$$\Delta_n = \gamma_0\Delta_{n-1} - (n-1)(n-2)\Delta_{n-2} \quad (n \geq 1),$$

$$C_k = \sum_{l_1 + 3l_3 + 5l_5 + \ldots + sl_s = k}\frac{l!}{l_1!l_3!l_5!\ldots l_s!}F_l f_1^{l_1}f_3^{l_3}f_5^{l_5}\ldots f_s^{l_s},$$

$$s = k - \frac{1+(-1)^k}{2},$$

$$l = l_1 + l_3 + l_5 + \ldots + l_s, f_{2k-1} = (-1)^{k-1}\frac{\gamma a_0}{c(2k-1)} \quad (k \geq 1).$$

The expression $\left[\frac{k}{2}\right]$ denotes the integer part of the number $\frac{k}{2}$.

The pressure P_{liq} is determined by the following expression:

$$P_{\text{liq}} = P_\infty + \frac{\mu_{\text{liq}} U_\infty}{c^3}\left\{\frac{1+\lambda^2}{2\lambda}\arctan\frac{x}{\lambda}\frac{d^3G}{d\lambda^3}\right.$$

$$-\frac{1}{2\lambda}\left[\frac{1+\lambda^2}{\lambda^2+x^2}x + \frac{1-\lambda^2}{\lambda}\arctan\frac{x}{\lambda}\right]\frac{d^2G}{d\lambda^2}$$

$$-\frac{1}{\lambda}\arctan\frac{x}{\lambda}\frac{dG}{d\lambda} + \frac{1}{\lambda}\left[\frac{x}{\lambda^2+x^2} + \frac{1}{\lambda}\arctan\frac{x}{\lambda}\right]G$$

$$+\frac{1}{\mu_{\text{liq}}}\frac{d\mu_{\text{liq}}}{d\lambda}\left[\frac{1+\lambda^2}{2\lambda}\arctan\frac{x}{\lambda}\frac{d^2G}{d\lambda^2}\right.$$

$$\left.\left.-\frac{1+\lambda^2}{2\lambda}\left[\frac{x}{\lambda^2+x^2} + \frac{1}{\lambda}\arctan\frac{x}{\lambda}\right]\frac{dG}{d\lambda} + \frac{x}{\lambda^2+x^2}G\right]\right\}.$$

Here, $G(\lambda) = c^2 + A_1G_1 + A_2G_2$, $x = \cos\eta$.

The viscous drag acting on the spheroid is determined by the integration of expression (1.8) over the surface of spheroid; with allowance for Eqs. (2.3) and (2.4), it is equal to

$$\mathbf{F}_z = -4\pi\frac{\mu_\infty U_\infty}{c}A_2\exp\left\{-\frac{A\gamma}{c}a_0\arctan\lambda_0\right\}\mathbf{n}_z, \quad (2.7)$$

where $\mathbf{n}_z$ is the unit vector in the direction of the z-axis.

Note that force (2.7) was calculated under the assumption that the particle motion is uniform, which is possible only if the total force acting on the particle is zero. Since force (2.7) is proportional to the velocity and nullified together with it, the uniform motion of a heated oblate spheroid can be implemented only if we assume the existence of a certain external force that balances force (2.7).

The integration constants A_1 and A_2 in the expressions for the velocity components are determined from the boundary conditions at the surface of the spheroid. After their calculation, expression (2.7) may be represented in the form

$$\mathbf{F}_z = 6\pi a_0\mu_\infty K U_\infty \mathbf{n}_z, \quad (2.8)$$

where

$$K = \frac{2}{3}\frac{G_1^I}{\sqrt{1+\lambda_0^2}[G_2G_1^I - G_1G_2^I]}\exp\left\{-\frac{A\gamma}{c}\arctan\lambda_0\right\},$$

G_1^I and G_2^I are the first derivatives of the corresponding functions with respect to λ.

To obtain the expression for the hydrodynamic drag of a prolate spheroid, one should substitute λ by $i\lambda$ and c by ic in (2.8); here, i is the imaginary unit.

Thus, formula (2.8) makes it possible to estimate the hydrodynamic force acting on a uniformly heated spheroidal particle with allowance for the temperature dependence of viscosity expressed by formula (1.1).

Table 1. The temperature dependences of the coefficient K for different semiaxis ratios of the spheroid

a_0/b_0	T_s, K						
	273	283	303	333	343	353	363
0.73	0.947	0.705	0.393	0.163	0.121	0.089	0.065
0.9	0.980	0.727	0.397	0.158	0.116	0.086	0.062

Table 2. The dependences of the coefficient K on the semiaxis ratio for different temperatures

a_0/b_0	0.71	0.75	0.8	0.85	0.9	0.95	0.99
$K^{(1)}$	0.5822	0.7076	0.7137	0.7201	0.7266	0.7332	0.7386
$K^{(2)}$	0.1451	0.1614	0.1594	0.1585	0.1581	0.1582	0.1585

As an example, Tables 1 and 2 list the results of numerical calculations for the dependence of the coefficient K on the average surface temperature of the spheroid and on its semiaxis ratio. The calculations were conducted for granite particles with the radius $R = 2 \times 10^{-5}$ m suspended in water at $T_\infty = 273$ K; $K^{(1)}$ and $K^{(2)}$ correspond to $T_s = 283$ and 333 K, respectively; $A = 5.779$, $F_n = 0$, and $n \geq 1$. A numerical analysis shows that heating of the spheroid surface significantly affects the magnitude of the resistance force.

In the limit at $\gamma \longrightarrow 0$ (small temperature differences in the vicinity of the spheroid), $G_1 = \frac{1}{3\lambda^3}$, $G_1^I = -\frac{1}{\lambda^4}$, $G_2 = \frac{1}{\lambda}$, $G_2^I = -\frac{1}{\lambda^2}$, and $a_0 = b_0 = R$; the coefficient $K = 1$, and formula (2.8) passes into Stokes' formula for a solid spherical particle of radius R [8].

Let us consider the motion of a spheroidal particle in the gravitational field. A particle falling down in a viscous liquid owing to the force of gravity finally begins to move at a constant velocity, at which the force of gravity is balanced by hydrodynamic forces.

The force of gravity acting on the particle, with allowance for the buoyancy force, is equal to

$$F_g = (\rho_p - \rho_{\text{liq}})g\frac{4}{3}\pi a_0^2 b_0. \quad (2.9)$$

Equating Eqs. (2.8) and (2.9), we obtain an expression for the velocity of steady-state sedimentation of a uniformly heated spheroidal particle:

$$U_\infty = (\rho_p - \rho_{\text{liq}})g\sqrt{1+\lambda_0^2}$$

$$\times\frac{a_0b_0}{3\mu_\infty}\frac{G_1G_2^I - G_2G_1^I}{G_1^I}\exp\left\{\frac{A\gamma}{c}a_0\arctan\lambda_0\right\}.$$

Let us also point to some other problems (associated with the motion of a uniformly heated spheroidal particle) that may be directly solved on the basis of the above results. Let us consider a particle that has uniformly distributed heat sources (sinks) with a constant power density q_p acting inside it. To describe the motion of a particle with a uniform internal heat generation, one should supplement Eqs. (1.4) and (1.5) with an equation describing the temperature distribution inside the particle ($\Delta T_p = -q_p/\lambda_p$) and formulate the boundary conditions so as to consider the equality of temperatures and heat fluxes at the particle surface. The average surface temperature of the spheroid will be described by the following relationship:

$$T_s = T_\infty + \frac{\alpha_0 b_0}{3\lambda_{\text{liq}}} q_p.$$

If the heat sources (sinks) of a constant intensity I_0 are situated at the surface rather than in the bulk of the particle, the corresponding result may be obtained if we formally set

$$q_p = \frac{3}{4b_0} I_0 \left[2 + \frac{b_0^2}{\varepsilon a_0^2} \ln \frac{1+\varepsilon}{1-\varepsilon} \right],$$

where ε is the eccentricity, in all relationships referring to the case of uniform internal heat liberation.

REFERENCES

1. Malai, N.V., *Inzh.-Fiz. Zh.*, 1999, vol. 72, no. 4, p. 651.
2. Kassoy, D.R., Adamson, T.C., and Messiter, A.F., *Phys. Fluids*, 1966, vol. 9, no. 4, p. 671.
3. Naidenov, V.I., *Prikl. Mat. Mekh.*, 1971, vol. 38, no. 1, p. 162.
4. Shchukin, E.R. and Malai, N.V., *Inzh.-Fiz. Zh.*, 1988, vol. 54, no. 4, p. 630.
5. Shchukin, E.R., Malai, N.V., and Yalamov, Yu.I., *Teplofiz. Vys. Temp.*, 1988, vol. 25, no. 5, p. 1020.
6. Bohren, C.F. and Huffman, D.R., *Absorption and Scattering of Light by Small Particles*, New York: Wiley, 1983. Translated under the title *Pogloshchenie i rasseyanie sveta malymi chastitsami*, Moscow: Mir, 1986.
7. Bretsznajder, S., *Wlasnosci gazow i cieczy* (Properties of Gases and Liquids. Engineering Methods of Calculation), Warsaw: Wydawnictwo Naukowo-Techniczne, 1962. Translated under the title *Svoistva gazov i zhidkostei. Inzhenernye metody rascheta*, Leningrad: Khimiya, 1966.
8. Happel, J. and Brenner, H., *Low Reynolds Number Hydrodynamics*, Englewood Cliffs: Prentice-Hall, 1965. Translated under the title *Gidrodinamika pri malykh chislakh Reinol'dsa*, Moscow: Mir, 1976.
9. Landau, L.D. and Lifshitz, E.M., *Mekhanika sploshnykh sred* (Mechanics of Continuous Media), Moscow: Nauka, 1958.