On relativistic electron bremsstrahlung in thin targets

T.H. Le, N.N. Nasonov *

Laboratory of Radiation Physics, NRU Belgorod State University, Russia

A B S T R A C T

The simple model of relativistic electron bremsstrahlung in a thin target is presented for conditions when its thickness less then a photon formation length. Results of our calculations obtained with account of an influence of target dielectric properties are in agreement with data at 25 GeV electron bremsstrahlung in thin layers of different media.

Keywords:
Bremsstrahlung
Formation length
Dielectric permeability

1. Bremsstrahlung formation length in the process of relativistic electron collision with an atom exceeds this atom size practically in a complete range of emitted photons. This circumstance opens the possibility to use a very simple and effective description of bremsstrahlung (B) [1]. Within the frame of approach [1] (approximation of angle-like trajectory of an emitting electron) it is sufficient to know only the emitting electron velocity before and after collision. Later the approach [1] has been applied for the description of B from fast electrons crossing a thin layer of a medium [2] (electron scattering angle was interpreted as the total multiple scattering angle and the layer thickness was assumed to be small as compared with the emission formation length). Studies have revealed some peculiarities of B in thin targets, in particular the effect of B suppression in small frequency range [2,3], observed experimentally [4,5] (the Landau–Pomeranchuk–Migdal (LPM) effect of B suppression due to the influence of emitting electron multiple scattering was observed in [4] for the first time).

It should be noted, that the target dielectric susceptibility was not taken into account in [1–3], and hence the contribution of such effects as the transition radiation and Ter-Mikaelian effect of B dielectric suppression [6] (this effect was observed experimentally as well [7]) were eliminated in sited works. Experiments with thin targets have shown an important role of effects being discussed. This is the reason of present work where we take into account the influence of medium dielectric properties on the relativistic electron B in thin targets and, at the same time, we preserve the simplicity of the B description proposed in [1,2].

Dielectric effects tend to the problem within the frame of the approximation [1,2]. Indeed, B cross section is calculated in [1,2] formally in the limit $\omega \to 0$, when the vacuum formation length $l_{coh} \approx 2\gamma^2/\omega$ increases indefinitely and can exceed any target thickness L. At the same time, in a medium $l_{coh} \approx 2\omega/\omega_0^2 \to 0$ for $\omega \to 0$ and the condition $l_{coh} \gg L$ cannot be fulfilled (here γ is Lorenz factor of a fast electron, ω_0 is plasma frequency of the target). Thus, the emission from a thin layer of a medium cannot coincide with B in small frequency range $\omega \to 0$ with account of dispersive properties of the medium. We show below, that, for example, B within the frame of angle-like trajectory approximation is suppressed in the simplest case of unbounded medium due to Ter-Mikaelian effect in the frequency range $\omega < \gamma\omega_0$ and coincides with its vacuum limit [1,2] in the frequency range $\omega > \gamma\omega_0$ only.

Relativistic system of units $\hbar = c = 1$ is used in our Letter.

2. Let us consider the simplest process of B from a single collision of incident electron with an atom in unbounded medium. Generalizing of the result [1] by taking into account the target dielectric permeability (we use the universal in X-ray range function $\varepsilon(\omega) = 1 - \omega_0^2/\omega^2$) one can obtain the following results for B spectrum:

$$\omega\frac{dN}{d\omega} = \frac{2e^2}{\pi}\left[\frac{1+(zx)^2/2(1+x^2)}{\sqrt{1+(zx)^2/4(1+x^2)}}\cdot\frac{2\sqrt{1+x^2}}{zx} \times \ln\left(\sqrt{1+\frac{(zx)^2}{4(1+x^2)}}+\frac{z}{x}2\sqrt{1+x^2}\right)-1\right] \equiv F_1(x,z),$$

$$x = \frac{\omega}{\gamma\omega_0}, \qquad z = \gamma\Psi, \qquad (1)$$

where Ψ is the scattering angle of the electron. Obviously, in the frequency range $\omega > \gamma\omega_0$ formula (1) converts to the result [2]

* Corresponding author.
E-mail address: nnn@bsu.edu.ru (N.N. Nasonov).

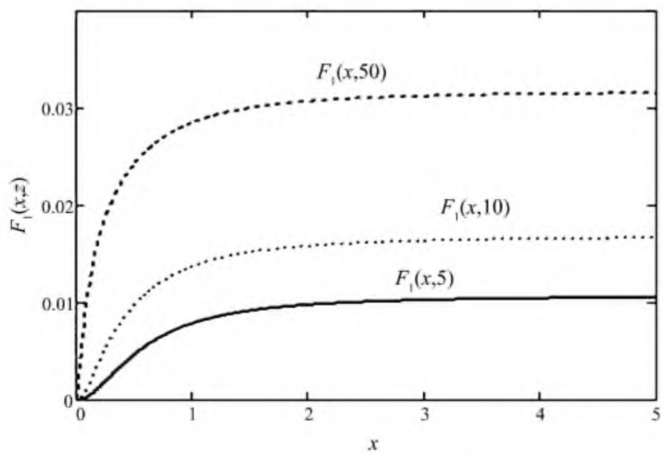

Fig. 1. Bremsstrahlung spectrum in low energy range in an unbounded medium. $x = \omega/\gamma\omega_0$, $z = \gamma\Psi$.

which does not depend on the frequency, but in small frequency range $\omega < \gamma\omega_0$ B spectrum is suppressed strongly by Ter-Mikaelian effect as it is demonstrated by curves presented in Fig. 1. Thus, results [1,2] are inapplicable in conditions under discussion.

3. Let us consider now an emission from a thin layer of a medium. Using angle-like trajectory approximation, we consider the simple model. Relativistic electrons fly into the layer moving uniformly to the middle of the target. Here electrons are scattered (each of the electrons is scattered individually on its angle Ψ) and fly off the target moving uniformly again. Emission amplitude follows in the case being considered from the general expression [8]

$$\mathbf{A}^{\mathrm{rad}} = \frac{e}{\pi}\left\{\frac{\mathbf{u}_{\mathrm{i}}}{\gamma^{-2}+\mathbf{u}_{\mathrm{i}}^2} - \frac{\mathbf{u}_{\mathrm{f}}}{\gamma^{-2}+\mathbf{u}_{\mathrm{f}}^2}\exp\left[\frac{i\omega}{2}\int_0^L dt\big(\gamma^{-2}-\chi(\omega)+u_t^2\big)\right] + \frac{i\omega}{2}\int_0^L dt\,\mathbf{u}_t\exp\left[\frac{i\omega}{2}\int_0^t d\tau\big(\gamma^{-2}-\chi(\omega)+u_\tau^2\big)\right]\right\} \times \exp\big(i\omega\chi(\omega)L/2\big), \quad (2)$$

taking into account the contribution of B and transition radiation as well as LPM and Ter-Mikaelian effects. Here L is the thickness of the layer, $\chi(\omega)$ is the dielectric susceptibility, $\mathbf{u}_t = \boldsymbol{\Psi}_t - \boldsymbol{\Theta}$, the angles $\boldsymbol{\Psi}_t$ and $\boldsymbol{\Theta}$ determine the current electron velocity $\mathbf{V}_t = \mathbf{e}(1-\Psi_t^2/2)+\boldsymbol{\Psi}_t$, $\mathbf{e}\boldsymbol{\Psi}_t = 0$ and the unit vector to the direction of emitted photon propagation $\mathbf{n} = \mathbf{e}(1-\Theta^2/2)+\boldsymbol{\Theta}$, $\mathbf{e}\boldsymbol{\Theta} = 0$, $\mathbf{e}$ is the normal to the layer surface, angles at inlet and outlet surface are signified by indexes i and f respectively.

Within the frame of the approximation being considered all integrals in (2) are calculated easily. Therefore, one can obtain the following expression for the emission amplitude:

$$\mathbf{A}^{\mathrm{rad}} = \frac{e}{\pi}\left\{\left[\frac{\boldsymbol{\Theta}}{\gamma^{-2}-\chi(\omega)+\Theta^2} - \frac{\boldsymbol{\Theta}}{\gamma^{-2}+\Theta^2}\right] - \left[\frac{\boldsymbol{\Theta}-\boldsymbol{\Psi}}{\gamma^{-2}-\chi(\omega)+(\boldsymbol{\Theta}-\boldsymbol{\Psi})^2} - \frac{\boldsymbol{\Theta}-\boldsymbol{\Psi}}{\gamma^{-2}+(\boldsymbol{\Theta}-\boldsymbol{\Psi})^2}\right] \times \exp\big\{\big[\big(\gamma^{-2}-\chi(\omega)+\Theta^2\big) + \big(\gamma^{-2}-\chi(\omega)+(\boldsymbol{\Theta}-\boldsymbol{\Psi})^2\big)\big]i\omega L/4\big\} - \left[\frac{\boldsymbol{\Theta}}{\gamma^{-2}-\chi(\omega)+\Theta^2} - \frac{\boldsymbol{\Theta}-\boldsymbol{\Psi}}{\gamma^{-2}-\chi(\omega)+(\boldsymbol{\Theta}-\boldsymbol{\Psi})^2}\right] \times \exp\big[\big(\gamma^{-2}-\chi(\omega)+\Theta^2\big)i\omega L/4\big]\right\}. \quad (3)$$

First and second items in (3) describe the transition radiation, emitted from inlet and outlet surfaces of the layer, third one corresponds to B mechanism. Let us pay attention that the result (3) coincides with its "vacuum limit" [1,2] in the frequency range where arguments of exponents are small (formation length exceeds essentially the thickness of the layer)

$$\mathbf{A}^{\mathrm{rad}} = \frac{e}{\pi}\left[\frac{\boldsymbol{\Theta}-\boldsymbol{\Psi}}{\gamma^{-2}+(\boldsymbol{\Theta}-\boldsymbol{\Psi})^2} - \frac{\boldsymbol{\Theta}}{\gamma^{-2}+\Theta^2}\right], \quad (4)$$

so that the emission spectrum is determined by B mechanism only in the frequency range under consideration.

In general case one should use the total formula (3). It is convenient to represent the formula for the emission spectrum following from (3) as a sum of "vacuum" item [2] and an interference item, including all dielectric and interference effects

$$\omega\frac{dN}{d\omega} = \frac{2e^2}{\pi}\left[\frac{1+z^2/2}{\sqrt{1+z^2/4}}\frac{2}{z}\ln\big(\sqrt{1+z^2/4}+z/2\big)-1\right] + F_2(x,y,z),$$

$$F_2(x,y,z) = \frac{2e^2}{\pi}\int_0^\infty dt\left\{\frac{(2x)^{-1}[t-xz^2/4]}{x+(2x)^{-1}+t+xz^2/4} \times\left[\frac{1}{x+t+xz^2/4}\frac{1}{\sqrt{(x+t+xz^2/4)^2-txz^2}} - \frac{1}{x+x^{-1}+t+xz^2/4} \times \frac{1}{\sqrt{(x+x^{-1}+t+xz^2/4)^2-txz^2}}\right] \times\big[1-\cos\big[y\big(x+x^{-1}+t+xz^2/4\big)\big]\big] - \left[\frac{1}{x+t}-\frac{1}{x+x^{-1}+t}\right] \times\left[\frac{x+x^{-1}-t+xz^2}{\sqrt{(x+x^{-1}+t+xz^2)^2-4txz^2}} - \frac{x+x^{-1}-t}{x+x^{-1}+t}\right] \times\big[1-\cos\big[\big(x+x^{-1}+t\big)y/2\big]\big]\right\}. \quad (5)$$

Interference item in (5) depends on the new parameter $y = \omega_0 L/2\gamma$ which is equal to the ratio of target thickness L to maximum value of emission formation length (with account of the dispersion of the dielectric permeability). This parameter is of the great importance in emission process. For example, it is easy to verify that there is no frequency range where the layer may be considered as a thin in case $y \gg 1$. On the other hand, in case $y \to 0$ the emission spectrum does not depend on the frequency and is determined by the formula [2].

4. Let us pay attention on experimental verification of the effects being discussed following from measured B spectra [9]. With this aim in view, we examine spectra [9] measured in conditions of thin layers ($y < 1$) and which were not analyzed in [3]. For quantitative comparison one should use the result (5) averaged over scattering angles. We use the simplest Gauss distribution

$$\left\langle\omega\frac{dN}{d\omega}\right\rangle = \int_0^\infty dz\frac{2z}{z_L^2}\exp\left(\frac{-z^2}{z_L^2}\right)\omega\frac{dN}{d\omega},$$

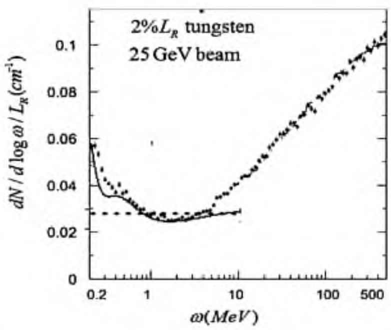

Fig. 2. Low energy spectrum of 25 GeV electron bremsstrahlung from the thin tungsten target ($L = 0.02L_R$).

$$z_L^2 = \left(\frac{21\ \text{MeV}}{m}\right)^2 \frac{L}{L_R} \approx 1700\frac{L}{L_R}, \tag{6}$$

here L_R is the radiation length.

Measured B spectrum from 25 GeV electrons in tungsten target ($L = 0.02$) is presented in Fig. 2 by dotted line. Results of our calculations are given by solid line. Dashed line shows the result of calculations on the base of "vacuum" model. The same but for uranium target ($L = 0.03$) is presented in Fig. 3. Theoretical predictions and data are in agreement.

5. Thus, the modification of the angle-like trajectory approximation being known in B physics is presented in our work to take into account the contribution of transition radiation and Ter-Mikaelian effect of dielectric B suppression. It is shown that the influence of such effects on B from a thin layer of a medium can be substantial. This conclusion is in quantitative agreement with data at 25 GeV electron B from thin targets [9].

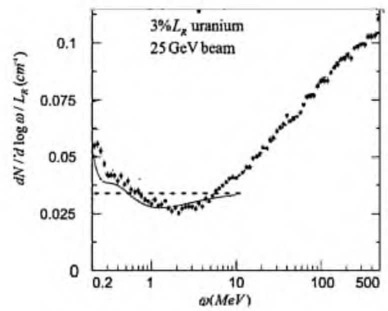

Fig. 3. The same, but for uranium target ($L = 0.03L_R$).

Acknowledgements

The authors are grateful to B.M. Bolotovsky and S.R. Klein stimulated our studies for helpful discussion of obtained results.

This work was supported in part by Ministry of Education and Science RF (s.c.02.740.11.0545 and s.c.16.518.11.7027).

References

[1] L.D. Landau, E.M. Lifshits, Classical Theory of Field, Pergamon, Oxford, 1975.
[2] N.F. Shulga, S.P. Fomin, JETP Lett. 27 (1978) 117.
[3] N.F. Shulga, S.P. Fomin, JETP 86 (1998) 32.
[4] P.L. Anthony, R. Becker-Szendy, P.E. Bosted, et al., Phys. Rev. Lett. 75 (1995) 1945.
[5] U.I. Uggerhoj, Y. Knudsen, S. Ballestrero, et al., Phys. Rev. D 53 (1996) 6265.
[6] M.L. Ter-Mikaelian, Dokl. Akad. Nauk SSSR 94 (1954) 1033 (in Russian).
[7] P.L. Anthony, R. Becker-Szendy, P.E. Bosted, et al., Phys. Rev. Lett. 76 (1996) 3550.
[8] N.N. Nasonov, NIM B 173 (2001) 203.
[9] S.R. Klein, Rev. Mod. Phys. 71 (1999) 1501.