Grazing incidence parametric X-ray emission

N.N. Nasonov [a,1], P. Zhukova [a], M.A. Piestrup [b], H. Park [b,*]

[a] Belgorod State University, 14 Studencheskaya, 308007 Belgorod, Russia
[b] Adelphi Technology Inc., 981 Industrial Road, Suite B, San Carlos, CA 94070, USA

Abstract

Parametric X-rays (PXR) from relativistic electrons incident at small angles on a surface of a crystalline target is considered as a method for increasing the X-ray yield. The yield can be increased by grazing incidence electrons than for perpendicular-incidence electrons by minimizing the photoabsorption of the emitted X-rays.

Keywords: Parametric X-rays; Grazing incidence; Photoabsorption

PXR is one of the most promising emission mechanisms for producing quasimonochromatic tunable X-rays. Nevertheless, the intensity of PXR must be increased for it to be practical [1]. Since the yield is limited by a photoabsorption, a search for an effective method to minimize its effect is of interest for increasing PXR yield.

One possible way to solve this problem is to use the effect of anomalous photoabsorption [2,3], but this does not result in a substantial increase in yield. Another possibility is to recycling the electrons through the thin PXR target using a cyclical accelerator thereby increasing the average current and total yield [4–7]. The efficiency of this promising approach is limited by thermal heating of the target, and, therefore, the possibility of increasing the yield using a simpler scheme with a single pass of the electrons through the target is of interest.

Another possible increase in yield can be achieved by using electrons that graze the surface of the target, as has been shown for the Cherenkov X-ray emission mechanism [8,9]. For the case of an electron incidence perpendicular to the target surface, the effective electron emission path L_{el} in the target is restricted by the emitted X-ray's photoabsorption length L_{ab}. However, when the electrons are at a grazing angle, L_{el} can exceed L_{ab} substantially [9]. Furthermore, the ratio L_{el}/L_{ab} can be particularly high for PXR due to possibly large emission angles. Note this effect can be achieved only for the asymmetrical diffraction case. The aim of this work is to derive a simple formulation for PXR spectral–angular distribution and demonstrate the advantages of this approach.

It should be noted that the analogous approach known as PXR under conditions of extremely asymmetric diffraction was presented earlier [10,11]. Within the frame of the approach being discussed an emitting electron beam moves inside the crystal parallel to its surface. Since emitted PXR photons fly off the crystal through this surface, the emission yield is proportional to the path of electrons in the target instead of the photoabsorption length L_{ab}. As in our scheme, the approach [10,11] allows to increase substantially PXR yield, but only the part of the electron beam cross-section with linear size of the order of L_{ab} can contribute to the formation of the emission yield.

* Corresponding author. Tel.: +1 650 598 9800x17; fax: +1 650 598 9400.

E-mail addresses: nnn@bsu.edu.ru (N.N. Nasonov), hpark@adelphitech.com (H. Park).

[1] Tel.: +7 722 315726.

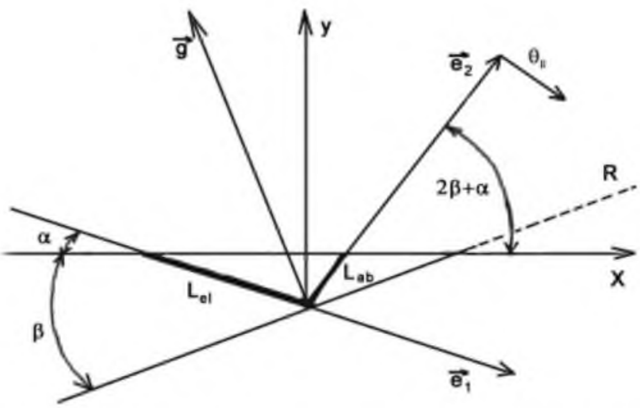

Fig. 1. The geometry of the emission process. R is the reflecting crystallographic plane, $\vec{g}$ is the reciprocal lattice vector, $\vec{e}_1$ is the axis of incident electron beam, $\vec{e}_2$ is the axis of the photon X-ray detector, the axis y coincides with the normal to crystal surface, L_{ab} is the photoabsorption length, L_{el} is the electron path in the target on which X-rays are emitted.

Let us consider PXR from relativistic electrons incident at the surface of crystalline target at small angle α as it is shown in Fig. 1. Using the Fourier-expansion for the crystal dielectric susceptibility $\chi(\omega,\vec{r}) = \chi_0(\omega) + \sum_{\vec{g}}\chi_{\vec{g}}(\omega)e^{i\vec{g}\vec{r}}$, where $\vec{g}$ is the reciprocal lattice vector, one can obtain for the Fourier-transform of electric field $\vec{E}_{\omega\vec{k}} = (2\pi)^{-4}\int dt d^3r\vec{E}(\vec{r},t)e^{-i\vec{k}\vec{r}+i\omega t}$ the following equation:

$$(k^2 - \omega^2(1+\chi_0))\vec{E}_{\omega k} - \vec{k}(\vec{k}\,\vec{E}_{\omega k}) - \omega^2\sum_{\vec{g}}\chi_{-\vec{g}}k\vec{E}_{\omega\,\vec{k}+\vec{g}} = 4\pi i\omega\vec{J}_{\omega\,\vec{k}}, \quad (1)$$

where $\vec{J}_{\omega\vec{k}}$ is the Fourier-transform of an emitting electron current density. Within the frame of well-known two-wave approximation of dynamical diffraction theory [12], Eq. (1) are reduced to simple system

$$\begin{aligned}&\vec{E}_{\omega\vec{k}} \approx \sum_{n=1}^{2}(\vec{e}_{\lambda0}\vec{E}_{\lambda0} + \vec{e}_{\lambda\vec{g}}\vec{E}_{\lambda\vec{g}}),\\&(\vec{k}^2 - \omega^2(1+\chi_0))\vec{E}_{\lambda0} - \omega^2\chi_{-\vec{g}}\alpha_\lambda\vec{E}_{\lambda\vec{g}} = 4\pi i\omega e_{\lambda0}\vec{J}_{\omega\vec{k}},\\&(\vec{k}_g^2 - \omega^2(1+\chi_0))\vec{E}_{\lambda\vec{g}} - \omega^2\chi_{\vec{g}}\alpha_\lambda\vec{E}_{\lambda0} = 0,\end{aligned} \quad (2)$$

where $\vec{k}_g = \vec{k}+\vec{g}, \alpha_1 = 1, \alpha_2 = \vec{k}\vec{k}_g/kk_g, \vec{e}_{10} = \vec{e}_{1\vec{g}} \sim [\vec{k}\vec{g}], \vec{e}_{20} \sim [\vec{k}\vec{e}_{10}], \vec{e}_{2\vec{g}} \sim [\vec{k}_g\vec{e}_{10}]$.

Eq. (2) describe the excited field inside the crystal. These equations, except when $\chi_0 = \chi_{\vec{g}} = \chi_{-\vec{g}} = 0$, describe the emission field in a vacuum outside the target. When determining field components in a vacuum, one should take into account the effect of total external reflection of the primary electromagnetic field associated with incident electron from the interface of the target. The role of this effect becomes substantial in the range of incidence angles α comparable with the critical angle $\sqrt{|\chi_0'|}$. For simplicity sake, assume that $\alpha > \sqrt{|\chi_0'|}$ in our calculations. This assumption allows one to neglect the reflected wave in the boundary conditions at the surface of the target. The emission field in a vacuum has the form

$$\vec{E}_{\lambda\vec{g}}^{\mathrm{vac}} = \alpha_\lambda(\vec{k}_{\vec{g}\|})\delta(\vec{k}_{\vec{g}y} - p), \quad p = \sqrt{\omega^2 - \vec{k}_{\vec{g}\|}^2}. \quad (3)$$

Using the general solutions of Eq. (2) and ordinary boundary conditions at the surface of the target one can determine the unknown coefficient a_λ in (3). The emission amplitude $A_{\lambda\vec{n}}$ follows from the Fourier-integral $E_\lambda^{\mathrm{Rad}} = \int d^3k_g e^{i\vec{k}_g\vec{n}r}E_{\lambda\vec{g}}^{\mathrm{vac}}$ calculated by the stationary phase method.

$$\begin{aligned}&A_{\lambda n} = \frac{e\omega^2\chi_g\alpha_\lambda}{2\pi n_y|\vec{V}_y|}\frac{e_{\lambda0}\vec{V}}{\Delta \pm \sqrt{\Delta^2 - \frac{\omega^2}{n_y^2}\chi_{\vec{g}}\chi_{-\vec{g}} - \alpha_\lambda^2\left(\frac{g_y}{\omega n_y} - 1\right)}}\\&\quad\times\left(\frac{1}{\xi_\perp - \xi_\pm} - \frac{1}{\xi_\perp - \frac{\frac{1}{2}g^2 - \omega\vec{n}\vec{g}}{g_y - \omega\vec{n}_y}}\right),\\&k_{gy} = \omega n_y + \xi_\pm,\\&\xi_\pm = \frac{\omega}{2n_y}\chi_0 + \frac{\omega n_y}{2(g_y - \omega n_y)}\\&\quad\times\left(\Delta \pm \sqrt{\Delta^2 - \frac{\omega^2}{n_g^2}\chi_g\chi_{-g} - \alpha_\lambda^2\left(\frac{g_y}{\omega n_y} - 1\right)}\right),\\&\omega n_y\Delta = \frac{1}{2}g^2 - g_y\omega n_y\left(1 + \frac{1}{2n_y^2}\chi_0\right) - g_x\omega n_x,\\&\xi_\perp = \frac{1}{V_y}(\omega(1 - \vec{n}\vec{V}) + \vec{g}\vec{V}),\end{aligned} \quad (4)$$

where $\vec{n}$ is the unit vector to the direction of emitted photon propagation, the sign $\pm$ in front of the radical in (4) is determined by the solution of dynamical diffraction task.

Using the definitions

$$\begin{aligned}&\vec{V} = \vec{e}_1\left(1 - \frac{1}{2}\gamma^{-2}\right), \quad \vec{n} = \vec{e}_2\left(1 - \frac{1}{2}\theta^2\right) + \bar{\theta}, \quad \vec{e}_2\bar{\theta} = 0,\\&\chi_0 = -\frac{\omega_0^2}{\omega^2} + i\chi_0'', \quad \chi_{\vec{g}} = \chi_{-\vec{g}} = -\frac{\omega_g^2}{\omega^2} + i\chi_g''\end{aligned} \quad (5)$$

and separating the total amplitude $A_{\lambda\vec{n}}$ into PXR and diffracted transition radiation amplitudes one can obtain from (4) the following expression for PXR spectral–angular distribution

$$\begin{aligned}&\omega\frac{dN_\lambda^{\mathrm{PXR}}}{d\omega d^2\theta}\\&= \frac{e^2}{\pi^2}\frac{\theta_\lambda^2\alpha_\lambda^2}{(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \theta^2)^2}\left|x \pm D_\lambda - \frac{\omega^2}{\omega_g^2}\left(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \theta^2 - i\chi_0''\frac{\sin\beta\cos(\beta+\alpha)}{\sin(2\beta+\alpha)}\right)\right|^{-2},\\&x = \frac{g^2}{2\omega_g^2}\left(\frac{\omega_B'}{\omega} - 1 + 2\frac{\omega_0^2}{g^2}\frac{\cos\beta\sin(\beta+\alpha)}{\sin(2\beta+\alpha)}\right),\\&\omega_B' = \frac{g^2}{2\vec{n}\vec{g}} \cong \frac{g}{2\sin(\beta+\alpha)}(1 + \theta_\|\cot(\beta+\alpha)),\\&D_\lambda = \sqrt{x^2 - \alpha_\lambda^2\frac{\sin\alpha}{\sin(2\beta+\alpha)} - 2i\frac{\omega^2}{\omega_g^2}\left(x\chi''\frac{\cos\beta\sin(\beta+\alpha)}{\sin(2\beta+\alpha)} - \chi_g''\alpha_\lambda^2\frac{\sin\alpha}{\sin(2\beta+\alpha)}\right)},\end{aligned} \quad (6)$$

where $\theta_1 = \theta_\perp, \theta_2 = \theta_\|, \theta^2 = \theta_1^2 + \theta_2^2, \omega_g^2 = \omega_0^2(F(g)/Z)(|S(\vec{g})|/N_0)e^{-\frac{1}{2}g^2U_T^2}$, ω_0 is the plasma frequency, $F(g)$ is the atomic form factor, Z is the number of electrons in an atom, $S(\vec{g})$ is the structure factor of an elementary call containing N_0 atoms, U_T is the meansquare amplitude of thermal vibrations of atoms.

Formula (6) takes into account an influence of dynamical diffraction effects in PXR. This result provide an analysis of the emission from high energy electrons. We consider the case of PXR from electrons with relatively small energies of the order of tens MeV that is of prime interest for X-ray source creating. Assuming that $\gamma \ll \gamma^* = \omega_B/\omega_0 = g/2\omega_0 \sin(\beta+\alpha)$, one can obtain the very simple formula

$$\omega \frac{dN_\lambda^{PXR}}{d\omega d^2\theta} = \frac{e^2\omega_g^4}{\pi g^2} \frac{\theta_\lambda^2 \alpha_\lambda^2}{(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \theta^2)^2} \frac{\sin(2\beta+\alpha)}{\omega \chi_0'' \sin\alpha} \delta(\omega - \omega_B') \tag{7}$$

Obviously, the obtained result differs from the ordinary formula for PXR spectral–angular distributions by the coefficient $\sin(2\beta+\alpha)/\sin(\alpha)$ only, so that all well-known PXR properties are preserved in the case being considered. Physical meaning of this coefficient is very simple. It is equal to the ratio of the effective electron path in the target L_{el} (PXR yield is formed on the distance L_{el}) to photon absorption length L_{ab} as may be inferred from Fig. 1.

The conditions best suited to increase PXR spectral–angular density are $\alpha \ll 1$ and $\beta = \pi/4$ in accordance with (7) and Fig. 1 (obviously, σ – polarization dominates). Such conditions can be realized easily in an experiment. As this takes place the emission density increases by a factor of $1/\alpha \gg 1$ (one should remember that $\alpha > \sqrt{|\chi_0'|}$ in obtained results). It should be noted that the multiple scattering of emitting electrons must be taken into account when calculating the parameters of X-ray source in conditions of grazing incidence under consideration.

Fig. 1 shows that the size of emitted X-ray beam can be large. This circumstance may be of great interest for X-ray lithography.

In order to estimate the total PXR yield one should integrate formula (7) over emitted photon energies and observation angles. The result of integration

$$N(\alpha,\beta) = \frac{2e^2\omega_g^4}{g^4}\left[\ln(1+\gamma^2\theta_d^2) - \frac{\gamma^2\theta_d^2}{1+\gamma^2\theta_d^2}\right] \cdot \left[\frac{\sin(2\beta+\alpha)\cdot\sin^2(\beta+\alpha)\cdot(1+\cos^2(2\beta+2\alpha))}{\sin(\alpha)\chi_0''\left(\frac{g}{2\sin(\beta+\alpha)}\right)}\right] \tag{8}$$

allows one to study a dependence of PXR yield on the orientation angles α and β. Here θ_d is the angular size of photon collimator.

As an example we consider the total number of photons N emitted from a relativistic electron penetrating a thick crystal of diamond (2 2 0). The dependence $N(\alpha)$ calculated by (8) for different β is illustrated by the curves presented in Fig. 2. The enhancement of PXR yield may be very substantial in accordance with obtained results. The field of application of the presented results is wider than that showed in Fig. 2 (see caption of Fig. 2). It should be noted that the unexpected dependence of the yield on the angle β (particularly in the range of small α) is caused by the strong dependence of a photoabsorption coefficient on the value of Bragg frequency $\omega_B(\beta)$, which increase rapidly with decreasing β.

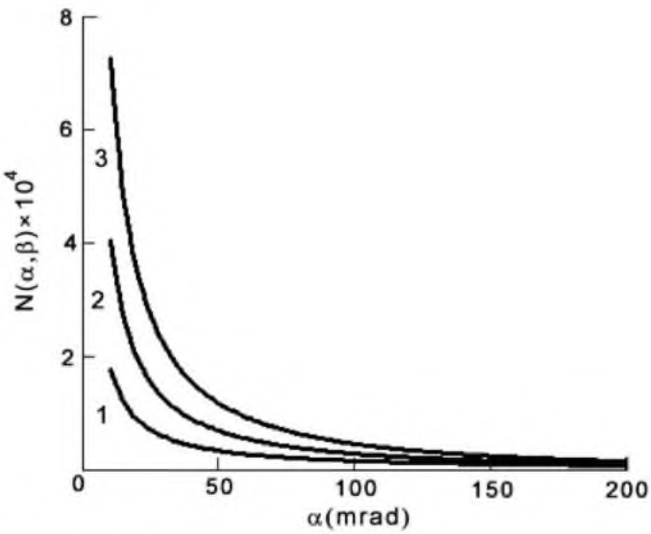

Fig. 2. Total number of emitted quanta versus a grazing angle. All curves are calculated in the case when $\gamma\theta_d = 2$. (1) $\beta = \pi/4$, the curve describes the yield correctly in the range $\alpha > 3.6$ mrad; (2) $\beta = \pi/6$, the curve describes the yield correctly in the range $\alpha > 2.6$ mrad; (3) $\beta = \pi/8$, the curve describes the yield correctly in the range $\alpha > 2$ mrad.

Acknowledgments

This work was supported by the United States National Institutes for Health under Small Business Innovation Research (SBIR) program (Grant No. 5-R44CA086545-03) and by the Russian Ministry of Education and Science under the program "Advancement of Science Potential of High Education" (Grant No. RNP.2.1.1.3263). The authors are grateful to the referee for useful remarks.

References

[1] R. Carr, Nucl. Instr. and Meth. B 122 (1994) 625.
[2] G. Garibian, C. Yang, Zh. Eksp. Toer. Fiz. 63 (1972) 1198.
[3] N. Nasonov, Phys. Lett. A 292 (2001) 146.
[4] M. Andreyashkin, V. Kaplin, M. Piestrup, S. Uglov, V. Zabaev, Appl. Phys. Lett. 72 (1998) 1385.
[5] V. Kaplin, S. Uglov, O. Bulaev, V. Goncharov, M. Piestrup, C. Gary, Nucl. Instr. and Meth. B 173 (2001) 3.
[6] M. Piestrup, L. Lombardo, J. Cremer, G. Retzlaf, R. Silzer, D. Skopik, V. Kaplin, Rev. Sci. Instr. 69 (1998) 2223.
[7] N. Nasonov, A. Kubankin, V. Kaplin, S. Uglov, M. Piestrup, C. Gary, Nucl. Instr. and Meth. B 227 (2005) 216.
[8] W. Knulst, Ph.D.Thesis, Technische Universiteit, Eindhoven, 2004.
[9] C. Gary, V. Kaplin, A. Kubankin, N. Nasonov, M. Piestrup, S. Uglov, Nucl. Instr. and Meth. B 227 (2005) 95.
[10] V.G. Baryshevsky, I.D. Feranchuk, A.O. Grubich, Nucl. Instr. and Meth. A 249 (1986) 306.
[11] V.G. Baryshevsky, I.D. Feranchuk, A.P. Ulyanenkov, Parametric X-ray Radiation: Theory, Experiment and Applications, Springer, Berlin, 2005.
[12] Z. Pinsker, Dynamic Scattering of X-rays in Crystals, Springer, Berlin, 1981.