

# VUV Resonant Transition Radiation from Relativistic Electrons

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**Abstract**—The modified radiator for generation of the resonant transition radiation from relativistic electrons is presented. This radiator consists of a set of small thin foils inclined with respect to the trajectory of an emitting electron. It is shown that the photoabsorption of the emitted photons is almost completely suppressed in the considered scheme. Therefore, the possibility of generating intense beams of quasimonochromatic vacuum ultraviolet (VUV) quanta by the proposed method appears.

## INTRODUCTION

Modern synchrotrons completely solve the problem of sources of quasimonochromatic X-ray beams for structural investigations of material [1]. However, these facilities are complex and expensive. Therefore, the problem of development and creation of alternative radiation sources using compact and cheap electron accelerators with particle energy of the order of 10 MeV, which is hundreds of times smaller than the characteristic electron energy in synchrotrons, is an urgent problem.

The resonant transition radiation (RTR) arising when a fast electron passes through a set of thin foils periodically arranged along the electron trajectory is one of the effective mechanisms for generation of quasimonochromatic photons by relativistic electrons [2, 3]. It should be kept in mind that the RTR mechanisms for the standard scheme of the photon radiation propagation along the electron radiation rate cannot be realized in the vacuum ultraviolet range (tens of eV) due to photoabsorption in the foils. The RTR photoabsorption can be suppressed by a special thin radiator in the form of a nanostructure layer with a period of  $\sim 100$  Å, and a number of layers of the order of several hundred [4]. However, to generate quanta with energy of tens of eV in this radiator, non-relativistic electron beams should be used (consequence of kinematical conservation laws for radiation), in this case, the negative effect of a substrate supporting a nanostructure sharply increases. For the RTR scheme the substrate (a thickness of which, as a rule, significantly exceeds the nanostructure thickness) should be located on the side of the incident electron beam to avoid the emitted photon absorption. In

this case, strong multiple electron scattering in the substrate takes place. The effect of multiple scattering can be eliminated for the scheme of parametric X-ray radiation in the Bragg scattering geometry (a substrate is located on the side of the electron beam leaving a nanostructure and therefore does not affect the radiation process) [5]. Nevertheless, the main disadvantage of the non-relativistic electron radiation for both considered schemes is low intensity.

In the present work, the modified RTR scheme of the relativistic electron, in which, the photoabsorption effect is completely suppressed, is considered. A radiator in this scheme consists of small sized foils obliquely installed along the axis of the electron beam, so that the photons emitted from each element of periodicity leave the system without absorption (Fig. 1). To effectively hold out photons, the large values of the coefficients of photon reflection from the foils are necessary, therefore it is important to underline that the realization of the considered approach is possible, namely in the vacuum ultraviolet range, in which the real part of the dielectric permeability  $\chi'(\omega)$  of a series of materials takes the large negative values of  $(\chi'(\omega) \approx -(0.1-0.4))$  [6] and, as a consequence, the effect of the total external reflection is possible.

The main aim of the work was to obtain simple equations describing the spectral-angular radiation distribution in the wave band where the electron waves emitted from different radiator periods interfere forming a sharp spectral peak. The relativistic scheme of units  $\hbar = c = 1$  is used in the calculations.



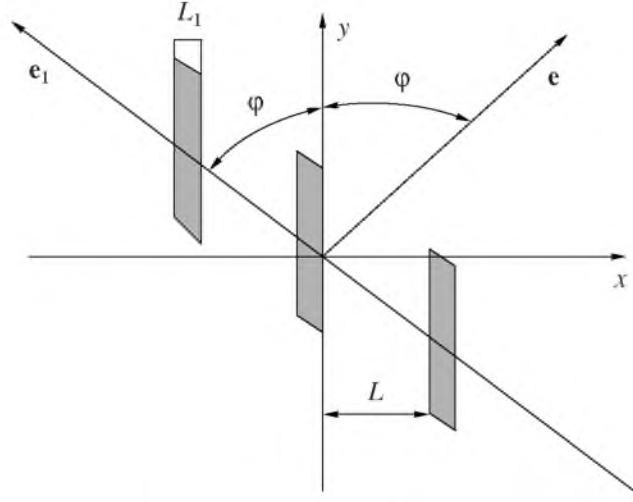


Fig. 1. Principal scheme of radiation source.  $\mathbf{e}_1$  is the moving direction of an emitting electron,  $\mathbf{e}$  is the photon detector axis.

### SPECTRAL-ANGULAR RADIATION DISTRIBUTION

Consider the structure of the electromagnetic field excited by a relativistic electron crossing the system of periodically arranged foils which is shown in Fig. 1. Based on the Maxwell equations, we will determine the Fourier transform of the electric field  $\mathbf{E}_{\omega\mathbf{k}} =$

$$\frac{1}{(2\pi)^4} \int dt d^3r e^{i\omega t - i\mathbf{k}\mathbf{r}} \mathbf{E}(\mathbf{r}, t),$$

and the wave equation for the transversal component of which  $E_{\omega\mathbf{k}}^{\text{tr}} = \sum_{\lambda=1}^2 \mathbf{e}_{\lambda\mathbf{k}} E_{\lambda\mathbf{k}}, \mathbf{k}\mathbf{e}_{\lambda\mathbf{k}} = 0, \mathbf{e}_{\lambda\mathbf{k}}\mathbf{e}_{\lambda'\mathbf{k}} = \delta_{\lambda,\lambda'}$  has a simple form in the space between adjacent foils,

$$(k^2 - \omega^2)E_{\lambda\mathbf{k}} = 4\pi i\omega \mathbf{e}_{\lambda\mathbf{k}} \mathbf{j}_{\omega\mathbf{k}}, \quad (1)$$

where  $\mathbf{j}_{\omega\mathbf{k}}$  is the Fourier transform of the current density of a fast electron. The analogous equation for the field in a foil differs from (1) only by renormalization of  $k^2 - \omega^2 \rightarrow k^2 - \omega^2 \varepsilon(\omega)$  in the left-hand side of the equation, and  $\varepsilon(\omega)$  is the dielectric permeability of the foil material.

Taking into account a strong absorption of the electromagnetic field in a material in the vacuum ultraviolet range we will assume that the thickness of the foil is sufficient for total field absorption. The radiation field is independently formed on each element of the radiator periodicity under the considered conditions (it is important to keep in mind the conservation of specified phase relations between the waves excited by a fast electron on different structure periods). The solution of Eq. (1) for the field excited in the  $l$ th vacuum gap between foils and the solution of the analogous equation for the field in the foils limiting the given gap has the form

$$E_{\lambda\mathbf{k}}^{(1,l)} = c_{\lambda\mathbf{k}}^{(l)} \delta(k_x - \sqrt{\omega^2 \varepsilon(\omega) - k_{\parallel}^2}) + \frac{4\pi i\omega}{k^2 - \omega^2 \varepsilon(\omega)} \mathbf{e}_{\lambda\mathbf{k}} \mathbf{j}_{\omega\mathbf{k}},$$

$$E_{\lambda\mathbf{k}}^{(2,l)} = a_{\lambda\mathbf{k}}^{(l)} \delta(k_x - \sqrt{\omega^2 - k_{\parallel}^2}) + b_{\lambda\mathbf{k}}^{(l)} \delta(k_x + \sqrt{\omega^2 - k_{\parallel}^2}) + \frac{4\pi i\omega}{k^2 - \omega^2} \mathbf{e}_{\lambda\mathbf{k}} \mathbf{j}_{\omega\mathbf{k}}, \quad (2)$$

$$E_{\lambda\mathbf{k}}^{(3,l)} = d_{\lambda\mathbf{k}}^{(l)} \delta(k_x + \sqrt{\omega^2 \varepsilon(\omega) - k_{\parallel}^2}) + \frac{4\pi i\omega}{k^2 - \omega^2 \varepsilon(\omega)} \mathbf{e}_{\lambda\mathbf{k}} \mathbf{j}_{\omega\mathbf{k}},$$

where  $\mathbf{k}_{\parallel} = \mathbf{e}_y \mathbf{k}_y + \mathbf{e}_z \mathbf{k}_z$ ,  $E^{(2,l)}$  is the field in vacuum,  $E^{(1,l)}$  and  $E^{(3,l)}$  are the fields in the foils. The unknown coefficients in (2) are determined from the standard boundary conditions for the field on the foil surface.

It is clear that the electromagnetic waves emitted from one element of the radiator periodicity should propagate approximately in the same direction. The considered condition results in the requirement only of a one-fold wave reflection from the second foil with respect to the course of movement of an emitting electron (Fig. 1). In this case, the radiation field leaving the fixed element of the radiator periodicity we are interested in is described by the expression  $a_{\lambda\mathbf{k}}^{(2,l)} \delta(k_x -$

$\sqrt{\omega^2 - k_{\parallel}^2})$ . It is important to note that the expression  $E^{(2,l)}$  for the field in vacuum (2) should be used for boundary conditions on the first foil with respect to the course of movement of a fast electron to determine the coefficient  $a_{\lambda\mathbf{k}}^{(2,l)}$ . In this expression we should take  $a_{\lambda\mathbf{k}}^{(2,l)} = 0$ . Taking the polarization vectors  $\mathbf{e}_{\lambda\mathbf{k}}$  in the form

$$\mathbf{e}_{1\mathbf{k}} = \frac{[\mathbf{k}_{\parallel}, \mathbf{e}_x]}{k_{\parallel}}, \quad \mathbf{e}_{2\mathbf{k}} = \frac{\mathbf{k}_{\parallel} k_x - \mathbf{e}_x k_{\parallel}^2}{k_{\parallel} k}, \quad (3)$$

we obtain the following expressions for  $a_{\lambda\mathbf{k}}^{(2,l)}$ :

$$\begin{aligned}
a_{1\mathbf{k}_\parallel}^{(1,l)} &= \frac{i\omega e}{2\pi^2|v_x|} (\mathbf{e}_{1\mathbf{k}} \mathbf{v}) \left( \frac{1}{k^2 - \omega^2} - \frac{1}{k^2 - \omega^2 \varepsilon(\omega)} \right) \left[ \frac{k_* - \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}}{\sqrt{\omega^2 - k_\parallel^2} + \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}} \right. \\
&\times \left. \frac{\sqrt{\omega^2 - k_\parallel^2} - \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}}{\sqrt{\omega^2 - k_\parallel^2} + \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}} - \frac{k_* + \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}}{\sqrt{\omega^2 - k_\parallel^2} + \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}} \exp[-i(k_x + \sqrt{\omega^2 - k_\parallel^2})L] \right] \\
&\times \exp[-i(k_x + \sqrt{\omega^2 - k_\parallel^2})Tl + 2i\sqrt{\omega^2 - k_\parallel^2}L], \\
a_{1\mathbf{k}_\parallel}^{(2,l)} &= \frac{i\omega e}{2\pi^2|v_x|} (\mathbf{e}_{2\mathbf{k}} \mathbf{v}) \left( \frac{1}{k^2 - \omega^2} - \frac{1}{k^2 - \omega^2 \varepsilon(\omega)} \right) \left[ \frac{\frac{\omega^2 \varepsilon(\omega)}{k_*^2 + k_\parallel^2} k_* - \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}}{\varepsilon(\omega) \sqrt{\omega^2 - k_\parallel^2} + \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}} \right. \\
&\times \left. \frac{\varepsilon(\omega) \sqrt{\omega^2 - k_\parallel^2} - \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}}{\varepsilon(\omega) \sqrt{\omega^2 - k_\parallel^2} + \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}} - \frac{\frac{\omega^2 \varepsilon(\omega)}{k_*^2 + k_\parallel^2} k_* - \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}}{\varepsilon(\omega) \sqrt{\omega^2 - k_\parallel^2} + \sqrt{\omega^2 \varepsilon(\omega) - k_\parallel^2}} \exp[-i(k_x + \sqrt{\omega^2 - k_\parallel^2})L] \right] \\
&\times \exp[-i(k_x + \sqrt{\omega^2 - k_\parallel^2})Tl + 2i\sqrt{\omega^2 - k_\parallel^2}L],
\end{aligned} \tag{4}$$

where  $L$  is the distance between the adjacent foils,  $T = L + L_1$  is the structure period ( $L_1$  is the foil thickness),  $k_x = k_* = (\omega - \mathbf{k}_\parallel \mathbf{v}_\parallel)/v_x$ ,  $\mathbf{v}$  is the velocity of the emitting electron ( $\mathbf{v} = \mathbf{e}_x v_x + \mathbf{v}_\parallel$ ,  $\mathbf{e}_x \mathbf{v}_\parallel = 0$ ).

To determine the component of the radiation amplitude  $A_{\lambda\mathbf{n}}^{(l)}$  corresponding to the contribution of the  $l$ th radiator period it is necessary to calculate the Fourier integral by the stationary phase method:

$$E_\lambda^{(l)} \int d^3k e^{i\mathbf{k}\mathbf{n}r} a_{\lambda\mathbf{k}}^{(2,l)} \delta(k_x - \sqrt{\omega^2 - k_\parallel^2}) \rightarrow A_{\lambda\mathbf{n}}^{(l)} \frac{e^{i\omega r}}{r}, \tag{5}$$

$$A_{\lambda\mathbf{n}}^{(l)} = -2\pi i \omega n_x a_{\lambda\omega\mathbf{n}_\parallel}^{(2,l)},$$

where  $\mathbf{n}$  is the unit vector in the radiation direction ( $\mathbf{n} = \mathbf{e}_x n_x + \mathbf{n}_\parallel$ ,  $\mathbf{e}_x \mathbf{n}_\parallel = 0$ ). Since the emitted photons in the considered case of the relativistic values of energy for

emitting electrons ( $\gamma = (1 - v^2)^{-\frac{1}{2}} \gg 0$ ) are mainly concentrated in the vicinity of the direction of the specular reflection, Fig. 1, the calculation of amplitude  $A_{\lambda\mathbf{n}}^{(l)}$  following from (4) and (5) can be substantially simplified. Introducing the two-dimensional observation angle  $\theta$  into consideration according to the expression

$$\mathbf{n} = \mathbf{e} \left( 1 - \frac{1}{2} \theta^2 \right) + \theta, \quad \mathbf{e}\theta = 0, \tag{6}$$

and summing the elementary amplitudes  $A_{\lambda\mathbf{n}}^{(l)}$ , we obtain a rather simple expression for the total amplitude  $A_{\lambda\mathbf{n}}$

$$\begin{aligned}
A_{\lambda\mathbf{n}} &\equiv \sum_l A_{\lambda\mathbf{n}}^{(l)} = \frac{e}{\pi} \theta_\lambda R_\lambda \left[ \frac{1}{\gamma^{-2} + \theta^2} + \frac{1}{\gamma^{-2} - \chi + \theta^2} \right] \\
&\times \left[ 1 - \exp\left( \frac{i\omega L}{2 \sin \varphi} (\gamma^{-2} + \theta^2) \right) \right] \\
&\times \frac{1 - \exp\left( \frac{i\omega T}{2 \sin \varphi} (\gamma^{-2} + \theta^2) N_0 \right)}{1 - \exp\left( \frac{i\omega T}{2 \sin \varphi} (\gamma^{-2} + \theta^2) \right)},
\end{aligned} \tag{7}$$

where  $\theta_1 = \theta_\perp$ ,  $\theta_2 = \theta_\parallel$ ,  $N_0$  is the number of periods in a radiator, and the reflection coefficients  $R_\lambda$  are determined by the expressions

$$\begin{aligned}
R_1 &\equiv \frac{\sin \varphi - \sqrt{\chi + \sin^2 \varphi}}{\sin \varphi + \sqrt{\chi + \sin^2 \varphi}}, \\
R_2 &\equiv \frac{(1 + \chi) \sin \varphi - \sqrt{\chi + \sin^2 \varphi}}{(1 + \chi) \sin \varphi + \sqrt{\chi + \sin^2 \varphi}}.
\end{aligned} \tag{8}$$

Expressions (7) and (8) completely determine the properties of the considered radiation. Before consideration of the spectral-angular distribution of emitted

quanta  $\frac{dN}{d\omega d^2\theta} = \sum_{\lambda=1}^2 \left( \frac{1}{\omega} \right) |A_{\lambda\mathbf{n}}|^2$ , consider the dependence of the reflection coefficients  $R_\lambda$  on the quantum energy  $\omega$ , and on the orientation angle of the electron velocity relative to the foil plane  $\varphi$ . It is easily seen that the value of dielectric susceptibility  $\chi \sim 10^{-6}$ ,



and reflection coefficients  $R_\lambda$  in the frequency range typical of X-ray analysis ( $\omega \sim 10$  keV) can be significant only in a very narrow range of reflection angles  $\varphi < 10^{-3}$  (in this case it is necessary to take into account the dependence of  $R_\lambda$  on the observation angle  $\theta$  omitted in (8)). On the other hand, the dielectric susceptibility for a series of materials has large negative values in the frequency range of the order of tens of eV. For example, the real part of carbon susceptibility changes from  $-0.9$  to  $-0.2$  in the range  $30 \text{ eV} \leq \omega \leq 60 \text{ eV}$ , and the molybdenum susceptibility changes from  $-0.45$  to  $-0.2$  in the frequency range  $50 \text{ eV} \leq \omega \leq 80 \text{ eV}$ . In this case, the expression under the root in (8) becomes negative in a wide range of values of the orientation angle  $0 < \varphi < 30^\circ$ , which means an appearance of the effect of total external reflection for the considered material. The indicated circumstance supposes a significant radiation yield in the framework of the considered method.

### DISCUSSION OF THE OBTAINED RESULTS

Analyze the spectral-angular radiation distribution

$$\omega \frac{dN}{d\omega d^2\theta} = \frac{4e^2}{\pi^2} \sum_{\lambda=1}^2 |R_\lambda|^2 \theta_\lambda^2 \left| \frac{1}{\gamma^{-2} + \theta^2} - \frac{1}{\gamma^{-2} - \chi + \theta^2} \right|^2 \times \sin^2 \left[ \frac{\omega L}{4 \sin \varphi} (\gamma^{-2} + \theta^2) \right] \frac{\sin^2 \left[ \frac{\omega T}{4 \sin \varphi} (\gamma^{-2} + \theta^2) N_0 \right]}{\sin^2 \left[ \frac{\omega T}{4 \sin \varphi} (\gamma^{-2} + \theta^2) \right]}, \quad (9)$$

following immediately from (7). According to the above considerations the radiation yield can be substantial for the fixed value of the orientation angle  $\varphi$  only in the frequency range  $\omega$  in which the condition of the total external reflection of the electromagnetic field from a foil  $|\chi(\omega)| > \sin^2 \varphi$  is performed (in this case  $|R_\lambda| \approx 1$ ). Note also the possibility of simplifying substantially the characteristic factor  $\left| \frac{1}{\gamma^{-2} + \theta^2} - \frac{1}{\gamma^{-2} - \chi + \theta^2} \right|^2$  describing the transition radiation at the vacuum-foil interface. Actually, in the considered case of rather large values of the susceptibility modulus  $\chi(\omega)$  of an order of several tenths, it is easy to satisfy the condition  $\gamma^{-2} \ll |\chi|$  (this condition is already valid for the energy of the emitting electron of the order of several MeV). Since the transition radiation is concentrated within a narrow angular cone with the opening angle  $\gamma^{-1}$ , the considered factor can be replaced by the simpler one  $(\gamma^{-2} + \theta^2)^{-2}$ . Then the last factor in (9) describing the interference summation of elementary waves emitted by a fast electron from different radiator periods shows that the considered radiation in the specified direction is a set of coherent peaks with the width of the relative line  $N_0 \gg 1$  when  $\Delta\omega/\omega \sim 1/N_0 \ll 1$ . Taking into account this circumstance, Eq. (9) can be easily integrated with respect to angles, and the radiation

spectrum recorded by a detector is obtained; the detector axis coincides with  $\mathbf{e}$  ((6) and Fig. 1) and the angular size is  $\theta_d$

$$\omega \frac{dN}{d\omega} = \frac{4e^2}{\pi^2} N_0 \left( 1 - \frac{\omega}{\omega_n} \right) \frac{1}{n} \sin^2 \left( \pi \frac{L_1}{T} n \right) \sigma(\omega_n - \omega) \times \sigma \left( \omega - \frac{\omega_n}{1 + \gamma^2 \theta_d^2} \right), \quad (10)$$

where  $\omega_n = \frac{4\pi\gamma^2 n \sin \varphi}{T} = \Omega_0 n$ ,  $L_1$  is the foil thickness,

$\sigma(x) = 1$  for  $x > 0$ , and  $\sigma(x) = 0$  for  $x < 0$ . Take note of some of the features of the radiation spectrum following from (10). The maximum of the spectral density for the  $n$ th peak and its position in the radiation spectrum substantially depend on the collimator angular size  $\theta_d$ . In addition, it should be kept in mind that the dependence of the intensity of the coherent peak on the harmonic number  $n$  generally is not a monotonously decreasing one (first of all, the monotonous character is observed for small values of the parameter  $L_1/T$ ). The considered properties of the coherent peak follow immediately from the expressions

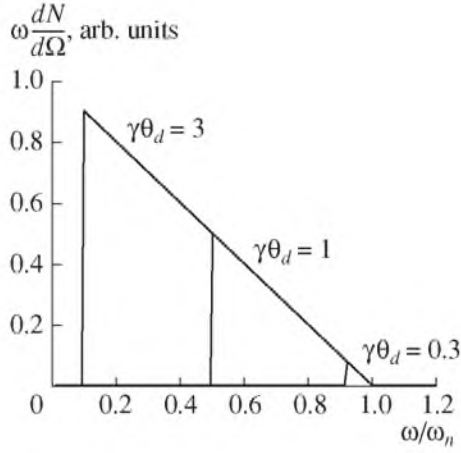
$$\left( \omega \frac{dN_n}{d\omega} \right)_{\max} = \frac{4e^2}{\pi^2} N_0 \frac{\gamma^2 \theta_d^2}{1 + \gamma^2 \theta_d^2} \pi \frac{L_1}{T} F(\eta_n), \quad (11)$$

$$\eta_n = \pi \frac{L_1}{T} n, \quad F(\eta_n) = \frac{\sin^2(\eta_n)}{\eta_n}, \quad \omega_{n \max} = \frac{\omega_n}{1 + \gamma^2 \theta_d^2}$$

and from the spectral dependence curves for the assigned peak (10) plotted in Fig. 2 for different values of the parameter  $\gamma\theta_d$ .

A sharp difference between the character of the effect of the size of the photon collimator on the spectral properties for the considered radiation and the standard RTR is caused by the substantial difference in the conditions of the interference summation of elementary waves in the framework of the considered radiation mechanisms. If the conditions for longitudinal coherence (along the axis of an emitted photon beam) are identical for both radiation mechanisms, then the conditions for transverse coherence substantially differ. For the usual RTR the transverse coherence is automatically provided, since the axis of an emitted photon beam coincides with the axis of an emitting electron beam. On the other hand, the elements of radiator periodicity in the proposed method of radiation generation are shifted relative to each other in the transverse direction; therefore, the properties of the resulting radiation substantially depend on the degree of overlap of the wave fields emitted from different radiator periods. The spectrum of waves measured on the assigned periodicity element of a radiator is broadened towards the frequencies  $\omega$  being smaller than  $\omega_n$  when the size of the angular collimator increases. For the usual RTR the





**Fig. 2.** Dependence of the spectral distribution of the RTR coherent peak on the photon collimator size for different values of the parameter  $\gamma\theta_d$  indicated in the figure.

best conditions for coherent summation are realized for the waves propagating along the axis of electron beam coinciding with the collimator axis and having the frequencies  $\omega$  close to  $\omega_n$ . In this case, the spectral maxima of coherent RTR peaks should be realized without

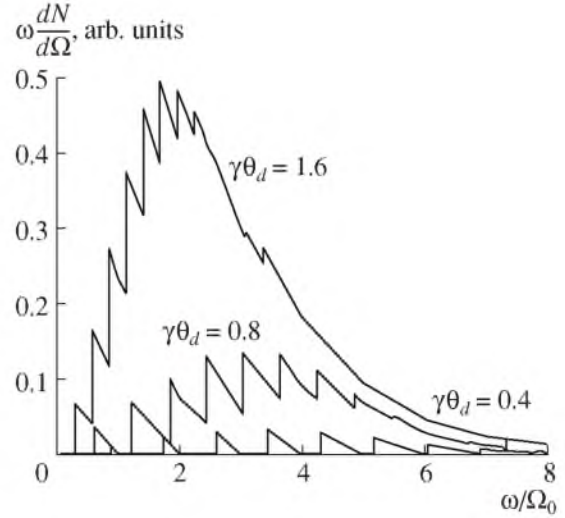
taking into account the dispersion  $\chi(\omega) \approx -\frac{\omega_0^2}{\omega^2}$  near the

frequencies  $\omega = \omega_n$  (dispersion substantially changes a spectrum). On the other hand, a degree of overlap in the proposed method increases for elementary waves emitted at the angle of the photon collimator axis, therefore the radiation intensity increases according to (11) with an increase in the of the collimator angular size and the spectral maximum of the coherent peak shifts to a small frequency range.

Note one more important property of the spectral distribution (1) determined by the angular size of the photon collimator  $\theta_d$  (more precisely by the value of the parameter  $\gamma\theta_d$ ). It is easy to ascertain that the spectral distributions of neighboring harmonics corresponding to the numbers  $n$  and  $n-1$  do not overlap when the following condition is fulfilled:

$$\gamma^2\theta_d^2 < \frac{1}{n-1}. \quad (12)$$

According to (12) all harmonics overlap for the weakly collimated radiation ( $\gamma\theta_d > 1$ ). Thus, to obtain the spectrally narrow X-ray peaks it is necessary to collimate an emitted photon flux which is related to a decrease in the radiation intensity. The radiation spectrum calculated by Eq. (10) for the fixed values of the parameter  $\Omega_0$  and the ratio  $L_1/T$  for the different values of the parameter  $\gamma\theta_d$  is shown in Fig. 3. It should be kept in mind that the number of harmonics contributing into the emitted photon spectrum is limited by the condition  $|\chi(\Omega_0 n)| > \sin^2\varphi$ .



**Fig. 3.** Dependence of the RTR spectrum on the photon collimator size. The curves were calculated for the value of the parameter  $L_1/T = 0.1$  and different values of the parameter  $\gamma\theta_d$  indicated in the figure.

Coming back to (11), note that the maximum of function  $F(\eta_n) \approx 0.72$  is obtained for the value of argument  $\eta_n \approx 1.17$ , therefore the optimal value of the harmonic number  $n_{opt}$  is an integer being nearest to the value  $1.17T/\pi L_1$ . It should be remembered that Eqs. (10) and (11) are valid when  $|\chi(\omega)| > \varphi^2$ . This condition limits the frequency range  $\omega$  from above, in which the effective generation is possible.

Integration of distribution (10) with respect to the frequency  $\omega$  allows us to determine a number of quanta  $N_n$  emitted in the  $n$ th coherent peak

$$N_n = \frac{4e^2 L_1}{\pi T} N_0 F(\eta_n) \left[ \ln(1 + \gamma^2\theta_d^2) - \frac{\gamma^2\theta_d^2}{1 + \gamma^2\theta_d^2} \right]. \quad (13)$$

Eq. (13) predicts the possibility of creating quasi-monochromatic radiation sources with a very high intensity. For example, at  $N_0 \approx 100$ ,  $L_1/T \approx 0.1$ ,  $\gamma\theta_d \approx 3$ , and  $F(\eta_n) \approx 0.72$ , a number of emitted quanta reaches the value of 0.66, which exceeds the intensity of soft X-ray sources based on the electron Cherenkov radiation in the vicinity of photoabsorption edges for a series of materials [7] and considered now as one of the most intense sources by almost two orders of magnitude.

We also want to note the constructive features of a reactor used in the proposed method of soft X-ray generation. We assume that the radiator cross section is completely filled with emitting electrons. In this case, the electromagnetic waves emitted from the surface of the second foil with respect to the course of movement of an emitting electron will not come to the first of two foils composing the element of radiator periodicity when the condition

$$2L = L_{\perp} \tan\varphi, \quad (14)$$



is fulfilled which connects the distance between foils  $L$  with the transverse foil size  $L_{\perp}$  shown in Fig. 1. The analysis shows that radiators with the period  $T \approx L$  of an order of tens of micrometers are required for generation of quasimonochromatic quanta in the vacuum ultraviolet range by the proposed method using 10–30 MeV electron beams. According to (14) the foil transverse size  $L_{\perp}$  shown in Fig. 1 and reaching the value of hundreds of micrometers corresponds to these periods. In this case the radiator transverse size proves to be smaller (tens of micrometers).

Thus, to realize the proposed method in practice it is necessary to solve the complex problem of the formation of an electron beam with a very small transverse size. One can propose the alternative approach based on the use of a cyclic accelerator with a radiator as an internal target. In this case, it is possible to gradually direct an electron beam to a target–radiator varying magnetic field and not losing the beam electron.

### CONCLUSIONS

The analysis performed in the present work shows the possibility in principle to create an intense source of quasimonochromatic quanta in the vacuum ultraviolet range with the yield of  $5 \times 10^{-2}$  photons per electron based on the modified scheme of resonant transition radiation.

The direction of radiation from a periodical radiator to the velocity of an emitting electron at a large angle is the distinctive feature of the scheme. It results in the effective suppression of photoabsorption, the main limiting factor in the resonant transition radiation in the low frequency range. It is important that the conditions of total external reflection of photons from foils composing a radiator, which is necessary to realize the considered radiation mechanism, can be performed for the foils made of a material with a small atomic number (for example graphite). This circumstance is very

important from the viewpoint of a decrease in the negative effect of multiple electrons scattering in a radiator.

Construction simplicity and the possibility of using electron beams with a relatively small energy of 10 MeV are advantages of the considered source.

The substantial disadvantage is the necessity to form an electron beam with a small cross section of an order of tens of micrometers. The alternative approach can be based on the use of a cyclic accelerator with a radiator as an internal target. In this case, the gradual direction of the beam to a target–radiator varying magnetic field helps to avoid electron losses.

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### REFERENCES

1. G. V. Fetisov, *Synchrotron Radiation: Techniques for Analyzing Materials Structure* (Fizmatlit, Moscow, 2007) [in Russian].
2. M. L. Ter-Mikaelyan, *Influence of the Medium on Electromagnetic Processes at High Energies* (AN Arm. SSR, Yerevan, 1969) [in Russian].
3. G. M. Garibyan and Yan Shi, *X-ray Transition Radiation* (AN Arm. SSR, Yerevan, 1983) [in Russian].
4. N. N. Nasonov, V. V. Kaplin, S. R. Uglov, et al., *Phys. Rev. E* **68**, 036 504 (2003).
5. N. N. Nasonov, A. S. Kubankin, P. N. Zhukova, et al., *Nucl. Instrum. Methods Phys. Res. B* **254**, 259 (2007).
6. B. L. Henke, E. M. Gullikson, and J. C. Davis, *At. Data Nucl. Data Tables* **54** (1993).
7. W. Knulst, O. J. Luiten, M. J. Van der Wiel, and J. Verhoeven, *Appl. Phys. Lett.* **79**, 2099 (2001).