On the Eikonal Approximation in the Theory of Transition Radiation

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Abstract—The transition radiation of relativistic electrons in nonuniform media is considered. Based on the equivalent photon method and the eikonal approximation in wave mechanics, a method for describing this process is proposed. For the case in which the permittivity depends on several coordinates, equations for the spectral-angular density of transition radiation are obtained. The main results obtained in the Born and eikonal approximations of the theory of transition radiation are compared. The equations obtained are used to analyze the transition radiation process for a fiberlike target.

INTRODUCTION

Transition radiation arises when a charged particle crosses the interface between two media with different dielectric properties [1–3]. As a rule, this process is described by joining the fields (generated by this particle in the medium) at the interface. However, such an approach to the description of transition radiation can be developed only for media with interfaces of the simplest shapes, namely, flat, spherical, or cylindrical ones [1–4]. In addition, it is usually assumed that the permittivity of each medium is constant. For media with boundaries of complicated or smeared shapes, other approaches to the description of this process must be developed.

One of such approaches for ultrarelativistic particles in the high-frequency range uses the Born expansion of the radiation amplitude in terms of the small deviation of the permittivity from unity [2, 3]. In the first order of the perturbation theory in this parameter, the radiation amplitude is determined by the Fourier component of the spatial electron density distribution in the medium. However, as the frequency of the radiated photon decreases, the applicability condition for this expansion soon becomes violated. In the problem under consideration, it is therefore necessary to use methods that can be used beyond the limits of the Born perturbation theory.

In this paper, we study whether it is possible to use the eikonal approximation to describe the process of transition radiation of relativistic electrons in a medium with nonuniform permittivity. An approximate method for describing a transition radiation process uses a representation of the particle field in the form of a localized packet of free electromagnetic waves and the eikonal approximation to describe the scattering process of this packet by the nonuniformities of the medium permittivity.

At first, we derive general equations for the spectral-angular radiation density of a relativistic electron traveling with a constant velocity in a medium with a nonuniform permittivity and consider the limiting case for these equations corresponding to the first approximation of the Born perturbation theory in the small deviation of the permittivity from unity. Then, we discuss whether it is possible to consider this problem by using the eikonal approximation. The obtained equations are used in the problems of transition radiation in thin layers of materials and for a fiberlike target. We obtain conditions under which it is possible to study these processes by the use of the eikonal approximation. In this paper, we also compare the main characteristics of transition radiation and bremsstrahlung in material thin layers. In this case, emphasis is on the conditions under which the transition mechanism contributes mainly to radiation.

PROCEDURE

Let us consider a particle with charge e moving with a constant velocity $\mathbf{v}$ in a nonuniform medium with permittivity $\varepsilon_\omega(\mathbf{r})$. In this case, the Fourier component of the electric field $\mathbf{E}_\omega(\mathbf{r}) = \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}, t)\exp(i\omega t)dt$ generated by a propagating particle in a target medium satisfies the equations [2]

$$(\Delta + \omega^2\varepsilon_\omega)\mathbf{E}_\omega = \nabla(\nabla\mathbf{E}_\omega) - 4\pi e i\omega\delta(\boldsymbol{\rho})\exp(i\omega z/v)\mathbf{v}/v, \quad (1)$$

$$\mathrm{div}\,\varepsilon_\omega \mathbf{E}_\omega(\mathbf{r}) = 4\pi e\delta(\boldsymbol{\rho})\exp(i\omega z/v) \quad (2)$$

(we use the system of units in which the velocity of light c is taken as unity).

We can show [5] that the spectral-angular radiation density can be described by

$$\frac{dE}{d\omega do} = \frac{\omega^2}{(8\pi^2)^2}|\mathbf{k}\times\mathbf{I}|^2, \quad (3)$$

where ω and $\mathbf{k}$ are the frequency and the wave vector of the radiated wave, respectively,

$$\mathbf{I} = \int d^3 r \exp(-i\mathbf{k}\mathbf{r})(1-\varepsilon_\omega(\mathbf{r}))\mathbf{E}_\omega(\mathbf{r}). \quad (4)$$

Let us consider the transition radiation in the high-frequency range, where the target permittivity is determined by the relationship

$$\varepsilon_\omega(\mathbf{r}) \approx 1 - \omega_p^2/\omega^2, \quad \omega \gg \omega_p, \quad (5)$$

where $\omega_p = \sqrt{4\pi e^2 n(\mathbf{r})/m}$ is the plasma frequency, m and e are the electron mass and charge, and $n(\mathbf{r})$ is the electron density in the target. In this case, the solution of Eqs. (1) and (2) can be sought as an expansion in terms of the small quantity $(1-\varepsilon_\omega)$. In the first order of this expansion (which corresponds to the first Born approximation), the solution of Eqs. (1) and (2) is the Coulomb field of the particle in a vacuum:

$$\mathbf{E}_\omega^{(C)}(\mathbf{r}) = \frac{2e\omega}{v^2\gamma}\exp(i\omega z/v) \times\left\{\frac{\boldsymbol{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right) - i\frac{\mathbf{v}}{v}\frac{1}{\gamma}K_0\left(\frac{\omega\rho}{v\gamma}\right)\right\}, \quad (6)$$

where $K_n(x)$ is the modified Bessel function of the third kind (the Macdonald function) and $\gamma = (1-v^2)^{-1/2}$ is the Lorentz factor of the particle. Thus, expression (4) with the substitution $\mathbf{E}_\omega(\mathbf{r}) = \mathbf{E}_\omega^{(C)}(\mathbf{r})$ corresponds to the Born approximation in the theory of transition radiation. It can be easily seen that the typical values of the transverse (normal to $\mathbf{v}$) component of the intensity of the Coulomb field of the relativistic particle exceed those of the longitudinal component by a factor γ. For the spectral-angular density of the transition radiation, we can therefore restrict ourselves only to the transverse component of the vector $\mathbf{I}$ up to the terms of the order of γ^{-2} in expression (3). In this case, we have

$$\frac{dE}{d\omega do} = \frac{\omega^4}{(8\pi^2)^2}|\mathbf{I}_\perp^{(B)}|^2, \qquad \mathbf{I}_\perp^{(B)} = \int d^3 r\exp(-i\mathbf{k}\mathbf{r})(1-\varepsilon_\omega(\mathbf{r}))\mathbf{E}_\omega^{(C)}(\mathbf{r})_\perp. \quad (7)$$

Using the Born approximation in the theory of transition radiation, one can study radiation for targets of sufficiently complicated shapes. Thus, the authors of [5, 7, 8] studied radiation for fiberlike targets, nanotubes, and capillaries in a continuous medium. Here, having in mind the subsequent comparison of the results (obtained in the Born and eikonal approximations) for the spectral-angular radiation density and the applicability limits for these approximations, we consider the simplest problem of transition radiation for the normal impact of a particle on a uniform plane-parallel plate of thickness L. The Born approximation in this problem yields the following result:

$$\mathbf{I}_\perp^{(B)} = -\frac{4\pi e}{v}\frac{\omega_p^2/\omega^2}{\frac{\omega}{v}-k_z}\frac{\mathbf{k}_\perp}{k_\perp^2+\left(\frac{\omega}{v\gamma}\right)^2} \times\left\{\exp\left[i\left(\frac{\omega}{v}-k_z\right)L\right]-1\right\}, \quad (8)$$

$$\frac{dE}{d\omega do} = 2\frac{e^2}{\pi^2}\left(\frac{\omega_p^2}{\omega^2}\right)^2\frac{\theta^2}{(\theta^2+\gamma^{-2})^4} \times\left\{1-\cos\left[(\theta^2+\gamma^{-2})\frac{\omega L}{2}\right]\right\}. \quad (9)$$

As we can see below, the Born approximation in the theory of transition radiation is valid under the following condition:

$$\omega_p^2 l/\omega \ll 1, \quad (10)$$

where l is the length of the region in which the particle interacts with the permittivity nonuniformity. This inequality becomes invalid with increasing l or decreasing ω; and therefore, it becomes necessary to study the transition radiation process outside the applicability range of the Born approximation. The eikonal approximation is one of the methods that can be used beyond the scope of the Born perturbation theory.

However, a direct application of the eikonal approximation to the theory of transition radiation raises difficulties related to the facts that Eq. (1) for the electric field contains the current density of the particle (the last term in Eq. (1)) and that the problem becomes multidimensional in the case of a target of sufficiently complicated geometry (for example, the transition radiation problem for a fiberlike target is two-dimensional).

The authors of [2, 9] tried to overcome these difficulties by constructing the Green's function for Eq. (1). The author of [2] constructed a semiclassical Green's function for Eq. (1) to fit the one-dimensional problem of the transition radiation of relativistic particles in a medium with a nonuniform permittivity. However, the one-dimensional character of the problem was used in the method described in [2].

The authors of [9] proposed a method for constructing Green's function for Eq. (1) with the particle current in the case of multidimensional problems, and this method is valid under certain conditions similar to the applicability conditions for the eikonal approximation in quantum mechanics. Although, using this method, one can go beyond the scope of the Born theory of transition radiation for multidimensional problems, it nevertheless turns out to be sufficiently complicated and cumbersome in certain particular problems. This method was only used in calculations of the energy lost by a relativistic charged particle crossing a plate in a random medium.

In this paper, we develop the theory of transition radiation based on the equivalent photon method [10]. Within this theory, the transition radiation process is regarded as a scattering process of the particle electromagnetic field by the nonuniformities of the medium permittivity; in this case, the particle field is represented as a wave packet of free electromagnetic waves. Using such an approach to the transition radiation problem, one can pass from the solution of Eq. (1) with a particle current to the following equation describing the evolution of a wave packet in a nonuniform medium:

$$(\Delta + \omega^2)\mathbf{E}_\omega = \nabla(\nabla\mathbf{E}_\omega) + \omega^2(1-\varepsilon_\omega)\mathbf{E}_\omega. \tag{11}$$

In this case, the initial state of the wave packet (before it enters the medium) is a packet obtained by expanding the particle eigenfield in terms of the set of free electromagnetic waves. This leads to the construction of a solution of Eq. (11) in the eikonal approximation for the multidimensional case by using simple methods.

Taking the foregoing into account, we seek a solution of Eq. (11) in the form

$$\mathbf{E}_\omega(\mathbf{r}) = \exp(i\omega z)\mathbf{\Phi}(\mathbf{r}). \tag{12}$$

We assume that the function $\mathbf{\Phi}(\mathbf{r})$ varies sufficiently slowly in the space so that, to find it, one can neglect the second derivatives in Eq. (11). In this approximation, equation for $\mathbf{\Phi}(\mathbf{r})$ becomes

$$\begin{aligned}2i\frac{\partial\mathbf{\Phi}}{\partial z} &= -\omega\Phi_z\mathbf{e}_z + i\nabla\Phi_z \\ &+ i\mathbf{e}_z\mathrm{div}\mathbf{\Phi} + \omega(1-\varepsilon_\omega)\mathbf{\Phi}.\end{aligned} \tag{13}$$

Separating the longitudinal and transverse components, we have

$$\frac{\partial\Phi_x}{\partial x} + \frac{\partial\Phi_y}{\partial y} = -i\omega\varepsilon_\omega\Phi_z, \tag{14}$$

$$2i\frac{\partial\mathbf{\Phi}_\perp}{\partial z} = \omega(1-\varepsilon_\omega)\mathbf{\Phi}_\perp + i\nabla_\perp\Phi_z. \tag{15}$$

Substituting Φ_z from the first equation into the second one, we obtain

$$2i\frac{\partial\mathbf{\Phi}_\perp}{\partial z} = \omega(1-\varepsilon_\omega)\mathbf{\Phi}_\perp - \nabla_\perp\frac{1}{\omega\varepsilon_\omega}\left(\frac{\partial\Phi_x}{\partial x} + \frac{\partial\Phi_y}{\partial y}\right). \tag{16}$$

When deriving Eq. (13), we neglected the second derivatives of the function $\mathbf{\Phi}(\mathbf{r})$. The second term on the right-hand side of Eq. (16) can be neglected with the same accuracy. In this case, it is assumed that $\varepsilon_\omega(\mathbf{r})$ also varies sufficiently slowly in the transverse direction so that the derivative $\nabla_\perp(1/\varepsilon_\omega)$ can be neglected. By using the relations

$$\omega(1-\varepsilon_\omega)|\mathbf{\Phi}_\perp| \sim \omega\frac{\omega_p^2}{\omega^2}|\mathbf{\Phi}_\perp|,$$

$$\left|\nabla_\perp\frac{1}{\omega\varepsilon_\omega}\left(\frac{\partial\Phi_x}{\partial x} + \frac{\partial\Phi_y}{\partial y}\right)\right| \sim \frac{1}{\omega\rho_{\mathrm{eff}}^2}|\mathbf{\Phi}_\perp|,$$

where ρ_{eff} is the typical distance in the transverse direction along which the quantity $|\mathbf{\Phi}_\perp|$ varies significantly, we estimate the first and second terms on the right-hand side of Eq. (16) and obtain the following condition determining the smallness of the second term on the right-hand side of Eq. (16) compared to the first one:

$$\rho_{\mathrm{eff}} \gg \omega_p^{-1}. \tag{17}$$

If this condition is satisfied, Eq. (16) becomes

$$\frac{\partial\mathbf{\Phi}_\perp}{\partial z} = -i\frac{\omega}{2}(1-\varepsilon_\omega)\mathbf{\Phi}_\perp. \tag{18}$$

Inserting the solution of the last equation into (12), we obtain

$$\mathbf{E}_\omega(\mathbf{r})_\perp = \mathbf{E}_\omega^{(0)}(\mathbf{r})_\perp\exp\left\{-i\frac{\omega}{2}\int_{-\infty}^{z}(1-\varepsilon_\omega(\mathbf{r}))dz\right\}, \tag{19}$$

where $\mathbf{E}_\omega^{(0)}(\mathbf{r})_\perp$ is the field of the incident wave packet

$$\mathbf{E}_\omega^{(0)}(\mathbf{r})_\perp = \frac{2e\omega}{v^2\gamma}\exp(i\omega z)\frac{\boldsymbol{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right). \tag{20}$$

In order that condition (17) be satisfied, it is necessary that typical transverse distances along which $\varepsilon_\omega(\mathbf{r})$ varies considerably be at least equal to ρ_{eff}. Thus, our constructed solution is valid only in the case of a target with sufficiently smeared boundaries.

Substituting (19) into (4), we obtain

$$\begin{aligned}\mathbf{I}_\perp &= \frac{2e\omega}{v^2\gamma}\int d^3r\exp(i(\omega-k_z)z - i\mathbf{k}_\perp\boldsymbol{\rho})(1-\varepsilon_\omega(\mathbf{r})) \\ &\times\frac{\boldsymbol{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right)\exp\left\{-i\frac{\omega}{2}\int_{-\infty}^{z}(1-\varepsilon_\omega(\mathbf{r}))dz\right\}.\end{aligned} \tag{21}$$

The order of magnitude of the typical values of the distances ρ_{eff} mainly contributing to integral (21) is

$$\rho_{\mathrm{eff}} \sim \min(\gamma/\omega, 1/k_\perp),$$

where $k_\perp \approx \omega\theta$ and θ is the observation angle for radiation ($\theta \ll 1$). According to Eq. (17), Eq. (21) is therefore valid in the frequency range ω and for radiation angles determined by the inequalities

$$1 \gg \omega_p/\omega \gg \gamma^{-1}, \quad \omega_p/\omega \gg \theta. \tag{22}$$

For small radiation angles, the order of smallness of the exponent of the first exponential in (21) can be estimated as $\omega\theta^2 l/2$, where l is the target thickness along the particle velocity direction. Then, if the condition

$$\omega\theta^2 l/2 \ll 1 \tag{23}$$

is satisfied, the first exponential in (21) can be replaced by unity. In this case, integrating with respect to z, we obtain the following expression for $\mathbf{I}_\perp$:

$$\mathbf{I}_\perp^{(eik)} = i\frac{4e}{v^2\gamma}\int d^2\rho \exp(-i\mathbf{k}_\perp\boldsymbol{\rho}) \times \frac{\boldsymbol{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right)\left\{\exp\left[-i\frac{\omega}{2}\int_{-\infty}^{\infty}(1-\varepsilon_\omega(\mathbf{r}))dz\right]-1\right\}. \tag{24}$$

Putting this value of $\mathbf{I}_\perp^{(eik)}$ into (3), we obtain the spectral-angular density of the transition radiation in the eikonal approximation:

$$\frac{dE}{d\omega do} \approx \frac{\omega^4}{(8\pi^2)^2}\left|\mathbf{I}_\perp^{(eik)}\right|^2. \tag{25}$$

This equation is valid in the ranges of frequencies and radiation angles determined by inequalities (22) and (23).

The exponent entering the last exponential in (21) can be estimated in the following way:

$$\frac{\omega}{2}\int_{-\infty}^{\infty}(1-\varepsilon_\omega(\mathbf{r}))dz \sim \omega\frac{\omega_p^2}{\omega^2}l.$$

If this value is small as compared to unity, then expression (24) can be expanded in terms of the parameter $\omega_p^2 l/\omega$. In the first order of this expansion, $\mathbf{I}_\perp^{(eik)}$ coincides with the corresponding result $\mathbf{I}_\perp^{(B)}$ of the Born approximation.

As l increases and ω decreases, the inequality $\omega_p^2 l/\omega \ll 1$ becomes invalid. In this case, using (25), we can also describe the transition radiation outside the applicability range of the Born approximation. Indeed, as l increases and ω decreases, the condition $\omega_p^2 l/\omega \geq 1$ can always be satisfied, and the Born approximation ceases to be valid under this condition. However, this condition does not contradict inequalities (22) and (23) determining the applicability conditions for (24). We note that inequalities (22) and (23) can always hold if the electron energy is sufficiently high in the range of typical transition radiation angles $\theta \sim \gamma^{-1}$.

A particular form of the function $1 - \varepsilon_\omega(\mathbf{r})$ was not used in deriving (24); for this reason, this function can be used to study the transition radiation for targets of complicated geometries, such as dielectric fiber.

To determine the applicability limits of the obtained equations more precisely, we consider the transition radiation in the eikonal approximation for the normal impact of the ultrarelativistic particle on a nonuniform thin plate of thickness L. In this simplest one-dimensional case, the dielectric properties of the target are independent of ρ; therefore, condition (17) is necessarily satisfied. The calculation using (24) yields

$$\mathbf{I}_\perp^{(eik)} = \frac{8\pi e}{v\omega}\frac{\mathbf{k}_\perp}{k_\perp^2+\left(\frac{\omega}{v\gamma}\right)^2}\left\{\exp\left[-i\frac{\omega}{2}\frac{\omega_p^2}{\omega^2}L\right]-1\right\}. \tag{26}$$

In this case, we obtain the following expression for the spectral-angular radiation density in the range of small angles:

$$\frac{dE}{d\omega do} = 2\frac{e^2}{\pi^2}\frac{\theta^2}{(\theta^2+\gamma^{-2})^2}\left\{1-\cos\left[\frac{\omega_p^2}{\omega^2}\frac{\omega L}{2}\right]\right\}. \tag{27}$$

We compare the results obtained in the Born (8), (9) and eikonal (26), (27) approximations with the exact spectral-angular radiation density for the thin plate. The latter in the range of small angles has the form [1]

$$\frac{dE}{d\omega do} = 2\frac{e^2}{\pi^2}\left(\frac{\omega_p^2}{\omega^2}\right)^2\frac{\theta^2}{(\theta^2+\gamma^{-2})^2\left(\theta^2+\gamma^{-2}+\frac{\omega_p^2}{\omega^2}\right)^2} \times\left\{1-\cos\left[\left(\theta^2+\gamma^{-2}+\frac{\omega_p^2}{\omega^2}\right)\frac{\omega L}{2}\right]\right\}. \tag{28}$$

Under the conditions

$$\gamma^2\omega_p^2/\omega^2 \ll 1, \quad \omega_p^2 L/\omega \ll 1, \tag{29}$$

precise equation (28) for the transition radiation intensity transforms into (9) corresponding to the Born approximation. Under the conditions

$$(\theta^2, \gamma^{-2}) \ll \omega_p^2/\omega^2 \ll 1, \tag{30}$$

precise result (28) for the radiation intensity transforms into (27) corresponding to the eikonal approximation.

TRANSITION RADIATION FOR A FIBERLIKE TARGET

We now consider transition radiation produced in the case of fast charged particles hitting a dielectric fiberlike target at a small angle ψ to the fiber axis. The distribution of the electron density in the plane normal to the fiber axis is assumed to be Gaussian with root-mean-square radius R:

$$n(\mathbf{r}) = \frac{n_e}{2\pi R^2}\exp\left[-\frac{(x-z\psi)^2+(y-y_0)^2}{2R^2}\right]. \quad (31)$$

Here, n_e is the number of electrons per unit fiber length, the z-axis direction coincides with the direction of the particle velocity $\mathbf{v}$, the fiber axis is parallel to the (x, z) plane, and y_0 is the impact parameter of the hitting particle with respect to the fiber axis. In this case, the target thickness along the direction of the particle motion is $l \sim 2R/\psi$; for this reason, condition (23) becomes

$$\omega\theta^2 R/\psi \ll 1.$$

In this case, Eq. (24) yields

$$\mathbf{I}_\perp^{(eik)} = i\frac{4e}{v^2\gamma}\int d^2\rho\exp(-i\mathbf{k}_\perp\boldsymbol{\rho})\frac{\boldsymbol{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right)$$
$$\times\left\{\exp\left[-i\frac{\sqrt{2\pi}e^2n_e}{m\omega}\frac{1}{R\psi}\exp\left(-\frac{(y-y_0)^2}{2R^2}\right)\right]-1\right\}. \quad (32)$$

Integrating with respect to x, we find that

$$\mathbf{I}_\perp^{(eik)} = i\frac{4\pi e}{v\omega}\int_{-\infty}^{\infty}dy\exp(-ik_yy-|y|\sqrt{k_x^2+(\omega/v\gamma)^2})$$
$$\times\left(\frac{-ik_x\mathbf{e}_x}{\sqrt{k_x^2+(\omega/v\gamma)^2}}+\mathbf{e}_y\operatorname{sgn}y\right) \quad (33)$$
$$\times\left\{\exp\left[-i\frac{\sqrt{2\pi}e^2n_e}{m\omega}\frac{1}{R\psi}\exp\left(-\frac{(y-y_0)^2}{2R^2}\right)\right]-1\right\}.$$

The order of magnitude of the typical values of y mainly contributing to integral (33) is γ/ω, while that of the typical values of y_0 is R. In the limiting case of a thick fiber ($R \gg \gamma/\omega$), the t dependence of the last exponential in (33) can be neglected. In this case, we have

$$\mathbf{I}_\perp^{(eik)} = \frac{8\pi e}{v\omega}\frac{\mathbf{k}_\perp}{k_\perp^2+\left(\frac{\omega}{v\gamma}\right)^2}$$
$$\times\left\{\exp\left[-i\alpha\exp\left(-\frac{y_0^2}{2R^2}\right)\right]-1\right\}, \quad (34)$$

where $\alpha = \sqrt{2\pi}e^2n_e/m\omega R\psi$. Inserting this expression for $\mathbf{I}_\perp^{(eik)}$ into (3), we obtain

$$\frac{d\mathrm{E}}{d\omega do} = \frac{e^2\gamma^2}{\pi^2}\frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^2}$$
$$\times\left|\exp(-i\alpha e^{-y_0^2/2R^2})-1\right|^2. \quad (35)$$

Equation (35) describes the transition radiation of the electron hitting the fiber for a given impact parameter y_0. To describe the radiation of a uniform particle beam, the notion of radiation efficiency is introduced. It is defined by the relation

$$\frac{dK}{d\omega do} = \int dx_0dy_0\frac{d\mathrm{E}}{d\omega do}, \quad (36)$$

where x_0 and y_0 are the beam particle coordinates in the plane normal to the velocity vector of the beam particles. In the case under consideration, where the beam hits the target at a small angle to the axis of the long fiber, $dE/d\omega do$ is independent of x_0. In this case, radiation efficiency (36) can be written as

$$\frac{dK}{d\omega do} = L\psi\int dy_0\frac{d\mathrm{E}}{d\omega do}, \quad (37)$$

where L is the fiber length. Here we use the fact that the radiation under consideration is produced only by particles with coordinates x_0 in the range $\Delta x_0 = L\psi$.

Studying the transition radiation process for a fiber-like target in the Born approximation, the authors of [8] singled out the factor $Le^6n_e^2\gamma/m^2\omega\psi$ in the radiation efficiency; this factor determines the order of magnitude of the radiation intensity. Separating the same factor in (37), we can write radiation efficiency (37) in the form

$$\frac{dK}{d\omega do} = \frac{Le^6n_e^2\gamma}{m^2\omega\psi}F. \quad (38)$$

Here, we have

$$F = F^{(eik)} = \frac{2\sqrt{2}\gamma}{\pi\alpha^2\omega R}\frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^2}$$
$$\times\int_{-\infty}^{\infty}du\left|\exp(-i\alpha e^{-u^2})-1\right|^2 \quad (39)$$

and $u = y_0/R\sqrt{2}$. Using these equations, we can compare the results for the radiation efficiency obtained in the eikonal and Born approximations. As shown in [8], the radiation efficiency for the fiber with electron density distribution (31) is determined by (38) with the function F expressed as

$$F = F^{(B)}(\theta,\varphi) = \frac{2}{\pi}\exp\left[-\left(\frac{R\omega}{2\psi\gamma^2}(1+\theta^2\gamma^2)\right)^2\right]$$
$$\times\int_{-\infty}^{\infty}dq\exp\left[-\left(\frac{R\omega}{\gamma}q\right)^2\right]\Phi(q,\theta,\varphi), \quad (40)$$
$$\Phi(q,\theta,\varphi)$$
$$= \frac{(\gamma\theta\sin\varphi-q)^2+\left(\gamma\theta\cos\varphi-\frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}{\left[1+(\gamma\theta\sin\varphi-q)^2+\left(\gamma\theta\cos\varphi-\frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2\right]^2}.$$

If condition $\alpha \ll 1$ is satisfied, (39) can be expanded in terms of α. In the first approximation of this expansion, the function $F^{(eik)}$ becomes

$$F = F^{(eik)} = \frac{2\sqrt{2}}{\sqrt{\pi}} \frac{\gamma}{\omega R} \frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^2}. \qquad (41)$$

This equation for F coincides with the corresponding expression in (40) obtained in the Born approximation if this expression is expanded in terms of the parameters $R\omega\theta^2/\psi \ll 1$, $\gamma/\omega R \ll 1$, and $(\gamma\psi)^{-1} \ll 1$, for which Eq. (34) in the eikonal approximation was obtained. These expansions correspond to the case in which the radiation (in the case of a fiber) is assumed to be concentrated in the range of small angles $\theta \le \gamma^{-1} \ll \psi$ to the direction of the electron motion. The transverse size R of the fiber is large as compared to the transverse size $\sim\gamma/\omega$ of the particle Coulomb field in whose limits the Fourier components of the electron field with frequency ω are mainly concentrated.

Equation (39) demonstrates that, under the condition $\alpha \ge 1$, the transition radiation intensity obtained in the eikonal approximation turns out to be less than the corresponding result obtained in the Born approximation. At the same time, the shapes of the angular distributions obtained in the eikonal and Born approximations turn out to be identical.

We note that the order of magnitude of the parameter α in (34) coincides with that of the corresponding parameter in (26) if the layer thickness L in (26) is assumed to be equal to the length $\sim 2R/\psi$ of the region in which the electron effectively interacts with the fiber. Indeed, assuming that $n_e \sim \pi R^2 n$, where n is the average density of electrons in the fiber, we can write the parameter α in the form

$$\alpha \sim (\pi n e^2/\omega)(2R/\psi),$$

which, for $L \sim 2R/\psi$, coincides with the corresponding parameter $\omega_p^2 L/\omega$ in (26), up to a constant factor. Thus, in the limit $R \gg \gamma/\omega$, the radiation for a fiber is equivalent to that for a uniform plate whose thickness and density are determined by the average electron density and the effective fiber thickness for a given impact parameter y_0. Similar results were obtained for radiation in the case of a uniform cylindrical fiber [11] as well.

CONCLUSIONS

We have considered the process of transition radiation of relativistic electrons in a medium with a nonuniform permittivity. We have proposed an approach to the description of this process using an equivalent photon method and eikonal approximation to describe the propagation of electromagnetic waves in a nonuniform medium. Based on this approach, we have obtained general equations for the spectral-angular density of transition radiation without using a particular coordinate dependence of the permittivity. In the case where the permittivity depends on several coordinates, one can consider transition radiation in the range of small angles by using these equations. We have shown that, using the eikonal approximation in our problem, we can go beyond the scope of the Born theory of transition radiation, in which the expansion of the wave fields in terms of the small deviation of the permittivity from unity is used. We have stated the conditions under which it is possible to use the Born and eikonal approximations in the problem under consideration.

As an example of how the obtained equations can be used for multidimensional problems, we have considered transition radiation for a fiberlike target under the impact of particles at small angles to the fiber axis. In this case, we have compared the spectral-angular densities obtained in the Born and eikonal approximations. We have obtained the conditions under which it was necessary to go beyond the scope of the Born approximation in order to describe the process under study.

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