Parametric X-rays along the velocity direction of an emitting particle under conditions of the Cherenkov effect

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Abstract

Peculiarities in the parametric X-rays along the velocity direction of a relativistic electron emitting in a regime where the Cherenkov effect is important are considered. It is shown that the emission yield can be increased substantially due to the modification of the anomalous photoabsorption.

Keywords: Parametric X-rays; Anomalous photoabsorption; Cherenkov effect; Dynamical diffraction

1. Introduction

The theory of parametric X-rays (PXR) from fast-moving charged particles in a periodic medium predicts the existence of a PXR peak not only in the direction of Bragg scattering but also along the velocity direction of the emitting particle (Garibian and Yang, 1971; Baryshevsky and Feranchuk, 1971; Baryshevsky, 1997; Kubankin et al., 2003). Such an additional X-ray peak (the forward PXR) is of great interest because up to now the question concerning its nature as a Cherenkov–like or scattering process, is still open.

It should be noted that the forward PXR from crystals has only recently been the subject of investigations (see, e.g. the experimental observations of Kube et al., 2001; Aleinik et al., 2004). Because of this, only the Bragg emission process in the hard X-ray range, where the average dielectric susceptibility of the target $\chi_0(\omega)$ is negative, has been studied. Forward PXR in the soft X-ray range, where χ_0 can be positive in the vicinity of a photoabsorption edge of the target material, is considered in our paper. Macroscopic periodic nanostructures are best suited for the realization of such an emission process. The aim of this work is to describe the peculiarities in forward PXR appearing under conditions of the Cherenkov effect. Among other things, substantial enhancement of the PXR intensity is predicted in our paper.

2. Statement of the problem

Let us consider the structure of the electromagnetic field excited by a relativistic electron emitted from a target with a periodically changing dielectric susceptibility $\chi(\omega, \mathbf{r}) = \chi_0(\omega) + \sum_{\mathbf{g}\neq 0}\chi_{\mathbf{g}}(\omega)e^{i\mathbf{g}\cdot\mathbf{r}}$, in which the $\mathbf{g}$'s are reciprocal lattice vectors. The geometry of the emission process is presented in Fig. 1. A simple model of a semi-infinite target is used to describe the emission characteristics, so that the thickness of the target is larger than an absorption length. Starting from the general Maxwell equations for the Fourier-transform of an electric field $\mathbf{E}_{\omega\mathbf{k}} = (2\pi)^{-4} \int \mathrm{d}t\, \mathrm{d}^3 r \mathbf{E}(\mathbf{r}, t) \mathrm{e}^{\mathrm{i}\omega t - \mathrm{i}\mathbf{k}\cdot\mathbf{r}}$

$$(k^2 - \omega^2(1+\chi_0))\mathbf{E}_{\omega\mathbf{k}} - \mathbf{k}(\mathbf{k}\cdot\mathbf{E}_{\omega\mathbf{k}}) - \omega^2 \sum_{\mathbf{g}\neq\mathbf{0}} \chi_{-\mathbf{g}}(\omega)\mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\mathbf{V}\delta(\omega - \mathbf{k}\cdot\mathbf{V}), \quad (1)$$

we determine the excited field within the framework of the two-wave approximation of dynamical diffraction theory (Pinsker, 1984) often and with great success employed in theoretical studies of PXR. The quantity $\mathbf{V} = (V_x, V_y, V_z)$ in Eq. (1) is the velocity of an emitting electron. Reasoning that

$$\mathbf{E}_{\omega\mathbf{k}} \simeq \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda 0} E_{\lambda 0}, \quad \mathbf{k}\cdot\mathbf{e}_{\lambda 0} = 0,$$
$$\mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} \simeq \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda g} E_{\lambda \mathbf{g}}, \quad (\mathbf{k}+\mathbf{g})\cdot\mathbf{e}_{\lambda g} = 0, \quad (2)$$

where $\mathbf{e}_{10} = \mathbf{e}_{1\mathbf{g}} \sim \mathbf{k}\times\mathbf{g}$, $\mathbf{e}_{20} \sim \mathbf{k}\times\mathbf{e}_{10}$, $\mathbf{e}_{2\mathbf{g}} \sim (\mathbf{k}+\mathbf{g})\times\mathbf{e}_{10}$ are the polarization vectors, one can reduce (1) to the simple relationships

$$(k^2 - \omega^2(1+\chi_0))\mathbf{E}_{\lambda 0} - \omega^2 \chi_{-\mathbf{g}}\alpha_\lambda E_{\lambda\mathbf{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\mathbf{e}_{\lambda 0}\cdot\mathbf{V}\delta(\omega - \mathbf{k}\cdot\mathbf{V}), \quad (3)$$

$$((\mathbf{k}+\mathbf{g})^2 - \omega^2(1+\chi_0))\mathbf{E}_{\lambda\mathbf{g}} - \omega^2\chi_{\mathbf{g}}\alpha_\lambda E_{\lambda 0} = 0$$

well known in PXR theory. Here, $\alpha_1 = 1$, $\alpha_2 = (\mathbf{k}\cdot(\mathbf{k}+\mathbf{g}))/k|\mathbf{k}+\mathbf{g}|$.

In the case of a Laue scattering geometry and a semi-infinite target the solution of Eq. (3) can be determined

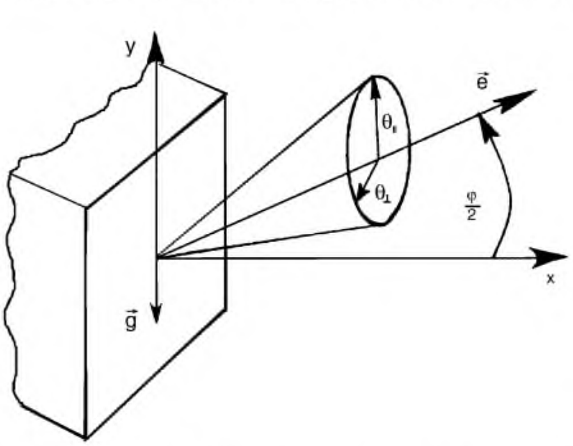

Fig. 1. The geometry of the emission process. **g** is the reciprocal lattice vector, **e** is the axis of the X-ray detector, $\varphi/2$ is the orientation angle, $\theta_\parallel$ and $\theta_\perp$ are the observation angles.

without regard for the free waves in the target. Then the components $E_{\lambda 0}$ are given by

$$E_{\lambda 0} = \frac{\mathrm{i}\omega e}{2\pi^2}\frac{((\mathbf{k}+\mathbf{g})^2 - \omega^2(1+\chi_0))(\mathbf{e}_{\lambda 0}\cdot\mathbf{V})\delta(\omega - \mathbf{k}\cdot\mathbf{V})}{(k^2 - \omega^2(1+\chi_0))((\mathbf{k}+\mathbf{g})^2 - \omega^2(1+\chi_0)) - \omega^4\chi_g\chi_{-g}\alpha_\lambda^2}. \quad (4)$$

The solution in the vacuum behind the target follows from (3) with $\chi_0 = \chi_g = \chi_{-g} = 0$

$$E_{\lambda 0}^{\mathrm{Vac}} = \frac{\mathrm{i}\omega e}{2\pi^2}\frac{(\mathbf{e}_{\lambda 0}\cdot\mathbf{V})\delta(\omega - \mathbf{k}\cdot\mathbf{V})}{(k^2 - \omega^2)} + a_{\lambda\mathbf{k}_\parallel}\delta\left(k_x - \sqrt{\omega^2 - k_\parallel^2}\right), \quad (5)$$

where $\mathbf{k}_\parallel = \mathbf{k}_y + \mathbf{k}_z$.

Determining the coefficient $a_{\lambda\mathbf{k}_\parallel}$ by boundary conditions at the surface of the target and calculating the Fourier-integral $E_\lambda^{\mathrm{Rad}} = \int \mathrm{d}^3 k \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} a_{\lambda\mathbf{k}_\parallel}\delta(k_x - \sqrt{\omega^2 - k_\parallel^2})$ in the wave-zone (at the distance from an emitting electron much greater than the emission formation length $l_{\mathrm{coh}} \approx (2/\omega)(\gamma^{-2} - \chi_0 + \theta^2)^{-1}$) by the stationary phase method one can obtain the following formula for the emission amplitude A_λ

$$E_\lambda^{\mathrm{Rad}} \to A_\lambda \frac{\mathrm{e}^{\mathrm{i}\omega r}}{r},$$

$$A_\lambda = -2\pi\mathrm{i}\omega n_x a_{\lambda\omega\mathbf{n}_\parallel} = \frac{e}{\pi}\frac{n_x}{V_x}(\mathbf{e}_{\lambda 0}\cdot\mathbf{V})\left[\frac{D + (2gn_y/\omega)(g/2\omega n_y - 1)}{D\cdot(D + 2(gn_y/\omega)(g/2\omega n_y - 1)) - \chi_g\chi_{-g}\alpha_\lambda^2} - \frac{1}{D+\chi_0}\right], \quad (6)$$

$$D = \frac{1}{V_x^2}(1 - n_y V_y)^2 - n_x^2 - \chi_0.$$

Here, $\mathbf{n} = (n_x, n_y, n_z)$ is the unit vector denoting the direction of the emitted photon propagation. Using angular variables $\theta_\parallel$ and $\theta_\perp$ and φ introduced in Fig. 1 one can simplify (6) and represent A_λ in the final form

$$A_\lambda = \frac{e}{2\pi}\theta_\lambda\left[\left(1 + \frac{\Delta}{\kappa_\lambda}\right)\frac{1}{\Delta_0 + \Delta - \kappa_\lambda} + \left(1 - \frac{\Delta}{\kappa_\lambda}\right)\frac{1}{\Delta_0 + \Delta + \kappa_\lambda} - \frac{2}{\gamma^{-2} + \theta^2}\right] \quad (7)$$

in which

$$\Delta_0 = \gamma^{-2} - \chi_0 + \theta^2, \quad \kappa_\lambda = \sqrt{\Delta^2 + \chi_g\chi_{-g}\alpha_\lambda^2},$$
$$\Delta = 2\sin^2\left(\frac{\varphi}{2}\right)\left(\frac{\omega_{\mathrm{B}}'}{\omega} - 1\right)$$

and where the first term is inversely related to the emission formation length l_{coh}, the second term describes an influence of dynamical diffraction effects, the last term is the so-called Bragg resonance defect, describing the deviation of the system of primary virtual photon and diffracted PXR wave from the condition of exact Bragg resonance, γ is the Lorentz factor of an emitting electron, $\omega'_B = \omega_B(1 - \theta_\| \mathrm{ctg}(\varphi/2))$, $\omega_B = g/2\sin(\varphi/2)$ is the Bragg frequency, $\theta_1 = \theta_\perp$, $\theta_2 = \theta_\|$.

3. Results and implications

Let us analyze the emission characteristics on the basis of Eq. (7). First of all it should be noted that dynamical diffraction effects with enhanced A_λ are only manifest in the narrow vicinity of the Bragg frequency ω_B, where the resonance defect Δ satisfies the condition $|\Delta| \leqslant \sqrt{\chi_g\chi_{-g}}\alpha_\lambda$. Indeed, outside this frequency range, the amplitude A_λ reduces to the traditional one

$$A_\lambda\Big|_{|\Delta| \gg \sqrt{\chi_g\chi_{-g}}\alpha_\lambda} \to \frac{e}{\pi}\theta_\lambda\left(\frac{1}{\gamma^{-2} - \chi'_0 + \theta^2 - \mathrm{i}\chi''_0} - \frac{1}{\gamma^{-2} + \theta^2}\right) \tag{8}$$

describing the contribution of the ordinary transition and Cherenkov radiation. The latter contribution is relevant in the frequency range, where $\chi'_0 - \gamma^{-2} - \theta^2 > 0$.

Our interest is in describing the emission properties under conditions when the Cherenkov and dynamical diffraction effects contribute simultaneously to the emission yield. As may be evident from (7), two branches of the forward PXR waves (two first terms in (7) correspond to the contribution of PXR branches, but these terms include the contribution of transition radiation as well) and the transition radiation determine the total yield. With the fundamental aspect of the problem being studied in mind we only consider the contribution of two branches of PXR without taking into account the transition radiation and interference items which have no impact on the conclusions of this work. Assuming $\chi''_0 \ll \chi'_0 \sim \Delta_0$ and using the limit $1/(x^2 + \alpha^2) \to (\pi/\alpha)\delta(x)$, one can obtain from (7) the following formula for the emission spectral-angular distribution:

$$\omega\frac{\mathrm{d}^3 N_\lambda}{\mathrm{d}\omega\,\mathrm{d}^2\theta} = \frac{e^2\theta_\lambda^2}{4\pi\chi''_0}\left[\left(1 + \frac{\Delta}{\kappa'_\lambda}\right)^2 \frac{\delta(\Delta'_0 + \Delta - \kappa'_\lambda)}{1 + \delta_\lambda\sqrt{\chi'_g\chi'_{-g}}\frac{\alpha_\lambda}{\kappa'_\lambda}} + \left(1 - \frac{\Delta}{\kappa'_\lambda}\right)^2 \frac{\delta(\Delta'_0 + \Delta + \kappa'_\lambda)}{1 - \delta_\lambda\sqrt{\chi'_g\chi'_{-g}}\frac{\alpha_\lambda}{\kappa'_\lambda}} + \cdots\right], \tag{9}$$

where Δ'_0 and κ'_λ are real parts of quantities Δ_0 and κ_λ. The parameter δ_λ describes the influence of the anomalous photoabsorption on PXR and is given by

$$\delta_\lambda = \frac{\chi''_g\chi'_{-g} + \chi'_g\chi''_{-g}}{2\sqrt{\chi'_g\chi'_{-g}}\chi''_0}\alpha_\lambda < 1. \tag{10}$$

It is easy to verify that the condition $\Delta'_0 + \Delta = \kappa'_\lambda \equiv \sqrt{\Delta^2 + \chi'_g\chi'_{-g}\alpha_\lambda^2}$, which determines the existence of emission described by the first item in (9), can only be fulfilled in the angular and frequency range, where $\Delta'_0 = \gamma^{-2} - \chi'_0 + \theta^2 > 0$ and the Cherenkov threshold is not reached. On the other hand, the emission due to the second term in (9) has the constraint $\Delta'_0 < 0$. Thus, this branch of the forward PXR only manifests itself under Cherenkov conditions.

It is of interest that the two terms in (9) can be combined into a single one

$$\omega\frac{\mathrm{d}^3 N}{\mathrm{d}\omega\,\mathrm{d}^2\theta} = \frac{e^2\theta_\lambda^2}{4\pi\chi''_0\sin^2(\varphi/2)} \times\left[\left(\frac{\chi'_g\chi'_{-g}\alpha_\lambda^2}{\Delta'_0}\right)^2 \frac{\delta(\omega'_B/\omega - 1 + (\Delta'_0)^2 - \chi'_g\chi'_{-g}\alpha_\lambda^2/4\sin^2(\varphi/2)\Delta'_0)}{(\Delta'_0 + \delta_\lambda\sqrt{\chi'_g\chi'_{-g}}\alpha_\lambda)^2 + \chi'_g\chi'_{-g}\alpha_\lambda^2(1 - \delta_\lambda^2)} + \cdots\right] \tag{11}$$

best suited for the study of the influence of anomalous photoabsorption on the emission properties.

It should be noted that for frequencies far from the Bragg frequency ($|\Delta| \gg \sqrt{\chi'_g\chi'_{-g}}\alpha_\lambda$), where the dynamical diffraction effects are small, the obtained formula reduces to

$$\omega\frac{\mathrm{d}^3 N}{\mathrm{d}\omega\,\mathrm{d}^2\theta} = \frac{e^2}{\pi}\frac{\theta_\lambda^2}{\chi''_0}\delta(\gamma^{-2} - \chi'_0 + \theta^2) \tag{12}$$

describing ordinary Cherenkov radiation (this result coincides with that following from the exact formula (7) neglecting the contribution of transition radiation). Summing over photon polarizations and integrating over observation angles one can obtain from (12) the classical Tamm–Frank formula for the spectrum of Cherenkov radiation.

In the vicinity of Bragg frequency ($|\Delta| \leqslant \sqrt{\chi'_g\chi'_{-g}}\alpha_\lambda$) the quantity $\gamma^{-2} - \chi'_0 + \theta^2$ cannot be equal to zero, and the argument of the δ-function in (11) reduces to the $\omega'_B/\omega - 1$ to $0(\gamma^{-1})$, because $\omega'_B/\omega - 1 \simeq \omega_B/\omega - 1 - \theta_\|\mathrm{ctg}(\varphi/2) \sim \gamma^{-2}$. Emission properties in this frequency range, where the contribution of dynamical diffraction effects is substantial, strongly depend on the behavior of the average dielectric susceptibility χ'_0.

Returning to formula (11) let us elucidate the role of anomalous photoabsorption in the emission process. It is significant that this role strongly depends on the state of the primary photon associated with the fast electron. Traditionally, $\chi'_0(\omega) = -\omega_0^2/\omega^2 \approx -\omega_0^2/\omega_B^2$ (this is the ordinary dielectric susceptibility of a medium in the X-ray domain away from the photoabsorption edge) and so the primary photon is the virtual photon of the fast electron Coulomb field. Eq. (11) clearly shows that the influence of anomalous photoabsorption is not very substantial because $|\chi'_g|$ is usually less than $|\chi'_0|$. Among other things, the change in PXR yield is limited even though $\delta_\lambda = 1$. The physical nature of such a small role of the anomalous photoabsorption in the PXR process is the following. Exact Bragg resonance between the primary virtual photon and the diffracted free photon of

PXR is not possible because of the difference between the dispersion laws for free and virtual photons, respectively. Indeed, the expression (13) for the resonance defect Δ following from the argument of δ-function in (11)

$$\Delta = \frac{(\gamma^{-2} - \chi_0' + \theta^2)^2 - \chi_g' \chi_{-g}' \alpha_\lambda^2}{2(\gamma^{-2} - \chi_0' + \theta^2)} \tag{13}$$

shows that $\Delta \neq 0$ if $\chi_0' < 0$. Because of this, the primary virtual wave and diffracted free wave cannot create the exact standing wave in the direction perpendicular to the reflecting plane (since minima of the intensity of such exact standing wave are located at atomic planes of the crystal, where the crystal electron density is maximum, the interaction of this wave with electrons of the crystal is strongly suppressed; this is the physics of anomalous photoabsorption) and so the effect of anomalous photoabsorption does not contribute in full measure to the formation of the forward PXR yield.

The situation changes dramatically under conditions of the Cherenkov effect, when $\gamma^{-2} - \chi_0' + \theta^2 < 0$. Both primary and diffracted photons are free and can create the standing wave. As this takes place, the contribution of anomalous photoabsorption is determined by the coefficient δ_λ (a necessary condition for anomalous photoabsorption in PXR, $\delta_\lambda \approx 1$, coincides with that in the physics of free X-ray scattering in crystals, Pinsker, 1984). As may be evident from (11) when $\delta_\lambda \approx 1$ the forward PXR spectral-angular distribution has a sharp maximum

$$\left(\omega \frac{d^3 N_\lambda}{d\omega\, d^2\theta}\right)_{\max} = \frac{e^2 \theta_\lambda^2}{4\pi \chi_0'' \sin^2(\varphi/2)} \frac{\delta(\omega_B'/\omega - 1)}{\delta_\lambda^2 (1 - \delta_\lambda^2)} \tag{14}$$

determined by the condition $\gamma^{-2} - \chi_0' + \theta^2 + \delta\sqrt{\chi_g' \chi_{-g}'}\alpha_\lambda = 0$. This condition corresponds to the exact Bragg resonance between the primary and the diffracted photons. Indeed, this condition coincides, as would be expected, with the condition $\Delta = 0$ (see (13)) in the case of $\delta_\lambda \approx 1$.

In contrast with the previous case $\chi_0' < 0$, the maximum value of (14) increases beyond all bounds when $\delta_\lambda \to 1$ (the forward PXR with σ polarization is most suitable for the observation of the predicted effect since $\alpha_\sigma \equiv \alpha_1 = 1$), and hence the enhancement of PXR intensity may be very substantial.

It is well to bear in mind that the condition $\chi_0' > 0$ is usually realized for soft X-rays ($\omega < 1$ keV), hence macroscopic periodic structures, such as multilayer nanostructures or periodic systems of nanotubes, hold the greatest promise for an experimental observation of the predicted forward PXR enhancement under Cherenkov effect conditions.

Acknowledgements

This work was accomplished in the context of the program "Advancement of the scientific potential of high education" by Russian Ministry of Education and Science (Grant RNP.2.1.1.3263) and Russian Fund of Basic Research (Grants: 03-02-16587 and 04-02-16583). The authors are very grateful to Dr. E. Tiesinga (NIST) for his clarity suggestions.

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