Anomalous photoabsorption in the parametric X-rays in conditions of Cherenkov effect

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Abstract

Parametric X-rays from relativistic electrons moving through a periodic medium is considered. It is shown that the emission yield can be increased substantially in conditions of Cherenkov effect.

Keywords: Parametric X-rays; Anomalous photoabsorption; Cherenkov effect

1. The effect of anomalous photoabsorption (Bormann effect) can increase the angular density of parametric X-rays (PXR) from relativistic electrons moving through a periodic medium [1], but the growth of total PXR yield was found to be not very substantial. This is due to the difference between dispersion laws for primary virtual photons of emitting particle Coulomb field and for real photons of emission field. Indeed, the condition of Bragg resonance cannot be asserted exactly for the ordinary PXR outside the frequency range of anomalous dispersion because of outlined difference. As a consequence, the discussed effect of anomalous photoabsorption provided as example of dynamical diffraction effects in PXR does not manifests in full measure.

It is believed that the influence of anomalous photoabsorption on PXR properties increases in conditions of Cherenkov effect because the Bragg resonance between primary Cherenkov wave and diffracted PXR wave is possible. The aim of this work is to check the presented idea theoretically.

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2. Let us consider an emission from relativistic electrons flying out a thick periodical target (the thickness of the target exceeds the photoabsorption length $l_{\rm ab} = 1/\omega\chi_0''$, χ_0'' is the imaginary part of average dielectric susceptibility of the target, the full susceptibility is given by the expression $\chi(\omega, \mathbf{r}) = \chi_0(\omega) + \sum'_{\mathbf{g}} \chi_{\mathbf{g}}(\omega) e^{i\mathbf{gr}}$, $\mathbf{g}$ are reciprocal lattice vectors). Defining the structure of Fourier-transform of the electric field $\mathbf{E}_\omega(\mathbf{r}) = (2\pi)^{-1} \int dt\, e^{i\omega t} \mathbf{E}(\mathbf{r}, t)$ within the frame of two-wave approximation of dynamical diffraction theory [2]

$$\mathbf{E}_\omega \simeq \sum_{\lambda=1}^{2} \left[\int d^3k\, e^{i\mathbf{kr}} \mathbf{e}_{\lambda 0} E_{\lambda 0} + \int d^3k_g\, e^{i\mathbf{k}_g\mathbf{r}} \mathbf{e}_{\lambda\mathbf{g}} E_{\lambda\mathbf{g}} \right], \tag{1}$$

one can reduce general Maxwell equations to well-known in the physics of PXR system:

$$\begin{aligned} &\big(k^2 - \omega^2(1+\chi_0)\big)\mathbf{E}_{\lambda 0} - \omega^2 \chi_{-\mathbf{g}} \alpha_\lambda E_{\lambda\mathbf{g}} = \frac{i\omega e}{2\pi^2} \mathbf{e}_{\lambda 0} \mathbf{V} \delta(\omega - \mathbf{kV}), \\ &\big(k_g^2 - \omega^2(1+\chi_0)\big)\mathbf{E}_{\lambda\mathbf{g}} - \omega^2 \chi_{\mathbf{g}} \alpha_\lambda E_{\lambda 0} = 0. \end{aligned} \tag{2}$$

Components $E_{\lambda 0}$ describe the electromagnetic field propagating along the velocity of emitting electron $\mathbf{V}$, whereas the components $E_{\lambda\mathbf{g}}$ correspond to the diffracted field. The following designations are used in (1) and (2): $\mathbf{e}_{\lambda 0}$ and $\mathbf{e}_{\lambda\mathbf{g}}$ are the polarization vectors, $\mathbf{e}_{10} = \mathbf{e}_{1\mathbf{g}} \sim [\mathbf{k}, \mathbf{g}]$, $\mathbf{e}_{20} \sim [\mathbf{k}, \mathbf{e}_{10}]$, $\mathbf{e}_{2\mathbf{g}} \sim [\mathbf{k}_g, \mathbf{e}_{10}]$, $\mathbf{k}_g = \mathbf{k} + \mathbf{g}$, $\alpha_1 = 1$, $\alpha_2 = \mathbf{e}_{20}\mathbf{e}_{2\mathbf{g}}$. In the case of semi-infinite target under consideration the solution of (2) is given by

$$E_{\lambda\mathbf{g}} = \frac{i\omega e}{2\pi^2} \frac{\omega^2 \chi_{\mathbf{g}} \alpha_\lambda (\mathbf{e}_{\lambda 0}\mathbf{V}) \delta(\omega - \mathbf{kV})}{\big(k^2 - \omega^2(1+\chi_0)\big)\big(k_g^2 - \omega^2(1+\chi_0)\big) - \omega^4 \chi_g \chi_{-g} \alpha_\lambda^2}. \tag{3}$$

PXR field in a vacuum outside the target $E_{\lambda\mathbf{g}}^{\rm Rad}$ is determined by the system (2) in the limit $\chi_0 = \chi_{\mathbf{g}} = \chi_{-\mathbf{g}} = 0$:

$$E_{\lambda\mathbf{g}}^{\rm Rad} = a_\lambda(\mathbf{k}_{g_\parallel}) \delta\Big(k_{g_x} - \sqrt{\omega^2 - k_{g_\parallel}^2}\Big). \tag{4}$$

The coefficient a_λ in (4) is determined by the ordinary boundary conditions at the surface of the target. Calculating the Fourier integral $E_\lambda^{\rm Rad} = \int d^3k_g\, e^{i\mathbf{k}_g\mathbf{r}} E_{\lambda g}^{\rm Rad}$ in wave-zone by the stationary phase method one can obtain the following formula for the emission amplitude A_λ:

$$\begin{aligned} A_\lambda &= -2\pi i\omega n_x a_\lambda(\omega\mathbf{n}_\parallel) = \frac{e}{\pi} \frac{n_x}{V_x} \frac{\chi_{\mathbf{g}} \alpha_\lambda \mathbf{e}_{\lambda 0} \mathbf{V}}{D \cdot \big(D + 2\frac{g}{\omega} n_y \big(\frac{g}{2\omega n_y} - 1\big)\big) - \chi_g \chi_{-g} \alpha_\lambda^2}, \\ D &= \left[\frac{1}{V_x^2} \left(1 + n_y V_y + 2 n_y V_y \left(\frac{g}{2\omega n_y} - 1 \right) \right)^2 - n_x^2 - \chi_0 \right], \end{aligned} \tag{5}$$

where $\mathbf{n} = \mathbf{e}_x n_x + \mathbf{e}_y n_y + \mathbf{e}_z n_z$ is the unit vector to the direction of emitted photon propagation.

Geometry of the emission process being considered is shown in Fig. 1. Introducing the new variables by the formulae

$$\begin{aligned} &\mathbf{V} = \mathbf{e}_1 \left(1 - \frac{1}{2}\gamma^{-2} \right), \\ &\mathbf{n} = \mathbf{e}_2 \left(1 - \frac{1}{2}\theta^2 \right) + \bar{\theta}, \qquad \mathbf{e}_2 \bar{\theta} = 0, \qquad \mathbf{e}_1 \mathbf{e}_2 = \cos\varphi = \alpha_2, \end{aligned} \tag{6}$$

one can obtain from (5) the formula

$$\omega \frac{d^3 N_\lambda}{d\omega\, d^2\theta} = \frac{e^2}{\pi^2} \frac{|\chi_g|^2 \Omega_\lambda^2 \alpha_\lambda^2}{\big[\Delta_0(\Delta_0 - 2\Delta) - \chi_g' \chi_{-g}' \alpha_\lambda^2\big]^2 + \big[2\chi_0''(\Delta_0 - \Delta) + \big(\chi_g'' \chi_{-g}' + \chi_g' \chi_{-g}''\big)\alpha_\lambda^2\big]^2}, \tag{7}$$

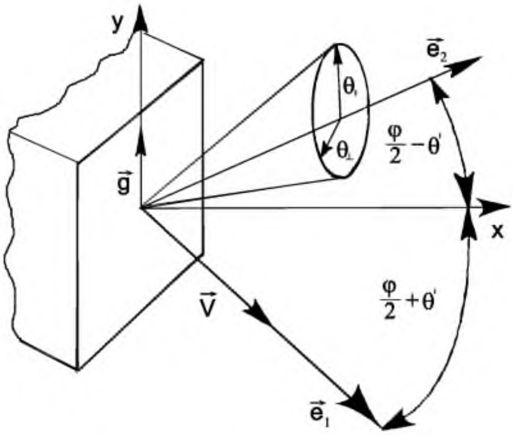

Fig. 1. The geometry of the emission process. **V** is the velocity of the emitting electron, $\mathbf{e}_2$ is the axis of the emitted photon beam, **g** is the reciprocal lattice vector, φ is the fixed emission angle, θ' is the orientation angle describing the turning of the target by the goniometer.

describing the emission spectral–angular distribution. Here

$$\begin{aligned}
&\Omega_1 = \theta_\perp, \qquad \Omega_2 = 2\theta' + \theta_\parallel, \qquad \chi_g = \chi_g' + i\chi_g'', \\
&\Delta_0 = \gamma^{-2} - \chi_0' + \Omega, \qquad \Delta = 2\sin^2\left(\frac{\varphi}{2}\right)\left(\frac{\omega_B'}{\omega} - 1\right), \\
&\omega_B' = \omega_B\left(1 + (\theta' + \theta_\parallel)\operatorname{ctg}\left(\frac{\varphi}{2}\right)\right), \qquad \omega_B = \frac{g}{2}\sin\left(\frac{\varphi}{2}\right).
\end{aligned} \tag{8}$$

3. Let us analyze the obtained result (7). Assuming imaginary parts of dielectric susceptibilities χ_0'' and χ_g'' much less than real parts χ_0' and χ_g' and using the well-known limit $(x^2 + \alpha^2)^{-1} \to (\pi/\alpha)\delta(x)$ one can represent the result (7) in more convenient form

$$\omega \frac{d^3 N_\lambda}{d\omega\, d^2\theta} = \frac{e^2\omega^2(\chi_g')^2}{4\pi \sin^2\left(\frac{\varphi}{2}\right)} \frac{1}{\omega\chi_0''} \frac{\Omega_\lambda^2 \alpha_\lambda^2 \delta(\omega - \omega_B')}{\left(\gamma^{-2} - \chi_0' + \Omega^2 + \kappa_\lambda\sqrt{\chi_g'\chi_{-g}'}\,\alpha_\lambda\right)^2 + \chi_g'\chi_{-g}'\alpha_\lambda^2\left(1 - \kappa_\lambda^2\right)}, \tag{9}$$

where the very important parameter κ_λ determining an influence of anomalous photoabsorption on PXR is given by

$$\kappa_\lambda = \frac{\chi_g''\chi_{-g}' + \chi_g'\chi_{-g}''}{2\sqrt{\chi_g'\chi_{-g}'}\,\chi_0''}\alpha_\lambda \approx 1. \tag{10}$$

Obviously, formula (9) is reduced to well-known formula for kinematic PXR [3] in the limit $\gamma^2\sqrt{\chi_g'\chi_{-g}'} \to 0$. There is no anomalous photoabsorption in this instance. On the other hand, the influence of anomalous photoabsorption can increase within the range of dynamic PXR $\gamma^2\sqrt{\chi_g'\chi_{-g}'} \geqslant 1$. It is easy to verify that the result (9) coincides with that obtained in [1] in the frequency range far from the vicinity of a photoabsorption edge of the target's material, where the dielectric susceptibility χ_0' is close to the quantity $-\omega_0^2/\omega^2$, ω_0 is the plasma frequency of the target. In accordance with (9) PXR intensity increases with increasing κ_λ but this enhancement is, in the instance, not very substantial even though $\kappa_\lambda = 1$ because the function $\gamma^{-2} - \chi_0' + \Omega^2 + \kappa_\lambda\sqrt{\chi_g'\chi_{-g}}\,\alpha_\lambda$ cannot be equal to zero in the case in question.

It can be seen that the situation is changed dramatically in the vicinity of a photoabsorption edge when the susceptibility χ_0' can take positive values and the function $\gamma^{-2} - \chi_0' + \Omega^2 + \kappa_\lambda \sqrt{\chi_g' \chi_{-g}}\, \alpha_\lambda$ can be equal to zero (obviously, this is possible in the angular and frequency range, where the Cherenkov threshold $\gamma^{-2} - \chi_0' + \Omega^2 < 0$ is achieved). As may be inferred from (9) $dN_\lambda / d\omega\, d^2\theta \to \infty$ if $\kappa_\lambda \to 1$ in conditions under consideration.

The discussed peculiarity can best be appreciated from the total PXR spectrum $dN_\lambda / d\omega$. Since the photon energy ω in argument of δ-function in (9) does not depend on the angle $\theta_\perp$, the best way to increase PXR yield without distortion of the emission spectrum is to use photons with σ-polarization. In this case spectral intensity is described by the formula

$$\omega \frac{dN_\sigma}{d\omega} = \int_{-\infty}^{\infty} d\theta_\perp \int_{-\frac{1}{2}\Delta\theta_\parallel}^{\frac{1}{2}\Delta\theta_\parallel} d\theta_\parallel\, \omega \frac{dN_\sigma}{d\omega\, d^2\theta} = \frac{e^2}{2\sqrt{2}\sin\varphi} \frac{(\chi_g')^2}{\chi_0''} \frac{\eta(\omega - \omega_B^{(-)})\eta(\omega_B^{(+)} - \omega)}{\sqrt{\sqrt{A^2 + B^2} + A}},$$

$$A = \gamma^{-2} - \chi_0' + \kappa_1 \sqrt{\chi_g' \chi_{-g}'} + \left(g' + \mathrm{tg}\left(\frac{\varphi}{2}\right)\left(1 - \frac{\omega_B}{\omega}\right)\right)^2, \qquad B = \sqrt{\chi_g' \chi_{-g}'}\sqrt{1 - \kappa_1^2},$$

$$\omega_B^{(\pm)} = \omega_B\left(1 + \left(\theta' \pm \frac{1}{2}\Delta\theta_\parallel\right)\mathrm{tg}\left(\frac{\varphi}{2}\right)\right). \tag{11}$$

The result (11) shows that PXR yield in conditions of the manifestation of anomalous photoabsorption ($\kappa_1 \approx 1$) depends strongly on the sign of the coefficient A in (11). Indeed, in the case $|A| \gg B$ best suited to producing of intense X-ray beams the ratio $dN_{\sigma(A<0)} / dN_{\sigma(A>0)} \sim |A|/B \gg 1$.

It should be noted that the condition $\chi_0' > 0$ is usually realized in the range of soft X-rays, so macroscopic multilayer nanostructures hold the greatest interest for experimental studies of the emission process being considered in this Letter.

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