Relative contribution of real and virtual photon diffraction to the parametric X-ray yield

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Abstract

Relative contributions of parametric X-rays and diffracted bremsstrahlung to the emission from relativistic electrons penetrating a crystal under Bragg scattering condition is studied theoretically. It is shown that the diffracted bremsstrahlung contribution may be essential under special conditions only.

1. Introduction

Multiple scattering exerts a dual action on the characteristics of parametric X-rays from relativistic electrons penetrating a crystalline target. On the one hand, multiple scattering can greatly extend the angular and spectral width of parametric X-rays, caused by the Bragg scattering of virtual photons from the fast particle's Coulomb field. On the other hand, bremsstrahlung diffracted by the same system of atomic planes which is responsible for the parametric X-rays, may contribute to the total emission yield. An influence of multiple scattering on the parametric X-ray properties is usually taken into account through averaging over scattering angles of the parametric X-ray cross-section calculated for a rectilinear trajectory of emitting particle. Obviously, the contribution of diffracted bremsstrahlung is lost within the framework of such approach. An exact statement of the discussed problem based on the kinetic equation approach was used in [1]. Unfortunately, the expansions used in [1] allow to describe the case of thin enough target only, where the influence of multiple scattering is small. Recently, the exact approach was used to analyze the influence of multiple scattering on the parametric X-ray spectral width [2,3], but the question concerning relative contributions of parametric X-rays and diffracted bremsstrahlung was not considered there.

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Thus, the problem of a correct description of parametric X-rays from a relativistic electron beam crossing a thick crystal has not been solved up till now. Incidentally it should be noted that the results of some experiments [4,5] point to a discrepancy between measured data and the theory based on averaging of the parametric X-ray cross-section. In accordance with [4,5], the diffracted bremsstrahlung contribution to total emission yield becomes substantial with the proviso that $\omega_B > \gamma\omega_p$. Here, ω_B is the Bragg frequency, in the vicinity of which the spectrum of parametric X-rays is concentrated, γ is the Lorentz factor of the radiating electron, ω_p is the plasma frequency of the target. Obviously, the presented condition $\omega_B > \gamma\omega_p$ means the absence of Ter-Mikaelian effect consisting in the bremsstrahlung suppression due to changing of emitted photon phase velocity because of the polarization of target's electrons by electromagnetic field [6]. It is this effect which is the reason why the diffracted bremsstrahlung contribution is small in the frequency range $\omega_B < \gamma\omega_p$ [4,5].

The aim of the present work consists in the development of an adequate model describing the parametric X-rays from relativistic electrons penetrating a thick crystal. We elucidate the experimental conditions for which the diffracted bremsstrahlung contribution may be essential. Among other things we show that the condition $\omega_B > \gamma\omega_p$ is not sufficient for a manifestation of the discussed effect. An influence of multiple scattering on the differential and integral characteristics of the parametric X-rays is studied in detail. Finally we show that the considered problem connects not only with the Ter-Mikaelian effect, but also with the second classical effect in bremsstrahlung physics, known as the Landau–Pomeranchuk–Migdal effect [7,8].

2. General expressions

Let us consider an emission from relativistic electrons moving in a crystal with the dielectric permeability $\varepsilon(\omega,\mathbf{r}) = 1 + \chi_0(\omega) + \sum_{\mathbf{g}}'\chi_{\mathbf{g}}(\omega)e^{i\mathbf{gr}}$. Here, $\mathbf{g}$ is the reciprocal lattice vector. Starting from Maxwell equations for the Fourier-transform $\mathbf{E}_{\omega\mathbf{k}} = (2\pi)^{-4}\int dt\, d^3r\,\mathbf{E}(\mathbf{r},t)e^{i\omega t - i\mathbf{kr}}$ of excited in the crystal electromagnetic field

$$(k^2 - \omega^2(1+\chi_0))\mathbf{E}_{\omega\mathbf{k}} - \mathbf{k}(\mathbf{k}\mathbf{E}_{\omega\mathbf{k}}) - \omega^2\sum_{\mathbf{g}}{}'\chi_{-\mathbf{g}}\mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} = 4\pi i\omega\mathbf{J}_{\omega\mathbf{k}}, \quad (1)$$

where $\mathbf{J}_{\omega\mathbf{k}}$ is Fourier-transform of the current density of emitting electron, one can obtain on the basis of well known methods of the dynamical diffraction theory [9] the following formula for emission field propagating along the direction of Bragg scattering:

$$E_{\lambda\mathbf{k}+\mathbf{g}} = \frac{4\pi i\omega^3\chi_{\mathbf{g}}\alpha_\lambda}{D}\mathbf{e}_{\lambda\mathbf{k}}\mathbf{J}_{\omega\mathbf{k}}, \quad (2a)$$

$$D = (k^2 - \omega^2(1+\chi_0))((\mathbf{k}+\mathbf{g})^2 - \omega^2(1+\chi_0)) - \omega^4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha_\alpha^2, \quad (2b)$$

where $\mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} = \sum_{n=1}^{2}\mathbf{e}_{\lambda\mathbf{k}+\mathbf{g}}E_{\lambda\mathbf{k}+\mathbf{g}}$, $\mathbf{e}_{\lambda\mathbf{k}}$ and $\mathbf{e}_{\lambda\mathbf{k}+\mathbf{g}}$ are the polarization vectors, $(\mathbf{k}, \mathbf{e}_{\lambda\mathbf{k}}) = (\mathbf{k}+\mathbf{g}, \mathbf{e}_{\lambda\mathbf{k}+\mathbf{g}}) = 0$, $\alpha_1 = 1$, $\alpha_2 = \mathbf{k}(\mathbf{k}+\mathbf{g})/k|\mathbf{k}+\mathbf{g}|$. It should be noted that the expressions analogous to (2) are well known in the theory of parametric X-rays [6,10–12].

To determine an emission spectral-angular distribution one should calculate Fourier-integral $E_\lambda^{\mathrm{Rad}} = \int d^3k_g e^{i\mathbf{k}_g\mathbf{n}-r}E_{\lambda\mathbf{k}+\mathbf{g}}$ in the wave-zone by the stationary phase method (here $\mathbf{k}_g = \mathbf{k}+\mathbf{g}$, $\mathbf{n}$ is the unit vector to the direction of emitted photon propagation). The result of integration has the form

$$E_\lambda^{\mathrm{Rad}} = \frac{4\pi^3 i\omega^3\chi_{\mathbf{g}}\alpha_\lambda}{\sqrt{\Delta'^2 + \omega^2\chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha_\lambda^2(1-\mathbf{ng}/\omega)}} \times \left[\mathbf{e}_{\lambda\mathbf{k}_+}\mathbf{J}_{\omega\mathbf{k}_+}e^{i\xi_+ r} - \mathbf{e}_{\lambda\mathbf{k}_-}\mathbf{J}_{\omega\mathbf{k}_-}e^{i\xi_- r}\right]\frac{e^{i\omega r}}{r}, \quad (3a)$$

$$\xi_\pm = \frac{1}{2(1-\mathbf{ng}/\omega)} \times \left(-\Delta' \pm \sqrt{\Delta'^2 + \omega^2\chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha_\lambda^2(1-\mathbf{ng}/\omega)}\right) + \frac{\omega}{\lambda}\chi_0, \quad (3b)$$

$$\Delta' = \frac{g^2}{2\omega} - \mathbf{ng}\left(1 + \frac{1}{2}\chi_0\right), \quad \mathbf{k}_\pm = (\omega + \xi_\pm)\mathbf{n} - \mathbf{g}. \quad (3c)$$

For the further analysis it is very convenient to introduce the angular variables $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}_t$ in accordance with

$$\mathbf{V}(t)\equiv\mathbf{V}_t=\mathbf{e}_1\left(1-\frac{1}{2}\gamma^{-2}-\frac{1}{2}\Psi_t^2\right)+\boldsymbol{\Psi}_t,\quad \mathbf{e}_1\boldsymbol{\Psi}_t=0, \tag{4a}$$

$$\mathbf{n}=\mathbf{e}_2\left(1-\frac{1}{2}\Theta^2\right)+\boldsymbol{\Theta},\quad \mathbf{e}_2\boldsymbol{\Theta}=0,\quad \mathbf{e}_1\mathbf{e}_2=\cos\varphi, \tag{4b}$$

where $\mathbf{V}(t)=\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r}(t)$ is the velocity of emitting electron, $\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\neq 0$ in the case under consideration because of multiple scattering. The variables $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}_t$ are shown in Fig. 1, where the geometry of considered emission process is presented.

Using (3) and (4) one can obtain the following formula for the spectral-angular distribution of the number of emitted photons:

$$\frac{\mathrm{d}^3N_\lambda}{\mathrm{d}\omega\,\mathrm{d}^2\Theta}=\frac{e^2\omega|\chi_\mathbf{g}|^2\alpha_\lambda^2}{8\pi^2}\frac{1}{\Delta^2+\chi_\mathbf{g}\chi_{-\mathbf{g}}\alpha_\lambda^2\cos\varphi}\cdot\mathrm{Re}\left\langle\int\mathrm{d}t\int_0^\infty\mathrm{d}\tau\,\Omega_{\lambda t}\Omega_{\lambda t+\tau}\mathrm{e}^{-\mathrm{i}\omega\tau}\times\left[\mathrm{e}^{\mathrm{i}\mathbf{k}_+(\mathbf{r}_{t+\tau}-\mathbf{r}_t)}+\mathrm{e}^{\mathrm{i}\mathbf{k}_-(\mathbf{r}_{t+\tau}-\mathbf{r}_t)}\right]\right\rangle, \tag{5}$$

where $\Delta=\Delta'/\omega_\mathrm{B}$, $\omega_\mathrm{B}=g/2\sin(\varphi/2)$ is the Bragg frequency, in the vicinity of which the parametric X-ray spectrum is concentrated, $\Omega_{1t}=\Theta_\perp-\Psi_{\perp t}$, $\Omega_{2t}=\Theta_\parallel+\Psi_{\parallel t}+2\Theta'$, Θ' is the orientation angle (see Fig. 1), the value $\Theta'=0$ corresponds to exact Bragg resonance orientation of the crystal relative to emitting electron velocity, $\alpha_1=1$, $\alpha_2=\cos\varphi$, the brackets $\langle\,\rangle$ mean the averaging over all possible trajectories of electrons in the target. In accordance with (5) two branches of electromagnetic waves propagating in the crystal contribute to total emission yield within the framework of used dynamical diffraction approach.

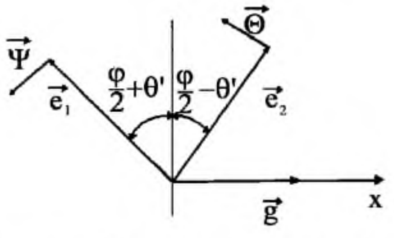

Fig. 1. Geometry of an emission process $\mathbf{g}$ is the reciprocal lattice vector, $\mathbf{e}_1$ is the axis of emitting electron beam, $\mathbf{e}_2$ is the axis of the photon detector, θ' is the orientation angle, $\Theta_\parallel$ and $\Psi_\parallel$ are the components of the observation angle $\boldsymbol{\Theta}$ and the angle $\boldsymbol{\Psi}$, describing beam spread.

Procedure of averaging of the expressions analogous to (5) is described in [6], where the influence of multiple scattering on the ordinary bremsstrahlung from relativistic electrons moving in amorphous medium has been considered in detail. Using the corresponding results [6] one can obtain from (5) the final formula for the total emission intensity

$$\frac{\mathrm{d}^4N_\lambda}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}^2\Theta}=\frac{e^2\omega|\chi_\mathbf{g}|^2}{4\pi^2}\frac{\Omega_{\lambda t}^2\alpha_\lambda^2}{\sigma_\lambda^2}\cdot\mathrm{Re}\int_0^\infty\mathrm{d}\tau\frac{\cos\frac{\omega}{2}\sigma_\lambda\tau}{\coth^2(\sqrt{2\mathrm{i}\omega q}\tau)}\times\exp\left[-\frac{\mathrm{i}\omega}{2}(\gamma^{-1}-\chi_0-\Delta)\tau-\sqrt{\frac{\mathrm{i}\omega}{8q}}\Omega_t^2\tanh\sqrt{(2\mathrm{i}\omega q)}\tau\right], \tag{6}$$

where $\sigma_\lambda^2=\Delta^2+\chi_\mathbf{g}\chi_{-\mathbf{g}}\alpha_\lambda^2\cos\varphi$, $q=\pi/e^2L_\mathrm{R}\gamma^2$, L_R is the radiation length.

3. Relative contribution of the parametric X-rays and the diffracted bremsstrahlung to total emission yield

Let us use the general result (6) to find out the conditions such that the diffracted bremsstrahlung contribution can be substantial. The emission angular density,

$$\frac{\mathrm{d}^2N}{\mathrm{d}^2\Theta}=\int_0^L\mathrm{d}t\int\mathrm{d}^2\Psi_t f(t,\boldsymbol{\Psi}_t)\int_0^\infty\mathrm{d}\omega\sum_{\lambda=1}^{2}\frac{\mathrm{d}^4N_\lambda}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}^2\Theta}, \tag{7}$$

is the most suitable characteristic for our purposes because this characteristic is very sensitive to the action of multiple scattering. Here L is the target's thickness, the distribution function $f(t,\boldsymbol{\Psi}_t)$ is presented as

$$f(t,\boldsymbol{\Psi}_t)=\frac{1}{\pi(\Psi_0^2+\Psi_\mathrm{Sc}^2t)}\exp\left(-\frac{\Psi_t^2}{\Psi_0^2+\Psi_\mathrm{Sc}^2t}\right), \tag{8}$$

where Ψ_0 is the initial angular spread of emitting electron beam, $\Psi^2_{Sc} = 4\pi/e^2 L_R \gamma^2$ (Ψ_{Sc} is the mean-square multiple scattering angle per unit length).

First of all let us integrate the intensity distribution (8) over emitted photon energies keeping in mind that the emission considered in this work is concentrated in the narrow vicinity of $\omega = \omega_B$ because the Bragg diffraction process extracts this segment of initially wide spectra of both real photons of bremsstrahlung and virtual photons from the emitting electron coulomb field. Taking into account that $\omega \approx \omega_B$ in (6) except "fast variable" $\Delta(\omega)$ (so-called resonance defect) one can perform the integration by the transformation of variables $\mathrm{d}\omega = \left(\frac{\mathrm{d}\Delta}{\mathrm{d}\omega}\right)^{-1}\mathrm{d}\Delta$. The result of integration has the following form:

$$\frac{\mathrm{d}^3 N_\lambda}{\mathrm{d}t\,\mathrm{d}^2\Theta} = -\frac{e^2\omega_B^4|\chi_{\mathbf{g}}|^2}{2\pi g^2}\frac{\Omega^2_{\lambda t}\alpha^2_\lambda}{\Omega^2_t\beta_\lambda} \times \mathrm{Im}\left\{(\beta_\lambda + \mathrm{i}\gamma^{-2} - \mathrm{i}\chi_0) \times \int_0^\infty \mathrm{d}\tau \exp\left[-\frac{\omega_B}{2}(\beta_\lambda + \mathrm{i}\gamma^{-2} - \mathrm{i}\chi_0)\tau - \sqrt{\frac{\mathrm{i}\omega_B}{8q}}\Omega^2_t \tanh(\sqrt{2\mathrm{i}\omega q}\tau)\right]\right\}. \quad (9)$$

The integration by parts over τ has been used when deriving this formula. The quantity β_λ in (9) is defined as $\beta^2_\lambda = \chi_{\mathbf{g}}\chi_{-\mathbf{g}}\alpha^2_\lambda \cos\varphi$ (we are considering the parametric X-rays for Laue geometry, so $\varphi < \pi/2$).

The simplest approximations of the dielectric susceptibilities χ_0 and $\chi_{\mathbf{g}}$ are used in our work:

$$\chi_0 = -\frac{\omega_p^2}{\omega^2} \approx -\frac{\omega_p^2}{\omega_B^2} = -\gamma_{TM}^{-2}, \quad (10a)$$

$$\chi_{\mathbf{g}}\chi_{-\mathbf{g}} = |\chi_{\mathbf{g}}|^2 = \frac{\omega_{\mathbf{g}}^4}{\omega^4}, \quad \omega_{\mathbf{g}}^2 = \omega_p^2 \frac{F(g)}{Z}\frac{|S(\mathbf{g})|}{N_C} \mathrm{e}^{-\frac{1}{2}g^2 u_T^2}, \quad (10b)$$

where $F(g)$ is the atom formfactor, Z is the number of electrons in an atom, $S(\mathbf{g})$ is the structure factor of a crystal elementary cell, containing N_C atoms, u_T is the mean-square amplitude of atom thermal vibrations, ω_p is the plasma frequency of the target's material, $m\gamma_{TM}$ is the characteristic energy of an emitting electron, corresponding to the manifestation of Ter-Mikaelian effect in bremsstrahlung (the emission of bremsstrahlung photon with the energy $\omega = \omega_B$ is suppressed due to Ter-Mikaelian effect, if $\gamma > \gamma_{TM}$).

The emission intensity (9) takes into account both parametric X-rays and diffracted bremsstrahlung contribution. In order to estimate the relative contribution of diffracted bremsstrahlung let us compare the result for the total emission angular density $\mathrm{d}^2N/\mathrm{d}^2\Theta$ which follows from (7), (8) and the exact formula for the emission intensity (9) with that following from above formulae (7), (8) and simplified formula (9), corresponding to the emission from a fast electron moving with constant velocity. The last case corresponds to the limit $q \to 0$ in (9), when this formula can be reduced to the angular distribution of ordinary parametric X-ray:

$$\frac{\mathrm{d}^3 N_\lambda}{\mathrm{d}t\,\mathrm{d}^2\Theta} \to \frac{\mathrm{d}^3 N_{0\lambda}}{\mathrm{d}t\,\mathrm{d}^2\Theta} = -\frac{e^2\omega_{\mathbf{g}}^4}{\pi g^2}\frac{1}{\omega_B}\frac{\Omega^2_{\lambda t}\alpha^2_\lambda}{(\gamma^{-2} + \gamma_{TM}^{-2} + \Omega_t^2)^2 + \beta_\lambda^2}, \quad (11)$$

where the coefficient β^2_λ in the denominator of presented formula describes an influence of dynamical diffraction effects on the parametric X-rays. Since $\beta^2_\lambda = \gamma_{TM}^{-4}(\omega_{\mathbf{g}}^4/\omega_p^4)\alpha^2_\lambda \cos\varphi < \gamma_{TM}^{-4}$ (see (10b)), such influence is not large (the changes in the parametric X-ray angular distribution are negligible in the range of emitting electron energies $\gamma < \gamma_{TM}$ and have a value of about 10–20% if $\gamma > \gamma_{TM}$, as it has been shown for the first time in [13]).

While calculating the quantities $\mathrm{d}^2N/\mathrm{d}^2\Theta$ and $\mathrm{d}^2N_0/\mathrm{d}^2\Theta$ we have restricted ourselves to the case of small incidence angle $\varphi \ll 1$, when $\alpha_2 = \cos\varphi \approx 1 = \alpha_1$. Under such conditions the diffracted bremsstrahlung contribution is substantial.

The final expression for $\mathrm{d}^2N_0/\mathrm{d}^2\Theta$ as it follows from (7), (8) and (11) can be presented in the form

$$\frac{1}{\gamma_{TM}^2}\frac{\mathrm{d}^2 N_0}{\mathrm{d}^2\Theta} = \frac{e^2\omega_p^4}{\pi g^2}\frac{L_{Sc}}{\omega_B}F_0(x, y, z, \delta, L/L_{Sc}), \quad (12a)$$

$$F_0 = \frac{\delta}{y^2}\,\mathrm{Im}\int_0^\infty \mathrm{d}t \frac{1+y^2+\mathrm{i}\delta}{\sqrt{(1+y^2+x^2+t+\mathrm{i}\delta)^2-4x^2t}} \times \left[E_1\left(\frac{t}{z^2+\frac{L}{L_{\mathrm{Sc}}}y^2}\right)-E_1\left(\frac{t}{z^2}\right)\right], \qquad (12b)$$

where $L_{\mathrm{Sc}} = \frac{e^2}{4\pi}L_{\mathrm{R}}$ is the multiple scattering angle ranges up to γ^{-1} at the distance L_{Sc}, $x = \gamma_{\mathrm{TM}}\Theta_0$, $\Theta_0^2 = \Theta_\perp^2 + (2\Theta' + \Theta_\parallel)^2$, $y = \gamma_{\mathrm{TM}}/\gamma$, $z = \gamma_{\mathrm{TM}}\Psi_0$, $\delta = \omega_{\mathrm{g}}^2/\omega_{\mathrm{p}}^2 < 1$.

The analogous calculations allow to obtain the following expression for $\mathrm{d}^2N/\mathrm{d}^2\Theta$:

$$\frac{1}{\gamma_{\mathrm{TM}}^2}\frac{\mathrm{d}^2N}{\mathrm{d}^2\Theta} = \frac{e^2\omega_{\mathrm{p}}^4}{\pi g^2}\frac{L_{\mathrm{Sc}}}{\omega_{\mathrm{B}}}F(x,y,z,\delta,v,L/L_{\mathrm{Sc}}), \qquad (13a)$$

$$F = -\frac{\delta}{y^2}\mathrm{Im}\Big[(1+y^2-\mathrm{i}\delta) \times \int_0^\infty \mathrm{d}t\coth(t)\exp\left(-\sqrt{\mathrm{i}}(1+y^2-\mathrm{i}\delta)\frac{v}{y}t\right) \times \left(E_1\left(\frac{\sqrt{\mathrm{i}}x^2\frac{v}{y}\tanh(t)}{1+\sqrt{\mathrm{i}}(z^2+\frac{L}{L_{\mathrm{Sc}}}y^2)\frac{v}{y}\tanh(t)}\right) - E_1\left(\frac{\sqrt{\mathrm{i}}x^2\frac{v}{y}\tanh(t)}{1+\sqrt{\mathrm{i}}z^2\frac{v}{y}\tanh(t)}\right)\right)\Big], \qquad (13b)$$

where the function F depends on the new parameter $v = \gamma_{\mathrm{LPM}}/\gamma_{\mathrm{TM}}$, $m\gamma_{\mathrm{LPM}} = m\sqrt{\omega_{\mathrm{B}}L_{\mathrm{Sc}}/2}$ is the characteristic energy of the Landau–Pomeranchuk–Migdal effect manifestation. It is easy to see that $\gamma_{\mathrm{LPM}}^{-1}$ is equal to the multiple scattering angle Θ_{coh} achievable at the distance of the order of the coherence length $l_{\mathrm{coh}} \approx 2\gamma^2/\omega_{\mathrm{B}}$ (photon with the energy ω_{B} is formed at this distance [14]). As this takes place, the value of Θ_{coh} exceeds the characteristic emission angle of relativistic particle and the emission of bremsstrahlung photon with energy $\omega = \omega_{\mathrm{B}}$ is suppressed due to Landau–Pomeranchuk–Migdal effect if $\gamma > \gamma_{\mathrm{LPM}}$.

The parameter v is crucial in determining the separate contributions of parametric and diffracted bremsstrahlung emission mechanisms to total emission yield. Close inspection of the function F in (13b) shows that the effective values of the variable of integration $t_{\mathrm{eff}} \ll 1$, if $v \gg 1$ independently on the value of the parameter y. Because of this, $\tanh(t) \approx t$ and the change of variables of integration $t' = \sqrt{i}\frac{v}{y}t$ allows us to reduce (13b) to more simple formula:

$$F \to -\frac{\delta}{y^2}\mathrm{Im}\left[(1+y^2-\mathrm{i}\delta)\int_0^\infty \frac{\mathrm{d}t'}{t'}\mathrm{e}^{-(1+y^2-\mathrm{i}\delta)t'} \times \left(E_1\left(\frac{x^2t'}{1+(z^2+\frac{L}{L_{\mathrm{Sc}}}y^2)t'}\right)-E_1\left(\frac{x^2t'}{1+z^2t'}\right)\right)\right]. \qquad (14)$$

The performed numerical analysis has shown that the function $F_0(x)$ in (12b) and the modified function $F(x)$ in (14) coincide. By this means the diffracted bremsstrahlung contribution is small under condition of $v \gg 1$ and the total emission yield is determined in the main by PXR in the case considered.

Within the range $v < 1$ the relation between PXR and diffracted bremsstrahlung contributions depends on the value of the parameter $y = \gamma_{\mathrm{TM}}/\gamma$. The curves presented in Fig. 2 demonstrate the small contribution of diffracted bremsstrahlung to total emission yield with the proviso that $y \ll 1$. On the other hand the diffracted bremsstrahlung can contribute significantly at $y \gg 1$, as it will be evident from Fig. 3, where the functions $F_0(x)$ and $F(x)$ differ significantly from each other.

To explain the physical nature of the parameter $v = \gamma_{\mathrm{LPM}}/\gamma_{\mathrm{TM}}$ let us note that the diffracted bremsstrahlung contribution can be substantial only

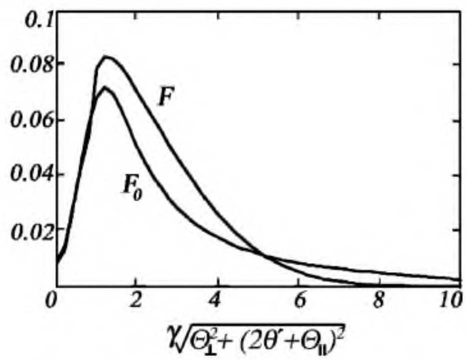

Fig. 2. The emission angular density with and without diffracted bremsstrahlung contribution. The presented functions $F(x)$ and $F_0(x)$ defined by (12) and (13) have been calculated for fixed values of the parameters $z = 0.2$, $\delta = 0.8$, $L/L_{\mathrm{Sc}} = 0.5$, $y = 0.3$ and $v = 0.5$.

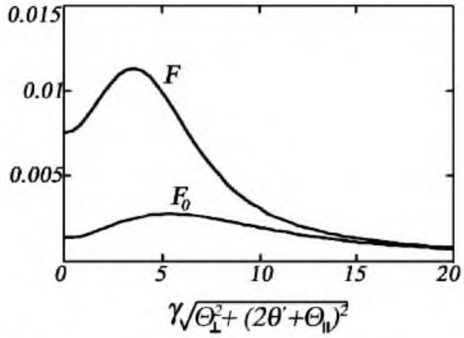

Fig. 3. The same as in Fig. 2, but for the value of the parameter $y = 5$.

when two conditions are fulfilled: $\gamma < \gamma_{\rm TM}$ and $\gamma > \gamma_{\rm LPM}$. Obviously, the first condition implies that the bremsstrahlung suppression due to Ter-Mikaelian effect is negligibly small. To explain the second one it is necessary to note that the trajectory of emitting electron is close to straight line at the distance of the order of the coherence length $l_{\rm coh}$, if $\gamma < \gamma_{\rm LPM}$, because in the case in question the corresponding multiple scattering angle achievable at the coherence length $\Theta_{\rm coh} \approx \gamma_{\rm LPM}^{-1}$ is smaller than the characteristic emission angle of relativistic particle γ^{-1}. As this takes place, the structure of emitting electron's electromagnetic field consisting of both virtual photons of the electron Coulomb field and free photons of the bremsstrahlung is close to that for the electron moving along the rectilinear trajectory with a constant velocity. As a consequence, the structure of the diffracted by crystalline atomic planes electromagnetic field differs little from the ordinary PXR field. On the other hand, in the case of $\gamma \gg \gamma_{\rm LPM}$ the angle of multiple scattering of emitting electron achievable at the coherence length exceeds substantially the value γ^{-1}. Therefore the bremsstrahlung quantum and the electron Coulomb field pull apart at the distance smaller than the coherence length $2\gamma^2/\omega$. On condition under consideration the structure of the electromagnetic field associated with emitting electron differs essentially from the equilibrium Coulomb field of this electron, resulting in an increase of the contribution of bremsstrahlung photons to the formation of the total emission yield. It is clear that the discussed conditions $\gamma > \gamma_{\rm LPM}$ and $\gamma < \gamma_{\rm TM}$ are fulfilled simultaneously with the proviso that $\gamma_{\rm LPM} < \gamma_{\rm TM}$ (or $\nu < 1$) only. In such a manner the contribution of diffracted bremsstrahlung to total yield of the parametric X-ray can be substantial if $\gamma_{\rm TM} > \gamma_{\rm LPM}$ or

$$\omega_{\rm B} > \frac{\omega_{\rm p}^2 L_{\rm Sc}}{2}. \quad (15)$$

It should be noted that $\omega_{\rm p}^2 L_{\rm Sc} \sim Z^{-1}$, therefore the contribution of diffracted bremsstrahlung can be observed more easily in the experiment with heavy crystals.

It is interesting to study the relative contributions of PXR and diffracted bremsstrahlung emission mechanisms to total non-collimated yield. Integrating the general expression (9) over observation angles, one can obtain the following formula:

$$\frac{{\rm d}N}{{\rm d}t} = \frac{e^2\omega_{\rm p}^4}{2g^2}\frac{1}{\omega_{\rm B}}\Phi(y,\delta,\nu), \quad (16a)$$

$$\Phi = -\delta\left[(1+y^2-{\rm i}\delta)\int_0^\infty {\rm d}t\coth(t) \times \exp\left(-\sqrt{\rm i}(1+y^2-{\rm i}\delta)\frac{\nu}{y}t\right) \times \left(1-\exp\left(-\sqrt{\rm i}x_d^2\frac{\nu}{y}\tan h(t)\right)\right)\right], \quad (16b)$$

which is valid for small orientation angles $\varphi < 1$. Here $x_d = \gamma_{\rm TM}\Theta_d$, Θ_d is the photon collimator angular size, which is assumed to be qreater than the angular size of emitting photon flux. The function $\Phi(y)$ should be compared with the analogous function $\Phi_0(y)$ describing the total emission yield without account of the diffracted bremsstrahlung contribution:

$$\frac{{\rm d}N_0}{{\rm d}t} = \frac{e^2\omega_{\rm p}^4}{2g^2}\frac{1}{\omega_{\rm B}}\Phi_0(y,\delta), \quad (17a)$$

$$\Phi_0 = \delta^2\ln\sqrt{\frac{(1+y^2+x_d^2)^2+\delta^2}{(1+y^2)^2+\delta^2}} - \delta(1+y^2) \times \arctan\left(\frac{\delta x_d^2}{(1+y^2)(1+y^2+x_d^2)+\delta^2}\right). \quad (17b)$$

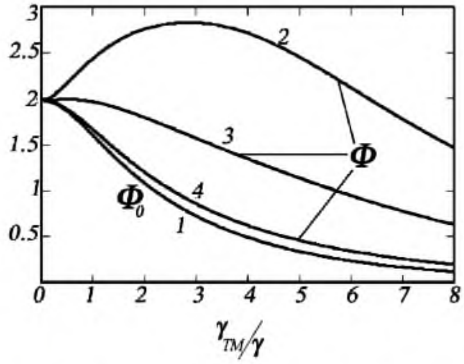

Fig. 4. The total emission intensity as a function of the emitting particle energy. The presented functions $\Phi(y)$ and $\Phi_0(y)$ defined by (16) and (17) have been calculated for fixed parameters $\delta = 0.8$, $x_d = 8$ and different values of the parameter $\nu = 0.3$ (curve 2), 0.5 (curve 3) and 1.3 (curve 4).

The result of a comparison is shown in Fig. 4, where the function $\Phi(y)$ calculated for different values of the parameter ν is presented as well as the function $\Phi_0(y)$. As indicated in the picture, the role of the coefficient ν in the formation of the emission intensity (16) is analogous to that for the emission angular density.

4. Conclusion

The performed analysis of the relative contribution of PXR and diffracted bremsstrahlung to total emission from relativistic electrons moving in a crystal is determined by the manifestation of two classical electrodynamical effects in the physics of relativistic particle bremsstrahlung in matter: Ter-Mikaelian effect of the bremsstrahlung suppression due to the polarization of medium electrons and Landau–Pomeranchuk–Migdal effect of the bremsstrahlung suppression due to multiple scattering of the emitting particle.

Since the emitted photon energy ω is approximately fixed in the discussed resonance emission process ($\omega \approx \omega_B$, ω_B is the Bragg frequency), an influence of the mentioned effects is determined by the relation between the emitting electron Lorentz factor γ and corresponding quantities $\gamma_{TM} = \omega_B/\omega_p$ (ω_p is the plasma frequency) and $\gamma_{LPM} = \sqrt{\omega_B L_{Sc}/2}$ ($L_{Sc} = e^2 L_R/4\pi$, L_R is the radiation length). Ter-Mikaelian effect appears within the range $\gamma > \gamma_{TM}$. Landau–Pomeranchuk–Migdal effect becomes essential, if $\gamma > \gamma_{LPM}$. In accordance with obtained results, the diffracted bremsstrahlung contribution to total emission yield can be substantial under condition of $\gamma_{TM} > \gamma_{LPM}$ only, when the conditions $\gamma < \gamma_{TM}$ and $\gamma > \gamma_{LPM}$ can be fulfilled simultaneously.

The nature of the condition $\gamma < \gamma_{TM}$ is evident in contrast with that for second condition $\gamma > \gamma_{LPM}$. The structure of the electromagnetic field of emitting electron consisting of both virtual photons from the electron Coulomb field and free photons of the bremsstrahlung is close to that for the electron moving along the rectilinear trajectory with a constant velocity if $\gamma < \gamma_{LPM}$. As a consequence, the structure of the diffracted total electromagnetic field differs little from the ordinary PXR field. On the other hand, the condition $\gamma > \gamma_{LPM}$ implies that the multiple scattering angle of emitting electron achievable at the distance $\sim l_{coh}$ exceeds the characteristic emission angle of relativistic particle γ^{-1} and therefore virtual and free photons of the electromagnetic field associated with the emitting electron pull apart at the distance smaller that l_{coh}. As this takes place, the structure of the electron's total electromagnetic field differs substantially from the structure of Coulomb field (among other things, the electron can emit several photons at the distance l_{coh}). Because of this, the diffracted bremsstrahlung relative contribution increases, if $\gamma > \gamma_{LPM}$.

In accordance with presented in this work numerical results, the diffracted bremsstrahlung can contribute substantially to both the emission angular density and the total emission yield, but it is not easy to fulfill the necessary condition $\gamma_{TM} > \gamma_{LPM}$.

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