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Spectrum of collimated X-rays emitted from relativistic electrons crossing an aligned crystal

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Abstract

Analytic description of X-ray emission from relativistic electrons crossing a thin aligned crystal is presented. The suggested model takes into account contributions of the transition radiation and coherent bremsstrahlung on atomic strings and has no limitations on the emitting particle energy.

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1. When a relativistic electron moves in a crystal at small angle Ψ relative to a set of parallel atomic strings an intensive emission appears. The yield of this emission (channeling radiation) is formed in the process of coherent scattering of channeling and above-barrier particles by atomic strings. The emitted photon energy range $\omega \geqslant \omega_c = 2\gamma^2\Psi_c/R$ (γ is the Lorentz factor of an emitting particle, Ψ_c is the critical angle of axial channeling, R is the screening radius in Fermi–Thomas atom model), where the channeling radiation spectrum has a maximum is usually the subject of theoretical and experimental studies. On the other hand very intensive emission was observed recently [1] in the small emitted photon energy range $\omega \ll \omega_c$ (emission from 500 MeV electrons crossing Si$\langle 110\rangle$ crystal was observed within the range 30 KeV $< \omega <$ 350 KeV $\ll \omega_c \approx 12$ MeV; the measured yield was more than ordinary bremsstrahlung by a factor of 8). Since the emission yield of channeling electrons is very small in the range $\omega \ll \omega_c$ the observed in the experiment [1] emission must be determined by the contribution of above-barrier particles.

Properties of above-barrier particle emission on a single atomic string have been studied well (see, for example, [2]), but in our case of small photon energies when the emission formation length $l_{\text{coh}} \approx 2\gamma^2/\omega$ exceeds the average electron path between consecutive collisions of emitting electron with atomic strings $\bar{l}_\perp/\Psi$ ($\bar{l}_\perp = 1/\sqrt{n_0 d}$, n_0 is the density of atoms in the crystal, d is the distance between neighbouring atoms in the string) it is necessary to take into account the correlations between emission on different strings.

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An influence of such correlations was considered in [3], where the suppression of nondipole coherent bremsstrahlung due to Landau–Pomeranchuk–Migdal effect was described as well as the suppression of dipole coherent bremsstrahlung due to azimuthal multiple scattering of electrons by atomic strings.

In contrast with [3], where noncollimated emission from relativistic electrons moving in an unbounded crystal was studied, the strongly collimated emission from a crystal with finite thickness is considered in our work. In addition to this we take into account the contribution of transition radiation and its interference with coherent bremsstrahlung. The final formula for emission spectral-angular distribution derived without limitations on emitting electron energy differs very essentially from that describing the ordinary coherent bremsstrahlung on a single atomic string.

2. Let us consider an emission from relativistic electrons crossing a layer of a medium with the thickness L. The general expression for Fourier-transform of emission field in wave-zone behind the layer has been obtained in [4] in the form

$$\mathbf{E}_\omega^{\mathrm{rad}} = \mathbf{A_n}\frac{e^{i\omega r}}{r}, \tag{1a}$$

$$\begin{aligned}\mathbf{A_n} = \frac{e}{\pi}\Bigg[&(\boldsymbol{\Psi}_i - \boldsymbol{\Theta})\left(\frac{1}{\gamma^{-2} + (\boldsymbol{\Psi}_i - \boldsymbol{\Theta})^2} - \frac{1}{\gamma_*^{-2} + (\boldsymbol{\Psi}_i - \boldsymbol{\Theta})^2}\right)\\ &- (\boldsymbol{\Psi}_f - \boldsymbol{\Theta})\left(\frac{1}{\gamma^{-2} + (\boldsymbol{\Psi}_f - \boldsymbol{\Theta})^2} - \frac{1}{\gamma_*^{-2} + (\boldsymbol{\Psi}_f - \boldsymbol{\Theta})^2}\right)\exp\left(\frac{i\omega}{2}\int_0^L dt\,\left(\gamma_*^{-2} + \left(\boldsymbol{\Psi}_t - \boldsymbol{\Theta}^2\right)\right)\right)\\ &- \int_0^L dt\left(\frac{d}{dt}\frac{\boldsymbol{\Psi}_t - \boldsymbol{\Theta}}{\gamma_*^{-2} + (\boldsymbol{\Psi}_t - \boldsymbol{\Theta})^2}\right)\exp\left(\frac{i\omega}{2}\int_0^t d\tau\,\left(\gamma_*^{-2} + (\boldsymbol{\Psi}_\tau - \boldsymbol{\Theta})^2\right)\right)\Bigg],\end{aligned} \tag{1b}$$

where $\gamma_* = \gamma/\sqrt{1 + \gamma^2\omega_0^2/\omega^2}$, ω_0 is the target's plasma frequency, two-dimensional angular variables $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}_t \equiv \boldsymbol{\Psi}(t)$ determine the unit vector $\mathbf{n}$ in the direction of emitted photon observation ($\mathbf{n} = \mathbf{e}_z(1 - \frac{1}{2}\Theta^2) + \boldsymbol{\Theta}$, $\mathbf{e}_z\boldsymbol{\Theta} = 0$) and the emitting particle velocity $\mathbf{v}(t) = \mathbf{e}_z(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\Psi_t^2) + \boldsymbol{\Psi}_t$, $\mathbf{e}_z\boldsymbol{\Psi}_t = 0$, $\mathbf{e}_z$ is the normal to the target's surface, the angles Ψ_i and Ψ_f represent the initial and final values of the emitting electron scattering angle Ψ_t.

It should be noted that the first and second terms in (1b) describe the contribution of transition radiation to total emission yield (these terms tend to zero in the limit $\omega_0 \to 0$). The last term in (1b) corresponds to bremsstrahlung contribution (this term tends to zero if $\mathbf{v}(t) = \mathrm{const}$).

The general formula (1) can be applied to describe both amorphous and crystalline target. Let us consider the last case assuming the axis of atomic strings to be parallel to the axis $\mathbf{e}_z$. As this takes place the scattering angle $\boldsymbol{\Psi}_t$ ends up as an orientation angle between emitting electron velocity and string's axis. In the photon energy range $\omega \ll \omega_c$, or more correctly

$$\frac{\omega R}{2\langle\Psi\rangle}\left(\gamma_*^{-2} + (\langle\boldsymbol{\Psi}\rangle - \boldsymbol{\Theta})^2\right) \ll 1, \tag{2}$$

where $\langle\Psi\rangle$ is the average orientational angle for electron beam, characteristics of photons emitted in the process of electron collision with a single string depend strongly on the values of $\boldsymbol{\Psi}_t$ before and after such collision only. Considering the motion of emitting electrons in the crystal as a series of consecutive collisions with atomic strings and assuming the emitted photon flux to be strongly collimated along the axis $\mathbf{e}_z$ ($\Theta^2 \ll \gamma^{-2}$) one can obtain from

(1b) the following expression for emission amplitude:

$$\mathbf{A_n} \simeq \frac{e}{\pi}\left[\left(\frac{1}{\gamma^{-2}+\Psi^2} - \frac{1}{\gamma_*^{-2}+\Psi^2}\right)\left(\boldsymbol{\Psi}_i - \boldsymbol{\Psi}_f \exp\left(\frac{i\omega}{2}\left(\gamma_*^{-2}+\Psi^2\right)\sum_k \tau_k\right)\right)\right.$$
$$\left. - \frac{1}{\gamma_*^{-2}+\Psi^2}\sum_k (\boldsymbol{\Psi}_k - \boldsymbol{\Psi}_{k-1}) \exp\left(\frac{i\omega}{2}\left(\gamma_*^{-2}+\Psi^2\right)\sum_{j\leqslant k}\tau_j\right)\right], \tag{3}$$

where $\boldsymbol{\Psi}_{k-1}$ and $\boldsymbol{\Psi}_k$ are the values of $\boldsymbol{\Psi}$ before and after kth collision, τ_k is the time of free electron motion between $(k-1)$th and kth collisions.

The unique property of an electron coherent scattering by the average potential of atomic string has been used under transformations of (1b): $\Psi_{k-1}^2 = \Psi_k^2 \equiv \Psi^2$. It is well known that only azimuthal angle χ of the vector $\boldsymbol{\Psi}$ is changed due to electron collision with atomic string. Therefore

$$\boldsymbol{\Psi}_k = \Psi(\mathbf{e}_x \cos\chi_k + \mathbf{e}_y \sin\chi_k), \qquad \chi_k = \chi_i + \sum_{j\leqslant k}\Delta\chi_j, \tag{4}$$

where χ_i is the initial value of χ, $\Delta\chi_j$ is the change of azimuthal angle χ in jth collision.

Formula for emission spectral-angular distribution following from (3)

$$\omega\frac{dN_\Psi}{d\omega\, d^2\Theta} = \left\langle |\mathbf{A_n}|^2\right\rangle = \omega\frac{dN_\Psi^{\mathrm{tr}}}{d\omega\, d^2\Theta} + \omega\frac{dN_\Psi^{\mathrm{cb}}}{d\omega\, d^2\Theta} + \omega\frac{dN_\Psi^{\mathrm{int}}}{d\omega\, d^2\Theta}, \tag{5}$$

where the brackets $\langle\,\rangle$ mean the averaging over τ_j and $\Delta\chi_j$, describes the contribution of electrons with orientation angle Ψ to total emission yield. Averaging over τ_j in (5) is performed with account of independence of different accidental quantities τ_j. The same is true for the accidental quantities $\Delta\chi_j$.

Using the distribution function [3] $f(\tau) = \frac{1}{\bar\tau}\exp(-\tau/\bar\tau)$, $\bar\tau = \bar l_\perp/\Psi$, one can obtain the formula

$$\left\langle \exp\left(\frac{i\omega}{2}\left(\gamma_*^{-2}+\Psi^2\right)\sum_{j=1}^{k}\tau_j\right)\right\rangle = \left(\frac{1}{1-\frac{i\omega}{2}\bar\tau(\gamma_*^{-2}+\Psi^2)}\right)^k. \tag{6}$$

To average the quantity $|\mathbf{A_n}|^2$ over $\Delta\chi_j$ one should use the following formulas

$$\langle\cos\chi_k\rangle = \langle\cos\Delta\chi\rangle^k\cos\chi_i, \qquad \langle\sin\chi_k\rangle = 0,$$
$$\langle\cos\Delta\chi\rangle = \frac{1}{\bar l_\perp}\int_{-\infty}^{\infty} db\cos\bigl(\Delta\chi(b)\bigr), \qquad \Delta\chi = \pi - 2b\int_{\rho_0}^{\infty}\frac{d\rho}{\rho^2\sqrt{1-\frac{b^2}{\rho^2}+\frac{\Psi_c^2}{\Psi^2}f(\rho)}}, \tag{7}$$

where

$$1-\frac{b^2}{\rho_0^2}+\frac{\Psi_c^2}{\Psi^2}f(\rho_0) = 0,$$

b is the impact parameter of electron collision with a string, the string potential is defined as $\varphi(\rho) = \varphi_0 f(\rho)$, $f(0) = 1$, $\Psi_c^2 = (2e\varphi_0)/(m\gamma)$.

3. Let us analyze all terms in the general formula (5) separately. Using (6) and (7) one can obtain for the transition radiation contribution the following result:

$$\omega\frac{dN_\Psi^{\mathrm{tr}}}{d\omega\, d^2\Theta} = 2\frac{e^2}{\pi^2}\Psi^2\left(\frac{1}{\gamma^{-2}+\Psi^2} - \frac{1}{\gamma_*^{-2}+\Psi^2}\right)^2\left[1-\mathrm{Re}\left(\frac{\langle\cos\Delta\chi\rangle}{1-\frac{i\omega}{2}\bar\tau(\gamma_*^{-2}+\Psi^2)}\right)^{L/\bar\tau}\right]. \tag{8}$$

This formula can be reduced to more simple form

$$\omega\frac{dN_{\Psi}^{\mathrm{tr}}}{d\omega\, d^2\Theta} \simeq 2\frac{e^2}{\pi^2}\Psi^2\left(\frac{1}{\gamma^{-2}+\Psi^2}-\frac{1}{\gamma_*^{-2}+\Psi^2}\right)^2\left[1-\langle\cos\Delta\chi\rangle^{L/\bar{\tau}}\cos\frac{\omega L}{2}\left(\gamma_*^{-2}+\Psi^2\right)\right], \tag{9}$$

within the photon energy range

$$\frac{\omega\bar{\tau}}{2}\left(\gamma_*^{-2}+\Psi^2\right) \ll 1, \tag{10}$$

which is best matched to discussed influence of inter-string correlations on the coherent X-ray emission properties. Indeed, the condition (10) means that the emission formation length exceeds the average electron path between two consecutive collisions with atomic strings.

The most interesting result following from (9) consists in the suppression of an interference between transition radiation waves emitted from in and out-surfaces of the target due to emitting electron multiple scattering.

In contrast with (9) the contribution of coherent bremsstrahlung is proportional to the thickness of the crystalline target

$$\omega\frac{dN_{\Psi}^{\mathrm{cb}}}{d\omega\, d^2\Theta} = \frac{e^2}{\pi^2}\frac{\Psi^2}{\gamma_*^{-2}+\Psi^2}\frac{\omega^2}{\omega^2+\omega_*^2}\omega_* L, \tag{11}$$

where the critical frequency ω_* is determined by

$$\omega_* = \frac{4\langle\sin^2(\frac{1}{2}\Delta\chi)\rangle}{\bar{\tau}(\gamma_*^{-2}+\Psi^2)}. \tag{12}$$

Derived expression (11) differs from that describing the coherent bremsstrahlung of relativistic electrons on a single atomic string [2] in many respects. Difference between high-energy limit of (11) ($\omega \gg \omega_*$)

$$\omega\frac{dN_{\Psi}^{\mathrm{cb}}}{d\omega\, d^2\Theta} \to \frac{e^2}{\pi^2}\frac{4\Psi^2\langle\sin^2(\frac{1}{2}\Delta\chi)\rangle}{(\gamma_*^{-2}+\Psi^2)^2}\frac{L}{\bar{\tau}}, \tag{13}$$

and corresponding result of coherent bremsstrahlung theory is caused by off-axis (relative to electron beam axis) collimation of emitted photon flux.

In accordance with (11) coherent bremsstrahlung is suppressed within the range of emitted photon energies $\omega < \gamma\omega_0$ due to well-known Ter-Mikaelian effect. More interesting and unexpected result, following from (11), consists in the suppression of emission yield due to the saturation of emitting electron scattering angle, achieved on the emission formation length. Indeed, emission intensity on the frequency ω is proportional to the square of emitting particle scattering angle, achieved on the formation length $l_{\mathrm{coh}} \approx 2\gamma_*^2/\omega$, divided by l_{coh} [2]. Since l_{coh} increases when decreasing of ω, but the scattering angle is limited by the value 2Ψ (see (4)), emission intensity must be suppressed in small frequency range. The discussed suppression effect can be manifested in the range $\omega < \omega_*$ as it follows formally from (11), but its real observation is possible under condition $\gamma\omega_0 \ll \omega_*$ only, when the influence of Ter-Mikaelian effect is negligible.

The detailed description of discussed effect will be presented in further publications. It is significant that the result (11) is valid for both dipole and nondipole emission processes and consequently it takes into account the Landau–Pomeranchuk–Migdal effect. Thus the manifestation of LMP-effect in aligned crystals may differ very essentially from that taking place for ordinary bremsstrahlung in amorphous medium.

Let us consider now the interference term in the general formula (5). Formula

$$\omega\frac{dN_{\Psi}^{\text{int}}}{d\omega\, d^2\Theta} \simeq 4\frac{e^2}{\pi^2}\frac{\Psi^2}{\gamma_*^{-2}+\Psi^2}\left(\frac{1}{\gamma^{-2}+\Psi^2}-\frac{1}{\gamma_*^{-2}+\Psi^2}\right)\frac{\omega_*^2}{\omega^2+\omega_*^2}$$
$$\times\left[1-\langle\cos\Delta\chi\rangle^{L/\bar{\tau}}\cos\left(\frac{\omega L}{2}\left(\gamma_*^{-2}+\Psi^2\right)\right)+\frac{\omega}{\omega_*}\left\langle\cos^2\left(\frac{1}{2}\Delta\chi\right)\right\rangle\right.$$
$$\left.\times\langle\cos\Delta\chi\rangle^{L/\bar{\tau}}\sin\left(\frac{\omega L}{2}\left(\gamma_*^{-2}+\Psi^2\right)\right)\right] \tag{14}$$

shows a small contribution of such an interference in both low photon energy range $\omega < \gamma\omega_0$, where transition radiation dominates and high energy range $\omega > \omega_*$ where the total emission is determined in the main by the contribution of coherent bremsstrahlung. On the other hand, the contribution of (14) may be essential in the range $\omega \sim \gamma\omega_0$, ω_* for thin enough crystals when the contributions of transition radiation and coherent bremsstrahlung are comparable.

4. Let us concentrate attention on the coherent bremsstrahlung from above-barrier fraction of an electron beam crossing a crystal along a string axis to elucidate the peculiarities of LPM effect manifestation and to estimate the possibility to create an effective X-ray source based on such a kind of emission mechanism. Using the simplest distribution function for emitting electrons $f(\mathbf{\Psi}, t) = \exp[-\Psi^2/(\Psi_0^2+\Psi_s^2 t)]/\pi(\Psi_0^2+\Psi_s^2 t)$, one can obtain the following expression for the strongly collimated emission spectrum

$$\omega\frac{dN^{\text{cb}}}{d\omega\, d^2\Theta} \simeq \frac{2e^4\gamma L_R}{\pi^3\bar{l}_\perp}F(x), \tag{15a}$$

$$F = \int_0^\infty dy\, \frac{y^4\langle\sin^2(\frac{1}{2}\Delta\chi)\rangle}{(1+x^{-2}+y^2)^2+\frac{16\langle\sin^2(\frac{1}{2}\Delta\chi)\rangle^2}{\omega_0^2\bar{l}_\perp^2}\frac{y^2}{x^2}}\left[E_1\left(\frac{y^2}{\gamma^2\Psi_0^2+\gamma^2\Psi_s^2 L}\right)-E_1\left(\frac{y^2}{\gamma^2\Psi_0^2}\right)\right], \tag{15b}$$

where Ψ_0 is the initial angular spread of electron beam, $\Psi_s^2 = 1/\gamma^2 L_{sc}$, $L_{sc} \approx \frac{e^2}{4\pi}L_R$, L_R is the radiation length (the expression (15) takes into account the change of the orientation angle Ψ for emitting electrons due to incoherent multiple scattering of such electrons by crystalline atoms), $x = \omega/\gamma\omega_0$.

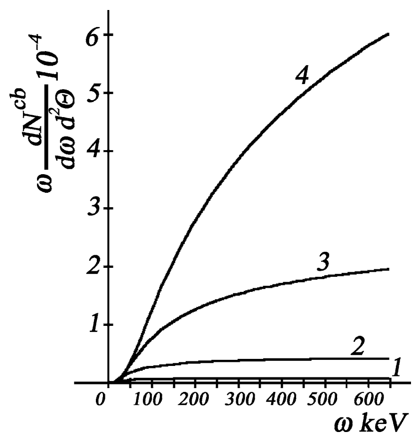

Fig. 1. The spectrum of collimated coherent bremsstrahlung from relativistic electron beam, crossing an aligned Si⟨110⟩ crystal. The curves have been calculated for the electron beam initial angular spread $\Psi_0 = 0.25 \times 10^{-3}$ rad, the electron path in the target $L = 0.523$ mm. The curves 1–4 correspond to the electron energies 250, 500, 1000 and 2000 MeV, respectively.

The function $\omega\, dN^{\mathrm{cb}}/d\omega\, d^2\Theta$, calculated by (15) for Si⟨110⟩ crystal with fixed thickness and different electron beam energies, is presented in Fig. 1. In accordance with presented curves the emission yield increases essentially when increasing of electron beam energy, but this process is attended by the widening of photon energy range, where the discussed suppression effect takes place.

The curve 2 has been calculated for experimental conditions [1]. The theory and data correspond to each other with an accuracy of about 30% within the photon energy range 30 keV $< \omega <$ 150 keV, but the discrepancy increases in range 150 keV $< \omega <$ 360 keV, where the calculated emission density saturates in contrast with obtained data showing the continuing growth of the emission density (it should be noted that statistical errors are large in this range). Such a discrepancy may be connected with the contribution of channeling particle emission in small photon energy range due to incoherent scattering processes.

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References

[1] E.A. Bogomazova, B.N. Kalinin, G.A. Naumenko, D.V. Padalko, A.P. Potylitsyn, A.F. Sharafutdinov, I.E. Vnukov, Nucl. Instrum. Methods B 201 (2003) 276.
[2] A.I. Akhiezer, N.F. Shulga, High Energy Electrodynamics in Matter, Gordon–Breach, Amsterdam, 1996.
[3] N.V. Laskin, A.S. Mazmanishvily, N.N. Nasonov, N.F. Shulga, Sov. Phys. JETP 62 (1985) 438.
[4] N.N. Nasonov, Nucl. Instrum. Methods B 173 (2001) 203.