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Coherent X-radiation of relativistic electrons in a single crystal under asymmetric reflection conditions

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Abstract

Coherent X-radiation of a relativistic electron crossing a single crystal plate with constant speed is considered in the two-wave approximation of the dynamic diffraction theory [Z. Pinsker, Dynamical Scattering of X-rays in Crystals, Springer, Berlin, 1984] in a Laue geometry. Analytical expressions describing the spectral-angular distribution of parametric X-radiation (PXR) and diffracted transition radiation (DTR) formed on a system of parallel atomic planes situated at an arbitrary angle δ to the surface of the crystal plate (asymmetric reflection) are derived. The dependences of the PXR and DTR spectral-angular density and their interference with angle δ are studied.

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1. Introduction

When a fast charged particle crosses a single crystal plate, its Coulomb field scatters on the set of parallel atomic planes of the crystal, giving rise to X-radiation (PXR) [2–4]. Nowadays there are two approaches to describe PXR, the kinematic [5,6] and dynamic [3,4,6] approaches. The kinematic approach takes into account the interaction of each atom only with the primary or refracted wave in the crystal. In contrast to the dynamic approach, the interaction of an atom with the wave field, formed in a crystal by the total scattering on all other atoms is neglected. That is, multi-wave scattering is not taken into account, in particular the interaction of the elementary modes with the diffracted wave. The dynamic approach was developed in [7,8] and effects have recently been considered in [9–11], where the scheme of symmetric reflection was studied. In symmetric reflection in a Bragg scattering geometry the crystal plate surface is parallel to the set of diffracting atomic planes and in a Laue scattering geometry it is perpendicular. Asymmetric reflection in PXR in a Bragg scattering geometry was considered in [12], in transition radiation (TR) and diffracted transition radiation (DTR) in [13,14].

In the present work coherent X-radiation by a relativistic electron crossing a single crystal plate in a Laue geometry is researched. Expressions are obtained for the spectral-angular distribution of PXR, DTR as well as a description of the interference between these two radiation mechanisms in the general case of asymmetric reflection. The distinctive feature of these expressions is that they take into account the orientation of the atomic planes of the crystal relative to plate surface (angle δ). Under a fixed relativistic electron incident angle (θ_B) to the set of parallel atomic planes of the crystal, the decrease of angle δ causes a widening of the PXR spectrum and a resultant increase of

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the integral (over frequency) output of radiation. The influence of reflection asymmetry on the angular density of DTR and interference of PXR and DTR on the summary angular density of radiation is also studied.

2. Radiation spectral-angular distribution

Let us consider the radiation of a fast charged particle crossing a single crystal plate with a steady speed $\boldsymbol{V}$ (Fig. 1). For this purpose, we will consider the equations for the Fourier image of the electromagnetic field

$$\mathbf{E}(\boldsymbol{k},\omega)=\int \mathrm{d}t\mathrm{d}^3r\mathbf{E}(\boldsymbol{r},t)\exp(\mathrm{i}\omega t-\mathrm{i}\boldsymbol{k}\mathbf{r}). \tag{1}$$

Since the Coulomb field of an ultrarelativistic particle can be accepted as transverse, the incident $E_0(\boldsymbol{k},\omega)$ and diffracted $E_{\boldsymbol{g}}(\boldsymbol{k},\omega)$, electromagnetic waves are defined by two amplitudes with different values of transverse polarization

$$\begin{aligned}\mathbf{E}_0(\boldsymbol{k},\omega)&=E_0^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(1)}+E_0^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(2)},\\ \mathbf{E}_{\boldsymbol{g}}(\boldsymbol{k},\omega)&=E_{\boldsymbol{g}}^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(1)}+E_{\boldsymbol{g}}^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(2)}.\end{aligned} \tag{2}$$

The unit vectors of polarization $\boldsymbol{e}_0^{(1)}$, $\boldsymbol{e}_0^{(2)}$, $\boldsymbol{e}_1^{(1)}$ and $\boldsymbol{e}_1^{(2)}$ are chosen in the following way. Vectors $\boldsymbol{e}_0^{(1)}$ and $\boldsymbol{e}_0^{(2)}$ are perpendicular to vector $\boldsymbol{k}$, and vectors $\boldsymbol{e}_1^{(1)}$ and $\boldsymbol{e}_1^{(2)}$ are perpendicular to vector $\boldsymbol{k}_{\boldsymbol{g}}=\boldsymbol{k}+\boldsymbol{g}$. Vectors $\boldsymbol{e}_0^{(2)}$, $\boldsymbol{e}_1^{(2)}$ are situated on the plane of vectors $\boldsymbol{k}$ and $\boldsymbol{k}_{\boldsymbol{g}}$ (π-polarization) and $\boldsymbol{e}_0^{(1)}$, $\boldsymbol{e}_1^{(1)}$ are perpendicular to this plane (σ-polarization); $\boldsymbol{g}$ is vector of the reciprocal lattice, defining a set of reflecting atomic planes. The system of equations for the Fourier transform images of the electromagnetic field in a two-wave approximation of dynamic theory of diffraction has the following form [15]:

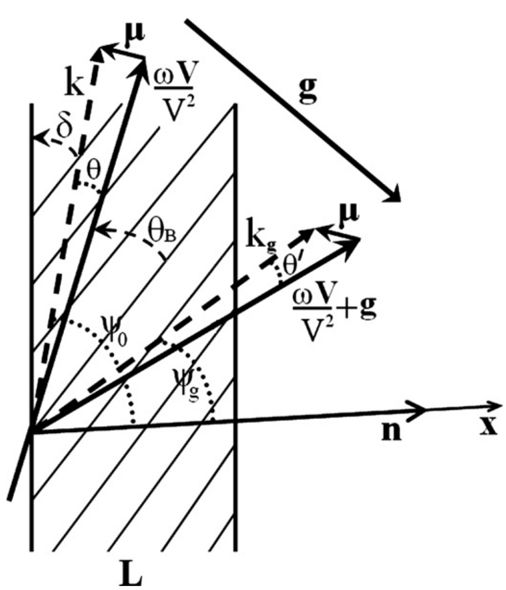

Fig. 1. Radiation process geometry.

$$\begin{cases}(\omega^2(1+\chi_0)-\boldsymbol{k}^2)E_0^{(s)}+\omega^2\chi_{-\boldsymbol{g}}C^{(s)}E_{\boldsymbol{g}}^{(s)}\\ \quad =8\pi^2\mathrm{ie}\omega\theta VP^{(s)}\delta(\omega-\boldsymbol{k}\boldsymbol{V}),\\ \omega^2\chi_{\boldsymbol{g}}C^{(s)}E_0^{(s)}+(\omega^2(1+\chi_0)-\boldsymbol{k}_{\boldsymbol{g}}^2)E_{\boldsymbol{g}}^{(s)}=0,\end{cases} \tag{3}$$

where $\chi_{\boldsymbol{g}}$, $\chi_{-\boldsymbol{g}}$ are the factors in the expansion of the dielectric susceptibility in the Fourier series by reciprocal lattice vectors $\boldsymbol{g}$

$$\chi(\omega,\boldsymbol{r})=\sum_{\boldsymbol{g}}\chi_{\boldsymbol{g}}(\omega)\mathrm{e}^{\mathrm{i}\boldsymbol{g}\mathbf{r}}=\sum_{\boldsymbol{g}}(\chi'_{\boldsymbol{g}}(\omega)+\mathrm{i}\chi''_{\boldsymbol{g}}(\omega))\mathrm{e}^{\mathrm{i}\boldsymbol{g}\mathbf{r}}. \tag{4}$$

We consider a crystal with the central symmetry ($\chi_{\boldsymbol{g}}=\chi_{-\boldsymbol{g}}$). $\chi'_{\boldsymbol{g}}$ is defined by the expression

$$\chi'_{\boldsymbol{g}}=\chi'_0(F(\boldsymbol{g})/Z)(S(\boldsymbol{g})/N_0)\exp\left(-\frac{1}{2}\boldsymbol{g}^2u_\tau^2\right), \tag{5}$$

where $\chi_0=\chi'_0+\mathrm{i}\chi''_0$ is the average dielectric susceptibility, $F(\boldsymbol{g})$ is the form factor of an atom containing Z electrons, $S(\boldsymbol{g})$ is the structure factor of a low-level cell containing N_0 atoms, u_τ is the mean-square amplitude of thermal atomic oscillations. In the present work the X-radiation frequency region is ($\chi'_{\boldsymbol{g}}<0$, $\chi'_0<0$).

The values $C^{(s)}$ and $P^{(s)}$ are defined in the set (3) as

$$\begin{aligned}&C^{(s)}=\boldsymbol{e}_0^{(s)}\boldsymbol{e}_1^{(s)},\quad C^{(1)}=1,\quad C^{(2)}=\cos 2\theta_{\mathrm{B}},\\ &P^{(s)}=\boldsymbol{e}_0^{(s)}(\boldsymbol{\mu}/\mu),\quad P^{(1)}=\sin\varphi,\quad P^{(2)}=\cos\varphi,\end{aligned} \tag{6}$$

where $\boldsymbol{\mu}=\boldsymbol{k}-\omega\boldsymbol{V}/V^2$ is the virtual photon momentum vector component perpendicular to the particle velocity vector $\boldsymbol{V}$ ($\mu=\omega\theta/V$, where $\theta\ll 1$ is the angle between vectors $\boldsymbol{k}$ and $\boldsymbol{V}$), θ_{B} is the angle between the electron velocity and a set of atomic planes in the crystal (Bragg angle), φ is the azimuth angle, counted from the plane formed by vectors $\boldsymbol{V}$ and $\boldsymbol{g}$, the value of reciprocal lattice vector is shown by expression $\boldsymbol{g}=2\omega_{\mathrm{B}}\sin\theta_{\mathrm{B}}/V$, ω_{B} is the Bragg frequency. The angle between vector $\frac{\omega\boldsymbol{V}}{V^2}+\boldsymbol{g}$ and the diffracted wave vector $\boldsymbol{k}_{\boldsymbol{g}}$ is defined as θ'. Equations set (3) under $s=1$ describes the fields of σ-polarization, and under $s=2$ the fields of π-polarization.

Let us solve the dispersion equation for X-rays in a crystal following from equations set (3):

$$(\omega^2(1+\chi_0)-\boldsymbol{k}^2)(\omega^2(1+\chi_0)-\boldsymbol{k}_{\boldsymbol{g}}^2)-\omega^4\chi_{-\boldsymbol{g}}\chi_{\boldsymbol{g}}C^{(s)^2}=0, \tag{7}$$

using standard methods of dynamic theory [1].

Let us now search for the wave vector projections $\boldsymbol{k}$ and $\boldsymbol{k}_{\boldsymbol{g}}$ to the x axis, coinciding with the vector $\boldsymbol{n}$ (see Fig. 1) as

$$\begin{aligned}\boldsymbol{k}_x&=\omega\cos\psi_0+\frac{\omega\chi_0}{2\cos\psi_0}+\frac{\lambda_0}{\cos\psi_0},\\ \boldsymbol{k}_{\boldsymbol{g}x}&=\omega\cos\psi_{\boldsymbol{g}}+\frac{\omega\chi_0}{2\cos\psi_{\boldsymbol{g}}}+\frac{\lambda_{\boldsymbol{g}}}{\cos\psi_{\boldsymbol{g}}}.\end{aligned} \tag{8}$$

For this purpose we use the well-known relation, connecting dynamic addition agents λ_0 and $\lambda_{\boldsymbol{g}}$ for X-rays [1]

$$\lambda_{\boldsymbol{g}}=\frac{\omega\beta}{2}+\lambda_0\frac{\gamma_{\boldsymbol{g}}}{\gamma_0}, \tag{9}$$

where $\beta = \alpha - \chi_0\left(1 - \frac{\gamma_g}{\gamma_0}\right)$, $\alpha = \frac{1}{\omega^2}(\boldsymbol{k}_g^2 - \boldsymbol{k}^2)$, $\gamma_0 = \cos\psi_0$, $\gamma_g = \cos\psi_g$, ψ_0 is the angle between the incident wave vector $\boldsymbol{k}$ and the vector normal to the plate surface n, ψ_g is the angle between the wave vector $\boldsymbol{k}_g$ and the vector n (see Fig. 1). The modules of the vectors $\boldsymbol{k}$ and $\boldsymbol{k}_g$ are

$$\boldsymbol{k} = \omega\sqrt{1+\chi_0} + \lambda_0, \quad \boldsymbol{k}_g = \omega\sqrt{1+\chi_0} + \lambda_g. \tag{10}$$

Let us substitute Eq. (8) into (7), taking into account Eq. (9), $\boldsymbol{k}_\parallel \approx \omega\sin\psi_0$ and $\boldsymbol{k}_{g\parallel} \approx \omega\sin\psi_g$. As a result, we will obtain the expression for the dynamic addition agents

$$\lambda_g^{(1,2)} = \frac{\omega}{4}\left(\beta \pm \sqrt{\beta^2 + 4\chi_g\chi_{-g}C^{(s)^2}\frac{\gamma_g}{\gamma_0}}\right),$$
$$\lambda_0^{(1,2)} = \omega\frac{\gamma_0}{4\gamma_g}\left(-\beta \pm \sqrt{\beta^2 + 4\chi_g\chi_{-g}C^{(s)^2}\frac{\gamma_g}{\gamma_0}}\right). \tag{11}$$

As $|\lambda_0| \ll \omega$, $|\lambda_g| \ll \omega$, we can show that $\theta \approx \theta'$ (see Fig. 1), and so later on θ will be used for all occasions.

It was suitable for us to express the solution of equations set (3) for the diffracted field in a crystal as

$$E_g^{(s)cr} = -\frac{8\pi^2 \mathrm{i}eV\theta P^{(s)}}{\omega} \times \frac{\omega^2\chi_{-g}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_g^2}\left(\lambda_g - \lambda_g^{(1)}\right)\left(\lambda_g - \lambda_g^{(2)}\right)}\delta\left(\frac{\omega\beta}{2} + \frac{\gamma_g}{\gamma_0}\lambda_0^* - \lambda_g\right) + E^{(s)^{(1)}}\delta(\lambda_g - \lambda_g^{(1)}) + E^{(s)^{(2)}}\delta(\lambda_g - \lambda_g^{(2)}) \tag{12}$$

where $\lambda_0 = \omega\left(\frac{\gamma^{-2}+\theta^2-\chi_0}{2}\right)$, $\gamma = \sqrt{1-V^2}$ is the Lorenz factor of the particle and $E^{(s)^{(1)}}$ and $E^{(s)^{(2)}}$ are free fields, corresponding to two solutions (11) of the dispersion equation (7).

The solution of equations set (3) for the field located in a vacuum forward of the crystal will be

$$E_0^{(s)\mathrm{vac}} = \frac{8\pi^2 \mathrm{i}eV\theta P^{(s)}}{\omega}\frac{1}{-\chi_0 - \frac{2}{\omega}\lambda_0}\delta(\lambda_0^* - \lambda_0) = \frac{8\pi^2 \mathrm{i}eV\theta P^{(s)}}{\omega} \times \frac{1}{\frac{\gamma_0}{\gamma_g}\left(-\chi_0 - \frac{2}{\omega}\frac{\gamma_0}{\gamma_g}\lambda_g + \beta\frac{\gamma_0}{\gamma_g}\right)}\delta\left(\frac{\omega\beta}{2} + \frac{\gamma_g}{\gamma_0}\lambda_0^* - \lambda_g\right). \tag{13}$$

Here the relation $\delta(\lambda_0 - \lambda_0) = \frac{1}{\frac{\gamma_0}{\gamma_g}}\delta\left(\frac{\omega\beta}{2} + \frac{\gamma_g}{\gamma_0}\lambda_0 - \lambda_g\right)$ following from Eq. (9) is used. For the field in a vacuum behind the crystal plate:

$$E_g^{(s)\mathrm{vac}} = E_g^{(s)\mathrm{Rad}}\delta\left(\lambda_g + \frac{\omega\chi_0}{2}\right), \tag{14}$$

where $E^{(s)\mathrm{Rad}}$ is the radiation field.

From the second equation of equations set (3) follows the expression connecting the diffracted and incident fields in the crystal:

$$E_0^{(s)\mathrm{cr}} = \frac{2\omega\lambda_g}{\omega^2\chi_g C^{(s)}}E_g^{(s)\mathrm{cr}}. \tag{15}$$

By using the standard boundary conditions

$$\int E_0^{(s)\mathrm{vac}}\mathrm{d}\lambda_g = \int E_0^{(s)\mathrm{cr}}\mathrm{d}\lambda_g, \quad \int E_g^{(s)\mathrm{cr}}\mathrm{e}^{\mathrm{i}\frac{\lambda_g}{\gamma_g}L}\mathrm{d}\lambda_g = \int E_g^{(s)\mathrm{vac}}\mathrm{e}^{\mathrm{i}\frac{\lambda_g}{\gamma_g}L}\mathrm{d}\lambda_g, \quad \int E_g^{(s)\mathrm{cr}}\mathrm{d}\lambda_g = 0, \tag{16}$$

we will obtain the expression for the radiation field divided into two items:

$$E_g^{(s)\mathrm{Rad}} = E_g^{(s)\mathrm{PXR}} + E_g^{(s)\mathrm{DTR}}, \tag{17a}$$

$$E_g^{(s)\mathrm{PXR}} = \frac{8\pi^2 \mathrm{i}eV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_g C^{(s)}}{2\omega\left(\lambda_g^{(1)} - \lambda_g^{(2)}\right)\frac{\gamma_0}{\gamma_g}} \times \left[\left(\frac{\omega}{2\frac{\gamma_0}{\gamma_g}\left(\lambda_g^* - \lambda_g^{(2)}\right)} - \frac{\omega}{2\lambda_0^*}\right)\left(1 - \mathrm{e}^{-\mathrm{i}\frac{\lambda_g^* - \lambda_g^{(2)}}{\gamma_g}L}\right) - \left(\frac{\omega}{2\frac{\gamma_0}{\gamma_g}\left(\lambda_g^* - \lambda_g^{(1)}\right)} - \frac{\omega}{2\lambda_0^*}\right)\left(1 - \mathrm{e}^{-\mathrm{i}\frac{\lambda_g^* - \lambda_g^{(1)}}{\gamma_g}L}\right)\right]\mathrm{e}^{\mathrm{i}\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}}, \tag{17b}$$

$$E_g^{(s)\mathrm{DTR}} = \frac{8\pi^2 \mathrm{i}eV\theta P^{(s)}}{\omega} \times \frac{\omega^2\chi_g C^{(s)}}{2\omega\left(\lambda_g^{(1)} - \lambda_g^{(2)}\right)\frac{\gamma_0}{\gamma_g}}\left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2\lambda_0^*}\right) \times \left(\mathrm{e}^{-\mathrm{i}\frac{\lambda_g^* - \lambda_g^{(1)}}{\gamma_g}L} - \mathrm{e}^{-\mathrm{i}\frac{\lambda_g^* - \lambda_g^{(2)}}{\gamma_g}L}\right)\mathrm{e}^{\mathrm{i}\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}}, \tag{17c}$$

where $\lambda_g^* = \frac{\omega\beta}{2} + \frac{\gamma_g}{\gamma_0}\lambda_0$.

Eq. (17b) describes the field of PXR, where the first branch of PXR is the more substantial because the real part of the denominator corresponding to this branch can become zero ($\mathrm{Re}(\lambda_g - \lambda_g^1)) = 0$) but the one corresponding to the second branch cannot ($\mathrm{Re}(\lambda_g - \lambda_g^2) \neq 0$).

Eq. (17c) describes the field of DTR, which appears as a result of diffraction of transition radiation, originating from the entrance surface of the crystal plate, on the same set of atomic planes in the crystal, which gives rise to PXR.

By substitution of Eq. (11) into (17b) and (17c), and by keeping only the first branch PXR and by using the well-known expression for the X-radiation spectral-angular density [15]

$$\frac{\mathrm{d}^2W}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}|E_{\mathrm{Rad}}^{(s)}|^2, \tag{18}$$

we obtain the formulae for the spectral-angular density of PXR, DTR and for the result of interference between these mechanisms:

$$\omega \frac{d^2 N^{(s)}_{\mathrm{PXR}}}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2} \frac{\theta^2 P^{(s)^2}}{(\theta^2+\gamma^{-2}-\chi'_0)^2} \left| \frac{1}{K^{(s)}} \frac{\xi^{(s)}(\omega) - \frac{i\rho^{(s)}(1-\varepsilon)}{2} - K^{(s)}}{\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}} \left(1 - e^{-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)}\right) \right|^2, \tag{19a}$$

$$\omega \frac{d^2 N^{(s)}_{\mathrm{DTR}}}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2} \theta^2 P^{(s)^2} \left(\frac{1}{\theta^2+\gamma^{-2}} - \frac{1}{\theta^2+\gamma^{-2}-\chi'_0} \right)^2 \left| \frac{e^{-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)} - e^{-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)+K^{(s)}}{\varepsilon}\right)}}{\frac{K^{(s)}}{\varepsilon}} \right|^2, \tag{19b}$$

$$\begin{aligned} \omega \frac{d^2 N^{(s)}_{\mathrm{INT}}}{d\omega\, d\Omega} = {} & \frac{e^2}{2\pi^2} \frac{\theta^2 P^{(s)2}}{(\theta^2+\gamma^{-2}-\chi'_0)} \left(\frac{1}{\theta^2+\gamma^{-2}} - \frac{1}{\theta^2+\gamma^{-2}-\chi'_0} \right) \frac{\varepsilon}{|K^{(s)}|^2} \mathrm{Re}\left(\frac{\xi^{(s)}(\omega) - \frac{i\rho^{(s)}(1-\varepsilon)}{2} - K^{(s)}}{\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}} \right. \\ & \left. \times \left(1 - e^{-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)}\right) \left(e^{ib^{(s)}\left(\sigma^{(s)} + \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)-K^{(s)*}}{\varepsilon}\right)} - e^{ib^{(s)}\left(\sigma^{(s)} + \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega)+K^{(s)*}}{\varepsilon}\right)} \right) \right), \end{aligned} \tag{19c}$$

where

$$\begin{aligned} & K^{(s)} = \sqrt{\xi^{(s)}(\omega)^2 + \varepsilon - 2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)}(\omega) + \kappa^{(s)}\varepsilon \right) - \rho^{(s)2}\left(\frac{(1-\varepsilon)^2}{4} + \kappa^{(s)2}\varepsilon \right)}, \\ & \xi^{(s)}(\omega) = \frac{\alpha}{2|\chi'_g|C^{(s)}} - \frac{\chi'_0(1-\varepsilon)}{2|\chi'_g|C^{(s)}} = \eta^{(s)}(\omega) + \frac{(1-\varepsilon)}{2\nu^{(s)}}, \\ & \nu^{(s)} = \frac{|\chi'_g|C^{(s)}}{|\chi'_0|}, \quad \rho^{(s)} = \frac{\chi''_0}{|\chi'_g|C^{(s)}|}, \\ & \eta^{(s)}(\omega) = \frac{\alpha}{2|\chi'_g|C^{(s)}} = \frac{2\sin^2\theta_B}{V^2|\chi'_g|C^{(s)}} \left(1 - \frac{\omega(1-\theta\cos\varphi\cot\theta_B)}{\omega_B} \right), \\ & \varepsilon = \frac{\gamma_g}{\gamma_0}, \quad \kappa^{(s)} = \frac{\chi''_g C^{(s)}}{\chi''_0}, \\ & \sigma^{(s)} = \frac{1}{|\chi'_g|C^{(s)}}(\theta^2+\gamma^{-2}-\chi'_0), \quad b^{(s)} = \frac{\omega|\chi'_g|C^{(s)}}{2}\frac{L}{\gamma_0}. \end{aligned} \tag{20}$$

$K^{(s)^*}$ is the complex conjugate to $K^{(s)}$. The parameter $b^{(s)}$ defines the role of the electron path length in the crystal L/γ_0 in the radiation process. Since the inequality $2\sin^2\theta_B/V^2|\chi'_g|C^{(s)} \gg 1$ holds true in the X-ray frequency region, the quantity $\eta^{(s)}(\omega)$ depends strongly on ω. Therefore, the $\eta^{(s)}(\omega)$ is convenient to use as the spectral variable for further analysis of PXR and DTR.

Let us note that the value $\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{(1-\varepsilon)}{2\nu^{(s)}}$ enters into the expressions instead of $\eta^{(s)}(\omega)$, where the second term appears due to the refraction effect under asymmetric reflection. In the symmetric case ($\varepsilon = 1$) this term becomes zero.

Let us present the parameter ε in a functional form $\varepsilon = \sin(\delta+\theta_B)/\sin(\delta-\theta_B)$, where δ is the angle between the entrance plate surface and the crystal plane. For an increase of angle δ the parameter ε increases and vice versa.

The expressions for the spectral-angular density of PXR and DTR are obtained on the basis of a two-wave approximation of dynamic diffraction theory, taking into account absorption of the radiation in matter and free orientation of the system of diffracting parallel atomic planes in the crystal to the external surface of the plate (angle δ).

In particular, when the crystal atomic planes are perpendicular to entrance surface of the plate ($\delta = \pi/2$, $\varepsilon = 1$), Eqs. (19a) and (19b) turn into expressions obtained in [9].

3. Spectral-angular density of PXR and DTR in a thin non-absorbing crystal

Let us consider a thin non-absorbing crystal. Assuming in Eq. 19 $\rho^{(s)} = \frac{\chi''_0}{|\chi'_g|C^{(s)}} = 0$, we obtain the following expression for the spectral-angular density of radiation:

$$\omega \frac{d^2 N^{(s)}}{d\omega\, d\Omega} = \frac{e^2 P^{(s)2}}{2\pi^2|\chi'_0|}(F^{(s)}_{\mathrm{PXR}} + F^{(s)}_{\mathrm{DTR}} + F^{(s)}_{\mathrm{INT}}) \tag{21}$$

$$F^{(s)}_{\mathrm{PXR}} = \frac{\frac{\theta^2}{|\chi'_0|}}{\left(\frac{\theta^2}{|\chi'_0|} + \frac{1}{\gamma^2|\chi'_0|} + 1 \right)^2} \left(\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}} - 1 \right)^2 \times \frac{1 - \cos\left(b^{(s)}\left(\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon} \right) \right)}{\left(\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon} \right)^2}, \tag{21a}$$

$$F_{\mathrm{DTR}}^{(s)}=\frac{\frac{\theta^{2}}{|\chi_0'|}}{\left(\frac{\theta^{2}}{|\chi_0'|}+\frac{1}{\gamma^{2}|\chi_0'|}+1\right)^{2}\left(\frac{\theta^{2}}{|\chi_0'|}+\frac{1}{\gamma^{2}|\chi_0'|}\right)^{2}}\times\frac{1-\cos\left(\frac{2b^{(s)}\sqrt{\xi^{(s)^{2}}+\varepsilon}}{\varepsilon}\right)}{\frac{\xi^{(s)^{2}}+\varepsilon}{\varepsilon^{2}}} \tag{21b}$$

$$F_{\mathrm{INT}}^{(s)}=\frac{2\frac{\theta^{2}}{|\chi_0'|}}{\left(\frac{\theta^{2}}{|\chi_0'|}+\frac{1}{\gamma^{2}|\chi_0'|}+1\right)^{2}\left(\frac{\theta^{2}}{|\chi_0'|}+\frac{1}{\gamma^{2}|\chi_0'|}\right)}\times\varepsilon\frac{\xi^{(s)}-\sqrt{\xi^{(s)^{2}}+\varepsilon}}{\xi^{(s)^{2}}+\varepsilon}\sin\left(\frac{b^{(s)}\sqrt{\xi^{(s)^{2}}+\varepsilon}}{\varepsilon}\right)\times\frac{\sin\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}}{\varepsilon}\right)\right)-\sin\left(\frac{b^{(s)}\sqrt{\xi^{(s)^{2}}+\varepsilon}}{\varepsilon}\right)}{\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^{2}}+\varepsilon}}{\varepsilon}}. \tag{21c}$$

Let us consider the dependence of the spectral-angular density of PXR on the orientation of the crystal plate surface relative to the set of parallel diffracting atomic planes (defined by the parameter ε). We suppose that the angle θ_{B} between the electron velocity and the reflecting atomic planes and also the electron path length in the crystal plate (L/γ_0) are fixed. Fig. 2 shows two orientations of the crystal plate surface from many possible ones, corresponding to a given fixed length of straight trajectory of relativistic electrons in the crystal plate ($\varepsilon = 1$ and $\varepsilon > 1$).

It is necessary to note that under these conditions the thickness of the plate will depend on the parameter $\varepsilon = \varepsilon(\theta_{\mathrm{B}}, \delta)$. For an increase of ε the plate thickness will decrease and accordingly the path length of photons in the crystal will decrease.

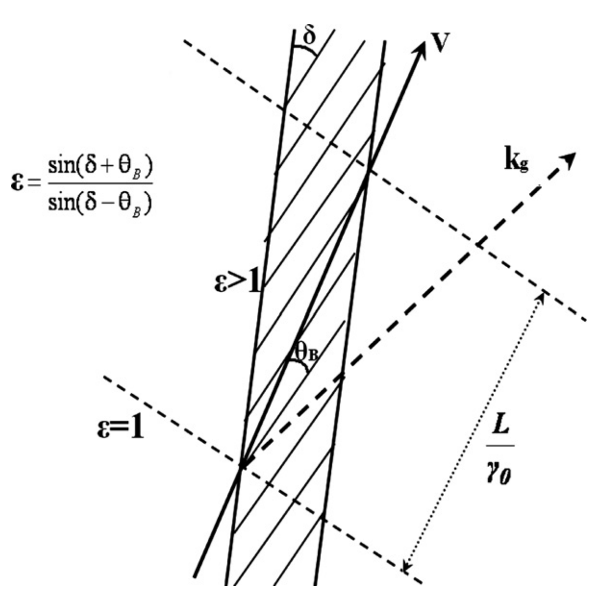

Fig. 2. Symmetric ($\varepsilon = 1$) and asymmetric ($\varepsilon > 1$) field reflection.

Let us extract from Eq. (21a) the part describing the PXR spectrum:

$$P_{\mathrm{PXR}}^{(s)}=\left(\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^{2}}+\varepsilon}}-1\right)^{2}\times\frac{1-\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^{2}}+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^{2}}+\varepsilon}}{\varepsilon}\right)^{2}},$$

$$\xi^{(s)}(\omega)=\eta^{(s)}(\omega)+\frac{(1-\varepsilon)}{2\nu^{(s)}},\quad \sigma^{(s)}=\frac{1}{\nu^{(s)}}\left(\frac{\theta^{2}}{|\chi_0'|}+\frac{1}{\gamma^{2}|\chi_0'|}+1\right). \tag{22}$$

It is evident from Eq. (22), that for a fixed electron energy and observation angle (fixed $\sigma^{(s)}$) the increase of ε (decrease of angle δ) leads to a widening of the PXR spectrum. This is explained by the real part of the fraction denominator $\mathrm{Re}\left[\sigma^{(s)}-\frac{\mathrm{i}\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right]$ in Eq. (22) changing more weakly under changing a quantity $\xi^{(s)}(\omega)$ when the value of ε is big. The curves describing the spectrum of PXR, plotted according to Eq. (22) for fixed electron energy γ and observation angle θ, are shown in Fig. 3. As evident from this figure the width of spectrum essentially depends on the parameter ε.

Let us consider the angular density of the radiation. For this purpose, we integrate Eq. (21) over the frequency function $\eta^{(s)}(\omega)$

$$\frac{\mathrm{d}N^{(s)}}{\mathrm{d}\Omega}=\frac{\mathrm{e}^{2}\nu^{(s)}P^{(s)^{2}}}{4\pi^{2}\sin^{2}\theta_{\mathrm{B}}}\left(R_{\mathrm{PXR}}^{(s)}+R_{\mathrm{DTR}}^{(s)}+R_{\mathrm{INT}}^{(s)}\right), \tag{23}$$

where

$$R_{\mathrm{PXR}}^{(s)}=\int_{-\infty}^{+\infty}F_{\mathrm{PXR}}^{(s)}\,\mathrm{d}\eta^{(s)}(\omega),\quad R_{\mathrm{DTR}}^{(s)}=\int_{-\infty}^{+\infty}F_{\mathrm{DTR}}^{(s)}\,\mathrm{d}\eta^{(s)}(\omega),$$
$$R_{\mathrm{INT}}^{(s)}=\int_{-\infty}^{+\infty}F_{\mathrm{INT}}^{(s)}\,\mathrm{d}\eta^{(s)}(\omega). \tag{24}$$

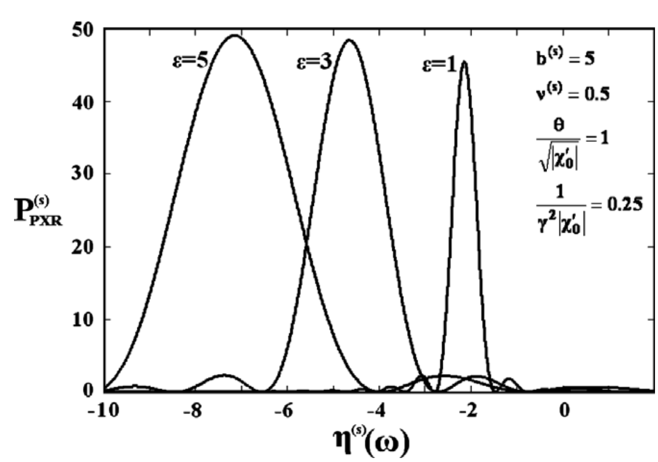

Fig. 3. PXR peak width dependence on parameter ε.

The curves, presented in Fig. 4 demonstrate a considerable increase of the PXR angular density under an increase of reflection asymmetry. In the case of an absorbing crystal this effect becomes stronger because of the decrease of the photon pass length under an increase of ε (see Fig. 2).

Now let us consider the influence of the reflection asymmetry on the angular density of DTR and on the interference of PXR and DTR. The curves of the dependence of $R_{\mathrm{DTR}}^{(s)}$ on the parameter $\theta/\sqrt{|\chi_0'|}$ are presented in Fig. 5. They demonstrate the essential increase of the angular density of DTR under a decrease of angle δ.

The curves of dependence $R_{\mathrm{PXR}}^{(s)}$, $R_{\mathrm{DTR}}^{(s)}$, $R_{\mathrm{INT}}^{(s)}$ and $R_{\mathrm{SUMM}}^{(s)} = R_{\mathrm{PXR}}^{(s)} + R_{\mathrm{DTR}}^{(s)} + R_{\mathrm{INT}}^{(s)}$ on $\theta/\sqrt{|\chi_0'|}$ are presented in Fig. 6, where the summary angular density $R_{\mathrm{SUMM}}^{(s)}$ reflects the effect of interference of these radiation mechanisms. One can see, that for a large enough energy of the electron, when the angular density of PXR and DTR become congruent quantities, interference can produce oscillations in the angular distribution of radiation for a certain reflection asymmetry.

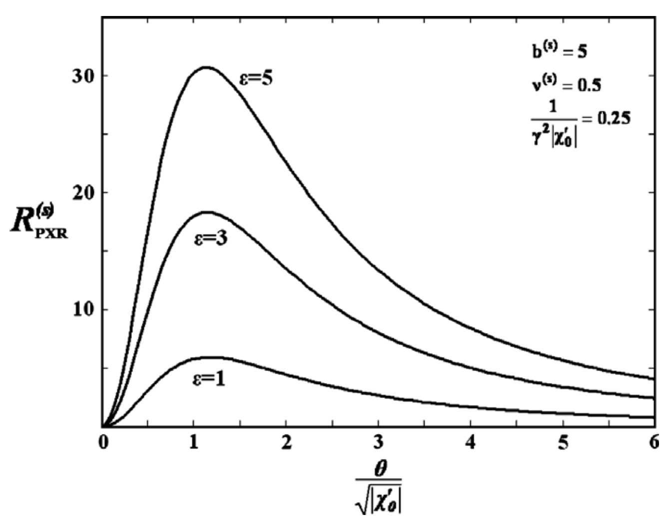

Fig. 4. Dependence of PXR angle density on asymmetry.

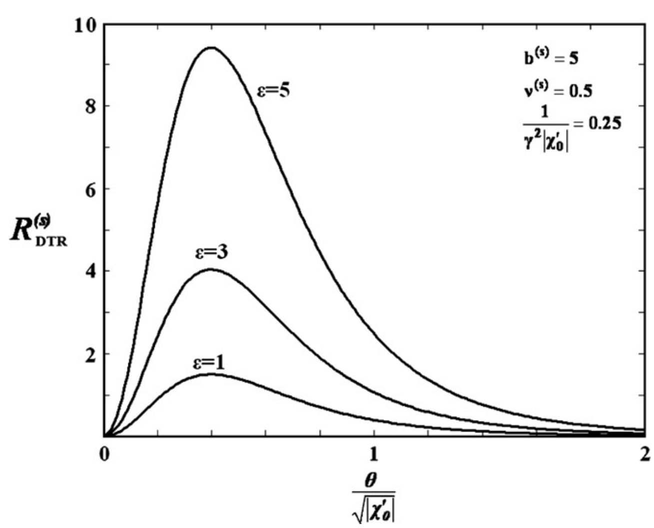

Fig. 5. Dependence of DTR angle density on asymmetry.

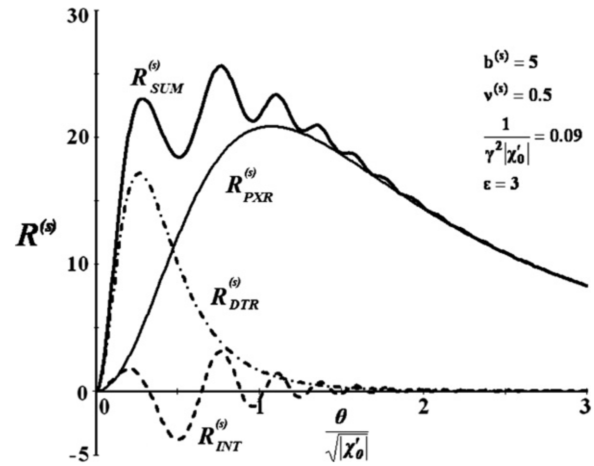

Fig. 6. Influence of PXR and DTR interference on summary angular density of radiation.

4. Conclusions

Analytical expressions of the spectral-angular distribution of PXR and DTR of a relativistic electron traversing a single crystal plate of finite thickness are obtained on the basis of a two-wave approximation of dynamic diffraction theory in a Laue scattering geometry for the general case of asymmetrical reflection. The expressions derived contain the angle between the crystal plate surface and the diffracting atomic planes of the crystal (angle δ) as one of the parameters of the theoretical model.

It has been shown that in a thin non-absorbing crystal under a decrease of angle δ, the spectrum of PXR becomes wider and as a result the PXR angular density essentially increases. It is shown that angular density of DTR also essentially increases under a decrease of angle δ. The influence of interference of PXR and DTR on the summary radiation angular density under conditions of asymmetric reflection are also shown.

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