KINETICS 0F UITRA-HIGH ENERGY LEPTONS CHANNELING

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The problem of dechanneling of ultrarelativistic leptons in aligned crystals at superhigh energies is investigated using the consecutive stochastic approach which guides to the Chapman-Kolmogorov kinetic equation. It is shown that the quantum recoil due to hard photons emission causes the specific dechanneling mechanism which becomes predominant for leptons at superhigh energies but was never discussed earlier. The complete solution of the dechanneling problem is obtained including the asymptotical distribution function over the transverse energies and the analytical expression for correspondent dechanneling length. The results obtained are applicable not only to leptons but to mesons, protons and other heavy ultrarelativistic particles.

I. Introduction

One of the main problems of modern physics of channeling is a correct description of the kinetics of channeling of ultrarelativistic leptons, mesons and protons. The steady growth of the interest to the channeling of high and superhigh energy leptons is observed now due to related directional effects in electromagnetic radiation and pair production as well as with respect to high-energy beams control technique. In view of such interest the study of superhigh energy leptons' flux kinetics during their passage through aligned crystals becomes very important as a background for various secondary effects prediction and investigation.

In present paper we propose an advanced stochastic model of channeled leptons' kinetics in crystals. This model starts from first principles and leads to the new type of kinetic equation in the theory of directional effects - the Chapman-Kolmogorov equation. As a result, this model deals with the whole spectrum of incoherent scattering factors including both small-angle and strong scattering mechanisms.

Moreover, the new dechanneling factor caused by emission of high-energy photons is introduced and an analytical solution of corresponding dechanneling problem is done. It is shown that abovementioned dechanneling factor becomes predominant for leptons of superhigh energies.

2. Dechanneling factors for ultrarelativistic leptons

Formerly (see, for example, refs. [1] to [3]) the kinetics of high-energy leptons' channeling was described on the basis of Fokker-Planck equations which were obtained in a semiphenomenological manner. This approach treats the channeled particle's incoherent scattering as a diffusion in the space of its transversal motion integrals - transverse energy $E_\perp$ and (in the case of axial channeling of negatively charged particles) its angular momentum M_z.

Consequently only so called *soft* dechanneling factors such as electronic and (with some reserve[1]) nuclear scattering can be taken into account by models confined to Fokker-Planck

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[1] * For axial channeling ofultrarelativistic electrons at energies $E \gg 1$ GeV rough analytical estimations [4] as well as the results of computer simulation [4-6] indicate that nuclear scattering in whole cannot be treated as soft scattering factor since it causes rather strong variation of transverse energy and especially of angular momentum of the particle each time it draws near to atomic rows.

kinetic equation.

In other words the diffusional mechanism of dechanneling cannot describe the particle's opportunity to be dechanneled by rare enough but strong acts of scattering each of which give rise to essential dechanneling probability P~1. These factors - so called *rigid* dechanneled factors - are well known in the physics of ion channeling in damaged crystals (the dechanneling caused by interstitials, voids and other crystal defects and impurities).

In the case of ultra-high energy region (E > 50 GeV) conventional soft dechanneling factors for relativistic leptons are substantially suppressed because of the decrease of their cross-sections with particles' energy *E*. On the other hand at such energies increases the influence of electromagnetic radiation emitted by leptons on their motion. Two electromagnetic effects - radiative energy losses and radiative cooling of transverse energy - were considered earlier to be the exclusive mechanisms of leptons' flux evolution in aligned crystals at ultra-high energies.

However, there is a never discussed dechanneling factor which is due to emission of photons not strictly ahead with respect to the direction of particle's motion which causes corresponding transversal quantum recoil. As we shall see later, this factor contributes both to "soft" diffusion dechanneling (due to soft photons radiation) as well as to "rigid" dechanneling (due to hard photons emission) and plays the main role in dechanneling processes at extremely high energies of particles.

3. Stochastic treatment of leptolls' channelihg and radiation

The starting-point of the model is the fact that the statistical ensemble of channeled particles is treated as a dynamical system under the influence of stochastic forces caused by incoherent scattering of various nature and by quantum recoil due to photons emission. The correspondent stochastic equation of transversal motion of leptons has the following form:

$$\gamma m\cdot\ddot{r} = -\nabla U(r) - \frac{\dot{r}}{c}\left\{\left(\frac{dE}{dz}\right)_e + \left(\frac{dE}{dz}\right)_\gamma\right\} + f_e(t) + f_T(t) + f_\gamma(t), \qquad (1)$$

where U(r) is Lindhard continuum potential [7] of atomic rows ($r=\{x,y\}$) or planes (r={y}), *m* and γ are leptons' mass and Lorentz-factor, *c* is the speed of light,

$$\left(\frac{dE}{dz}\right)_e = \frac{4\pi e^4 n_e}{mc^2\gamma}\ln\left(1.13\cdot\frac{\gamma^3 m^2 c^4}{2I^2}\right) \qquad (2)$$

and

$$\left(\frac{dE}{dz}\right)_\gamma = \frac{2e^2}{c^4}\cdot\gamma^4 w^2\cdot\lambda(\chi) \qquad (3)$$

are leptons' electronic [8] and radiative [9] energy losses, respectively; $n_e=n_e[r(t)]$ is the electron density distribution function in the transversal plane, $w=w[r(t)]$ is particle's transversal acceleration,

$$\lambda(\chi) \approx \frac{1-\alpha_3\chi^2\left(1+\alpha_4\chi^2\right)^{-2}}{(1+\alpha_1\chi)(1+\alpha_2\chi)^{1/3}}, \qquad \left(\chi = \frac{\lambda_c}{c^2}\gamma^2 w\right),$$

$\alpha_1=0.72$, $\alpha_2=15.7$, $\alpha_3=1.3$, $\alpha_4=0.94$, λ_c is Campion length of electron, $I\approx 13.6Z$ eV is the ionisatlon potential of crystal atom, Z is target atomic number.

The terms $f_e(t)$, $f_T(t)$ and $f_\gamma(t)$ are stochastic forces caused by electronic and nuclear multiple scattering and by emission of soft photons, respectively. For high enough energies all these forces are considered to be stochastic processes with independent variations, so their stochastic properties are completely determined by appropriate correlation functions. Starting from the first principles it is not so difficult to obtain the following expressions for all correlation functions to be considered.

For electronic scattering the correlation function has the form:

$$\langle f_e(t) f_e(t') \rangle = \frac{16\pi e^4 n_e}{c} \ln\left(1.13 \frac{\gamma^3 m^2 c^4}{2I^2}\right) \cdot \delta(t-t'). \qquad \textbf{(4)}$$

For nuclear multiple scattering [10]:

$$\langle f_T(t) f_T(t') \rangle = \frac{4\alpha}{c} r_e^2 E_0^2 Z^2 n \cdot p(r) \cdot \ln(183 \cdot Z^{-1/3}) \cdot \delta(t-t'), \qquad \textbf{(5)}$$

where E_0=21 MeV, α and r_e are fine structure constant and classical radius of electron, n is the target atomic density. The factor $p(r)$ takes into account thermal vibrations of crystal atoms, r has the meaning of the particle's distance to the nearest atomic row or plane;

$p(r) = (r_0^2 / u_\perp^2) \cdot \exp(-r^2 / u_\perp^2)$ for axial channeling,

$p(r) = p(y) = (r_0 / \sqrt{2\pi u_1}) \cdot \exp(-y^2 / 2u_1^2)$ for planar channeling; $2r_0$ is the typical distance between atomic rows or planes; $u_\perp = \sqrt{2u_1}$ is the r.m.s. amplitude of thermal vibrations.

For high enough energies E > $m^2c^4/U_0 \propto 10$ GeV (U_0 is the potential well depth) the coherence length of channeling radiation becomes less then the period of the particle's oscillations in the channel; therefore the emission of soft ($\hbar\omega$<<E) photons can be treated as Poisson stochastic process with high frequency. The appropriate correlation function has the following form:

$$\langle f_\gamma(t) f_\gamma(t') \rangle \approx \frac{\hbar e^2}{4c^6} w^3 \gamma^5 \cdot \delta(t-t'). \qquad (6)$$

It should be noted that in general the particle's acceleration $w[r(t)]$ as well as local electron and nuclear densities $n_e[r(t)]$ and p=[r(t)] depend on the position r in the transverse plane along with the particle's trajectory r(t), but further in order to obtain analytical solutions of kinetic equations we replace these functions with some effective constant values equal to their averages upon the particles' flux distribution in the channel.

4. Master kinetic equation

Standard methods of the theory of stochastic processes (see e.g. ref [1 1]) allow to obtain, using Equation (1) and correlators (4÷6), the following master kinetic equation (the Chapman-Kolmogorov equation) for the leptons' distribution function $F(E_\perp, t)$ on transverse energy:

$$\frac{\partial F(E_\perp, t)}{\partial t} = \frac{\partial}{\partial E_\perp}\left\{A(E_\perp)\cdot F(E_\perp, t) + D(E_\perp)\cdot \frac{\partial F(E_\perp, t)}{\partial E_\perp}\right\} - I_{st}(E_\perp, t). \qquad (7)$$

Three basic features differ this equation from those ones proposed earlier [1÷3]:

i. In the expressions for drift coefficients $A(E_\perp) = A_e(E_\perp) + A_\gamma(E_\perp)$ and diffusion coefficients $D(E_\perp) = D_T(E_\perp) + D_e(E_\perp) + D_\gamma(E_\perp)$ along with terms caused by electronic scattering

$$A_e = v\frac{8\pi c r_e^2 n_e}{\gamma} \ln\left(1.13 \frac{\gamma^3 m^2 c^4}{2I^2}\right), \qquad (8)$$

$$D_e = \nu \frac{8\pi e^4 n_e}{mc\gamma} \ln\left(1.13 \cdot \frac{\gamma^3 m^2 c^4}{2I^2}\right) \tag{9}$$

and incoherent nuclear scattering on thermal vibrations

$$D_T = \nu \frac{2\alpha r_e^2 E_0^2 Z^2}{mc\gamma} \cdot n \cdot p \cdot \ln(183 \cdot Z^{-1/3}) \tag{10}$$

the terms caused by emission of soft photons by channeled particles have appeared:

$$A_\gamma(E_\perp) \approx \nu \cdot \frac{4e^2 U_0}{3m^3 c^5 r_0^2} \cdot \gamma \cdot E_\perp , \tag{11}$$

$$D_\gamma(E_\perp) \approx \nu \cdot \frac{\hbar e^2 U_0}{8m^4 c^6 r_0^3} \cdot \gamma \cdot E_\perp , \tag{12}$$

where ν=l for axial and equals to 1/2 for planar channeling, respectively.

ii. The stoss integral $I_{st}(E_\perp, t) = I_{st}^T(E_\perp, t) + I_{st}^\gamma(E_\perp, t)$ takes into account the changes of transverse energy of lepton caused both by strong incoherent scattering on thermal vibrations:

$$I_{st}^T(E_\perp, t) \approx \mu \frac{\pi Z^2 e^4 n}{2\gamma \cdot mc \cdot |E_\perp|} \cdot p \cdot F(E_\perp, t) \tag{13}$$

and by emission of hard photons:

$$I_{st}^\gamma(E_\perp, t) \approx \mu \frac{33\sqrt{2} e^2 U_0}{8\pi\sqrt{3}\hbar mc^2 r_0} F(E_\perp, t) \int_{a(E_\perp)}^{\infty} x \cdot K_{1/3}(x) dx , \tag{14}$$

where $a(E_\perp) \approx \frac{8mcr_0}{3\hbar} \cdot \sqrt{\frac{mc^2}{\gamma \cdot U_0}} \cdot \sqrt{\frac{E_\perp}{U_0}}$, $K_{1/3}(x)$ is the McDonald function, μ = 1 for axial and equals to $(2\pi)^{-1}$ for planar channeling, respectively.

iii. The equation is one-dimensional, i.e. only the transverse energy is regarded as the integral of motion of unperturbated dynamical system. For planar channeling it's quite clear, whereas for axial channeling of electrons this assumption is based on the fact that due to the axial asymmetry of realistic axial channels [5] and the phenomenon of dynamical chaos [12] the angular momentum of electrons suffers intensive variations even in absence of random forces and cannot be regarded in general as adiabatic invariant of motion. Thus the model under consideration gives the unified description of the kinetics both of positively and negatively charged particles; moreover the one-dimensionality of the equation affords the opportunity to solve it analytically using some standard approximations.

5. Tile Solution of Dechanneling Problem: distribution on transverse energies and the dechanneling length

Because of limited phase space of proper channeling which Is determined by critical transverse energy the dechanneling problem is equivalent [13] to the Stum-Liouville problem for Eq. (7) with appropriate boundary and initial conditions:

$$F(E_{\perp c}, t) = 0, \quad F(E_\perp, 0) = F_{in}(E_\perp), \tag{15}$$

where $E_{\perp c}$ is the critical transverse energy correspondent to the escape of channeled particle from the bound state motion, $F_{in}(E_\perp)$ is the initial distribution on transverse energies.

The corresponding dechanneling length has the meaning of mean path length before the first event of transition from the bound state into the above-barrier state of motion.

Due to the different energy dependencies of Coulomb nuclear and electronic scattering cross-sections, from the one side, and the photon emission cross-section from the other side, the bound energy appears in the theory of kinetics ofleptons:

$$E_b = mc^2 \cdot \sqrt{\frac{8}{Z}} \cdot \frac{d}{r_e} \propto 50 \text{ GeV}, \tag{16}$$

where d is the lattice unit of crystal.

The bound energy E_b distinguishes the region of predomination of Coulomb incoherent scattering ($E<E_b$) from the region where the dechanneling is caused by photon emission processes ($E>E_b$). For the latter energy region (which we call superhigh energies) the radiative cooling effectively suppresses the electronic and nuclear scattering, so the sole mechanism of dechanneling consists in quantum recoil due to hard photons radiation.

The solution of the Sturm-Liouville problem for Eq. (7) gives us the following Boltzmann-type distribution function at superhigh energies $E>E_b$:

$$F(E_\perp, z) = \frac{C}{kT} \cdot \exp\left(-\frac{U_0 + E_\perp}{kT}\right) \cdot \exp\left(-\frac{z}{R_{ch}}\right) \tag{17}$$

with effective "transverse temperature"

$$T \approx \frac{3\hbar \cdot U_0}{32k \cdot mc \cdot r_0}, \tag{18}$$

and the dechanneling length R_{ch} is expressed by the formula:

$$\frac{1}{R_{ch}} = \frac{44\sqrt{2}e^2}{\pi\sqrt{3}\hbar^2 c^2} \cdot \nu \cdot \int_{-U_0}^{0} dE_\perp \left\{ \exp\left(-\frac{E_\perp}{kT}\right) \cdot \int_{a(E_\perp)}^{\infty} x K_{1/3}(x) dx \right\} \approx 1.12 \cdot \nu \cdot \frac{e^2 U_0}{\hbar m c^3 r_0}. \tag{19}$$

In Eq. (17) k is Boltzmann constant and the normalisation factor C depends on the initial transverse energy distribution $F_{in}(E_\perp)$ of particles penetrating into the channel:

$$C = \int_{-U_0}^{0} F_{in}(E_\perp) dE_\perp$$

The asymptotic form of the distribution of particles on transverse energy becomes easy to understand with regard to the strong effect of radiative cooling (which results in decrease of transverse energy) combined with relatively small fluctuations of $E_\perp$ due to incoherent scattering and the emission of numerous soft photons. Namely after it the dechanneling itself is predominantly caused by sufficiently rare emission of hard photons.

It should be mentioned that at superhigh energies $E>E_b$ the dechanneling length R_{ch} is only slightly dependent on particle's energy and does not depend at all on its charge sign. On the contrary, in energy region $E<E_b$ the dechanneling length is proportional to E and is substantially different for electrons and positrons due to the differences in flux-peaking effect. For superhigh energies the dechanneling length depends only on particle's mass and the typical value of gradient of Lindhard continuum axial or planar potential which tunes up the photon emission probability. One should point out that the dechanneling length for axial channeling is several times smaller than for planar channeling (due to differences in ν and U_0/r_0).

The rough numerical estimations according to Eq. (19) give the utmost value of R_{ch} for electrons and positrons of order of several tens up to hundred of microns in the limit of superhigh energy. The significance of photon emission dechanneling mechanism becomes evident while comparing with the dechannellng at GeV energy region: for 1GeV electrons In Si the corresponding dechanneling length is of order of 1 . . . 5 μm (see, e.g., ref. [6]) that in view of proportionality R_{ch} to energy would give enormous values at TeV energies. The existence of dechannellng due to quantum recoil at hard photons emission limits the unrestricted growth of dechanneling length with the increase of particle energy which allows to make theoretical predictions more close to experimental data. It should be mentioned however that due to the strong rechanneling effect which is out of the scope of present paper the effective dechanneling length is expected to be ten or more times greater than the R_{ch}, value calculated above. The detailed calculations of particles kinetics which takes into account rechannellng effect as well as the comparison with experimental data will be published elsewhere.

As a final note we would pay attention to the fact that our results are applicable not only to leptons but to mesons, protons and other heavy ultrarelativistic particles.

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