On the classical and quantum theories of the backward Compton scattering of the electromagnetic waves on the relativistic electron

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Abstract

The classical theory of backward Compton scattering of electromagnetic waves on the relativistic electron bunch is suggested. It was compared basic results of quantum and classical theories of that effect. The possibility of coherent effect at the backward Compton scattering of electromagnetic waves on the short electron bunch of relativistic electrons was shown. Conditions, at which this effect takes place, was obtained.

1. In the present Letter the process of backward Compton scattering of the laser wave on the relativistic electron bunch is considered. This process is of great interest because on its basis the creation of hard electromagnetic radiation sources is possible. It is related to the fact, that the spectrum distribution of scattered waves has a sharp maximum in the direction of electron bunch motion. The position of this maximum shifts rapidly with the increase of the electron energy to the region of high frequencies. Theoretical investigations of this process are usually made on the basis of the first significant order of perturbation theory of quantum electrodynamics, which takes into account the recoil effect of radiation [1,2].

Similar features of the spectrum distribution of electromagnetic waves exist also in the process of the coherent radiation of relativistic electrons in a crystal when the particles enter onto the crystalline plane at the small angle. This process was initially investigated on the basis of the Born approximation of quantum electrodynamics [3]. Later it had been shown, that the coherent effect takes place not only for the quantum, but also for classical consideration of the electron radiation process in the crystal. It was found significant, that the formulas of quantum and classical theories in a number of cases are the same, when the recoil effect in radiation is neglected [4].

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Below we show, that the similar situation takes place also in the case of description of the process of backward Compton scattering of the electromagnetic wave on the relativistic electron. It is shown, that in the region of low frequencies the description of this process can be made on the basis of formulas of classical theory of radiation, which were used earlier for the description of the process of coherent radiation of relativistic electrons in a crystal. The process of Compton scattering of electromagnetic wave on the relativistic electron in this case is actually reduced to the process relativistic electron radiation during its accelerated motion in the field of electromagnetic wave flying onto the electron. The obtained results are compared to the corresponding results of quantum theory. Here we analyze the conditions, under which quantum and classical formulas for spectrum distribution of scattered waves coincide.

Using the formulae of classical electrodynamics, we can simplify the consideration of complex processes in Compton scattering of electromagnetic waves on electron bunch. In the scope of this approach we have considered the possibility of coherent effect for the scattering of electromagnetic wave on the narrow bunch of relativistic electrons. Also the conditions of such possibility were obtained.

2. Spectral-angular density of the radiation by the electron moving in the external field along the trajectory $\vec{r}(t)$ is defined in classical electrodynamics by equation [4,5]

$$\frac{dE}{d\omega\, do} = \frac{e^2}{4\pi^2}|\vec{k}\times\vec{I}|^2, \qquad \vec{I} = \int_{-\infty}^{\infty} dt\,\vec{v}(t)e^{i(\omega t-\vec{k}\vec{r}(t))}, \tag{1}$$

where e is the electron charge, $\vec{v}(t)$ is the velocity vector of the electron, ω and $\vec{k}$ are correspondingly the frequency and the wave vector of the radiated wave, $\omega = |\vec{k}|$, do is the elementary solid angle in the direction of radiation. (Note, that we use the system of units, where the speed of light is equal to unity.) The trajectory of electron is defined by relativistic equation of motion

$$\dot{\vec{v}} = \frac{e}{\varepsilon}\left[\vec{E} + \vec{v}\times\vec{H} - \vec{v}(\vec{v}\vec{E})\right], \tag{2}$$

where $\varepsilon = m\gamma$ is the electron energy, m is its mass, γ is the Lorentz factor of the electron, $\vec{E}$ and $\vec{H}$ are the vectors of external electric and magnetic fields.

When the electron is moving in the field of the flat electromagnetic wave, vectors $\vec{E}$ and $\vec{H}$ in (2) can be represented as follows

$$\vec{E} = \mathrm{Re}\left(A\vec{e}e^{i(\Omega t-\vec{\Omega}\vec{r})}\right), \qquad \vec{H} = \vec{n}\times\vec{E}, \tag{3}$$

where Ω and $\vec{\Omega}$ are the frequency and the wave vector of the electromagnetic wave, $\vec{n}$ is the unit vector along wave propagation direction, $\vec{n} = \vec{\Omega}/\Omega$, A is the field amplitude, $\vec{e}$ is the unit vector of the wave polarization. Vector $\vec{e}$ is orthogonal to $\vec{n}$. In general case the problem of electron radiation in such field was considered in [6]. Formulae for spectral–angular distributions of radiation obtained there are rather complex. Below we consider one of the simplest variants of this problem, supposing that the electromagnetic wave moves towards the relativistic electron and that process of radiation can be considered in dipole approximation.

The deviation of the relativistic electron from the initial movement direction $\vec{v}_0$ in the field of the electromagnetic wave is small. Using the first approximation by this deviation the electron velocity vector $\vec{v}(t)$ can be written as follows

$$\vec{v}(t) \approx \vec{v}_0 + \vec{v}_\perp(t), \tag{4}$$

where $\vec{v}_\perp(t)$ is the transversal component of the electron velocity $\vec{v}(t)$, $v_\perp \ll v_0$. Using Eqs. (3), (4) and transformation

$$\vec{v}\times\vec{H} = \vec{v}\times(\vec{n}\times\vec{E}) = -\vec{E}(\vec{n}\vec{v}_0) \approx \vec{E},$$

we can obtain with accuracy up to the terms of the order $v_\perp^2/v^2$ the following equation for $\vec{v}_\perp(t)$

$$\dot{\vec{v}}_\perp(t) = \frac{2e}{\varepsilon}\mathrm{Re}\left(A\vec{e}e^{it(\Omega-\vec{\Omega}\vec{v}_0)}\right). \tag{5}$$

According to (5), the characteristic values of deviation angle of the electron in the field of electromagnetic wave are equal by the order of magnitude to

$$\vartheta_e \approx \frac{v_\perp}{v} \approx \frac{eA}{\varepsilon\Omega}. \tag{6}$$

When the condition $\gamma\vartheta_e \ll 1$, corresponding to the condition of dipolity of radiation by relativistic electron in the external field, Eq. (1) can be expanded by the parameter $\gamma\vartheta_e$. In the first approximation of such expansion Eq. (1) takes the form [4]

$$\frac{dE}{d\omega\,do} = \frac{e^2}{4\pi^2}\frac{\omega^2}{q^2}\left[1 - 4\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\cos^2\varphi\right] \times \left|\vec{W}(q)\right|^2, \tag{7}$$

where $\delta = \omega/(2\gamma^2)$, $q = \omega - \vec{k}\vec{v}_0 \geqslant \delta$. The angle φ in (7) is the azimuthal angle between the orthogonal to $\vec{v}_0$ component of the vector $\vec{k}$ and $\vec{W}(q)$, where $\vec{W}(q)$ is the Fourier component of the transversal component of the acceleration

$$\vec{W}(q) = \int\limits_{-\infty}^{\infty} dt\, e^{iqt}\,\dot{\vec{v}}_{\perp}(t). \tag{8}$$

Therefore, in dipole approximation of classical electrodynamics the spectral–angular radiation density is determined by the transversal component of particle acceleration in the external field. In the used method the field of initial electromagnetic wave is considered as an external field, which causes the accelerated movement of the electron. This acceleration induces the radiation. This method is a generalization of the method used earlier in classical electrodynamics for the description of scattering of electromagnetic waves on the rest electron [5,7] to the case of scattering on the moving electron. Such an approach is widely used for the investigation of relativistic electron radiation in crystals and ondulators (see [4]). So the extension of the present method to the problem of relativistic electron radiation in the field of the flat electromagnetic wave would allow us to consider uniformly the processes of electron radiation in the field of electromagnetic wave, crystal, ondulator and other fields, what is very important for the comparative analysis of radiation characteristics in various fields.

Substituting (5) in (7), we can obtain after averaging on polarizations of initial wave the following expression for spectral–angular density of radiation

$$\frac{dE}{d\omega\,do} = TA^2\frac{r_0^2}{2\pi\gamma^2}\frac{\omega^2}{q^2}\left(1 - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right) \times \delta(q - 2\Omega), \tag{9}$$

where T is the time of electron motion in the electromagnetic wave, $r_0 = e^2/m$ is the classical radius of the electron. Here we use the relation $\Omega - \vec{\Omega}\vec{v}_0 \approx 2\Omega$.

Delta-function $\delta(q - 2\Omega)$, consisting in (9), defines connection between characteristics of electromagnetic wave, in which electron is moving, and the frequency ω and angle θ of radiated wave. Taking into account the expression $q = \omega - \vec{k}\vec{v}_0 = \omega(1 - v\cos\theta)$, where the angle θ presents the angle between the wave vector of radiated wave $\vec{k}$ and the vector of initial direction of electron motion $\vec{v}_0$, we can obtain

$$\omega = \frac{2\Omega}{1 - v\cos\theta}. \tag{10}$$

This expression shows, that when the condition $\theta = 0$ holds, the maximum of transformation coefficient of frequency of scattering wave Ω in frequency ω of radiated wave is reached

$$\omega_{\max} \approx 4\Omega\gamma^2. \tag{11}$$

After integration on solid angles of radiation we can obtain the follows expression for the spectral density of radiation of relativistic electron in the field of unpolarized electromagnetic wave

$$\frac{dE}{d\omega} = T\frac{A^2 r_0^2}{\Omega}\frac{\omega}{\omega_{\max}}\left[1 - 2\frac{\omega}{\omega_{\max}}\left(1 - \frac{\omega}{\omega_{\max}}\right)\right], \quad \omega \leqslant \omega_{\max}. \tag{12}$$

When the condition $\omega > \omega_{\max}$ holds, the spectral density of radiation is equal to zero.

Expression (12) for the spectral density was received on the basis of classical theory of radiation of relativistic electron. Will compare this expression with corresponding results of quantum theory of radiation.

In the first approximation of quantum-electrodynamical theory of perturbations the cross section of Compton scattering of unpolarized wave on the relativistic electron, moving in opposite to wave direction, can be written as follows [8,9]

$$d\sigma = 2\pi r_0^2\frac{d\omega}{\omega}\frac{\omega}{\omega_m}\frac{\varepsilon'}{\varepsilon} \times \left[\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} - 4\frac{\omega}{\omega_m}\left(1 - \frac{\omega}{\omega_m}\right)\right], \tag{13}$$

where ε and ε' are energies of initial and final electrons, $\varepsilon' = \varepsilon - \omega$ and ω_m is maximal value of frequency of scattered photon. (We use the system of units, where the Plank constant is equal to unity.) Using conservation laws for energy and momentum we can obtain, that the frequency of scattered photon ω, consists in (13), constrains with the angle θ of photon radiation, by the following expression

$$\omega = \frac{2\Omega}{1 - v\cos\theta + (\Omega/\varepsilon)(1+\cos\theta)}. \tag{14}$$

Maximal value of ω is reaches for $\theta = 0$: it can be written in the form

$$\omega_m \approx \frac{4\varepsilon\varepsilon'_m\Omega}{m^2}, \tag{15}$$

where $\varepsilon'_m = \varepsilon - \omega_m$.

Obtaining the formulae (13), it was neglected by terms on the order of values of γ^{-2} and ϑ^{-2}. In addition to this, it was assumed that the condition $\omega \gg \Omega$ holds. Expression (13) takes place in the region, when the condition $\omega \leqslant \omega_m$ holds. If $\omega > \omega_m$, then conservation laws of energy and momentum in the considered process do not carry out and the scattering cross section is equal to zero.

Now consider electron interaction with electromagnetic wave with amplitude A. The density of energy flux of this wave through the unit plane, orthogonal the direction of wave propagation, is defines by Poynting vector $|\vec{S}| = A^2/(4\pi)$. The number of photons, participating in the process of its interaction with electron for the time T will be determined by equation

$$N_\gamma = \frac{A^2}{4\pi\Omega} ST, \tag{16}$$

where S is plane, orthogonal direction of motion of interacting photons. The probability of interaction of photons with electrons will be determined by $dw = d\sigma/S$. Multiplying this expression on ω and N_γ, we can obtain the following distribution of final photons by energies

$$\begin{aligned} \frac{dE_q}{d\omega} &= N_\gamma \frac{\omega}{S}\frac{d\sigma}{d\omega} \\ &= T\frac{A^2 r_0^2}{2\Omega}\frac{\varepsilon'}{\varepsilon}\frac{\omega}{\omega_m}\left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} - 4\frac{\omega}{\omega_m}\left(1 - \frac{\omega}{\omega_m}\right)\right), \\ \omega &\leqslant \omega_m. \end{aligned} \tag{17}$$

Comparing this expression with corresponding results of classical theory of radiation (12), we will see, that both formulas completely coincides, if it is possible to neglect of terms ω/ε in the quantum expression for the backward Compton scattering. Taking account the conservation law for energy for Compton scattering $\varepsilon + \Omega = \varepsilon' + \omega$ and noting, that the characteristic values of frequencies of scattered waves in the direction of electron motion $\omega \gg \Omega$, will see, that the condition $\omega \ll \varepsilon$ corresponds to neglection of recoil effect in radiation.

Hence, consideration of the scattering process of electromagnetic wave on the relativistic electron on the basis of born approximation of quantum electrodynamics and on the basis of dipole approximation of classical electrodynamics lead to similar results for the spectral–angular density of scattered waves in the case of small frequencies. In this case the process of scattering of electromagnetic wave on the relativistic electron could be considered as the process of bremsstrahlung by electron, when it motion in the external field is accelerated.

3. Consider now the possibility of coherent effect in radiation, when electromagnetic wave is scattered on the relativistic electron bunch. For this, consider on the basis of described above method the radiation of the electron bunch in the field of scattered electromagnetic wave.

The particle density in the bunch can be written as

$$\rho(\vec{r}) = e\sum_{n=1}^{N} \delta\big(\vec{r} - \vec{r}_n(t)\big), \tag{18}$$

where $\vec{r}_n(t)$ is the trajectory of nth particle of bunch in the external field and N is the number of particles in the bunch. The spectral–angular density of radiation will be determined by expression (1) with

$$\vec{I} = \vec{I}_N = \sum_n \int_{-\infty}^{\infty} dt\, e^{i(\omega t - \vec{k}\vec{r}_n(t))}\vec{v}_n(t). \tag{19}$$

Suggesting, that interaction between particles is small enough, trajectories of all particles in the bunch are similar, and the trajectory of nth particle $\vec{r}_n(t)$ in the bunch in the field of electromagnetic wave can be written as $\vec{r}_n(t) = \vec{r}(t) + \vec{r}_n^{\,0}$, where $\vec{r}(t)$ is the trajectory of the middle of the bunch and $\vec{r}_n^{\,0}$ is the initial position of this particle in the bunch. Using this expression, we can obtain the following expression for the spectral–angular density of radiation of the electron bunch in the electromagnetic wave

$$\frac{dE_N}{d\omega\, do} = \left|\sum_n e^{-i\vec{k}\vec{r}_n^{\,0}}\right|^2 \frac{dE_1}{d\omega\, do}, \tag{20}$$

where $dE_1/d\omega do$ is the spectral–angular density of radiation of the one of particles.

Eq. (20) must be averaged on electron positions in the bunch. If particles are situated at large distances enough, that if for $n \neq m$ the condition $|\vec{k}(\vec{r}_n - \vec{r}_m)| \gg 1$ hold, then after averaging we can obtain the following expression for spectral–angular density of radiation

$$\left\langle \frac{dE_N}{d\omega do} \right\rangle = N \frac{dE_1}{d\omega do}. \tag{21}$$

In this case it is possible to neglect by interference effect in radiation by different electrons and the spectral–angular density of radiation takes proportional to the number of particles in the bunch.

But if the condition

$$\left|\vec{k}(\vec{r}_n - \vec{r}_m)\right| \ll 1 \tag{22}$$

holds for $n \neq m$, then it is possible to neglect of phase shifts in radiation by different particles, and the spectral–angular density of radiation takes the following form

$$\left\langle \frac{dE_N}{d\omega do} \right\rangle = N^2 \frac{dE_1}{d\omega do}. \tag{23}$$

In this case phases of waves, emitted by different particles, coincide and the coherent effect in radiation takes place.

Consider now in detail conditions, when the coherent effect in radiation of electromagnetic waves by particles of bunch takes place. For this write down inequalities (22) separately for orthogonal and longitudinal components of bunch:

$$\left|\vec{k}_\perp(\vec{\rho}_n - \vec{\rho}_m)\right| \ll 1, \qquad \left|k_z(z_n - z_m)\right| \ll 1, \tag{24}$$

where $\vec{k}_\perp$ are components of the wave vector of emitted wave, orthogonal to $\vec{v}_0$. Taking into account, that characteristic angle of the radiation by relativistic electron $\theta \approx \gamma^{-1}$ is small enough, write down these inequalities in the form

$$\frac{\omega}{\gamma} L_\perp \ll 1, \qquad \omega L_p \ll 1, \tag{25}$$

where $L_\perp$ and L_p are characteristic orthogonal and longitudinal sizes of the bunch. Hence, it is necessary, that longitudinal size of the bunch was small in comparison with the length of emitted wave $\lambda \approx 1/\omega$, and orthogonal size was small in comparison with $\lambda\gamma$, for rising of the coherent effect in radiation.

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