Dynamical diffraction effects in the transition radiation of a relativistic electron crossing a thin crystal

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Abstract

Transition radiation (TR) of a relativistic electron crossing the *in* surface of a crystal may be scattered by a system of atomic planes. This effect may change essentially the interference between TR waves emitted from in and out surfaces of a crystalline target. The influence of the mentioned effect on total emission properties is studied in this work within the frame of the dynamical diffraction approach.

1. Introduction

When a fast charged particle crosses the boundary between two different media it emits the free photons of transition radiation (TR) [1]. In the case that such a particle crosses a layer of a medium the interference between TR waves emitted from *in* and *out* surfaces of a layer takes place. If the target possesses a crystalline structure the TR wave emitted from *in* surface of the target may be scattered by a system of atomic planes. This dynamical diffraction effect can change the condition of the interference between such wave and that emitted from *out* surface as it is shown in the present work.

The aim of this paper is the detailed analysis of TR wave Bragg scattering in Laue geometry. The possibility to obtain a single emission peak with very narrow spectral and angular width caused by the dynamical diffraction effect is shown.

The general formulae obtained in this work take into account the contribution of the parametric X-ray radiation (PXR) emitted in forward direction by a fast particle [2,3]. The possibility to separate TR contribution by the use of a photon collimator is shown for the case of high enough particle energy. On the other hand the problem of forward PXR observation is not solved up to now. Therefore the conditions of such PXR observation are discussed as well.

The analysis is performed in this work for both a thin non-absorbing crystal and a thick crystal in which the influence of a photoabsorption is essential. For the latter case the manifestation of the anomalous Borrmann absorption of the TR wave is predicted.

Numerical calculations and corresponding discussion of a possible experiment devoted to the observation of the narrow TR peak complete the study of this work.

It is important to note that the considered problems are analogous to that arising in the physics of PXR and diffracted TR [4–6]. However, in the above mentioned works the emission in the Bragg-scattering direction was studied, contrary to this work devoted to the emission in the forward direction.

2. General expressions

Let us consider the emission of a relativistic electron beam crossing a crystalline target with *in* surface and *out* surface placed in the planes $x = 0$ and $x = L$, respectively. Reflecting crystallographic plane is assumed to be parallel to the plane XZ (the reciprocal lattice vector $\boldsymbol{g}$ defining this crystallographic plane is directed along the axis $\boldsymbol{e}_y$). The mean beam velocity $\boldsymbol{v}$ is placed in the plane XY at the angle φ relative to reflecting crystallographic plane, $\boldsymbol{e}_y\boldsymbol{v} = -v\cos\varphi$. To find the Fourier transform of electric field

$$\boldsymbol{E}_{\omega\boldsymbol{k}} = (2\pi)^{-4}\int \mathrm{d}t\,\mathrm{d}^3r\boldsymbol{E}(\boldsymbol{r},t)\exp(\mathrm{i}\omega t - \mathrm{i}\boldsymbol{k}\boldsymbol{r})$$

inside the crystal with the dielectric permeability

$$\varepsilon(\omega,\boldsymbol{r}) = 1 + \chi_0(\omega) + \sum_{\boldsymbol{g}}{}' \chi_{\boldsymbol{g}}(\omega)\mathrm{e}^{\mathrm{i}\boldsymbol{g}\boldsymbol{r}}$$

($\chi_0(\omega)$ is the mean dielectric susceptibility of a crystal, $\chi_{\boldsymbol{g}}(\omega)$ is the structure amplitude of atomic scattering), we have the following Maxwell equation:

$$\begin{aligned}&\left(k^2 - \omega^2(1+\chi_0)\right)\boldsymbol{E}_{\omega\boldsymbol{k}} - \boldsymbol{k}(\boldsymbol{k}\boldsymbol{E}_{\omega\boldsymbol{k}})\\&\quad - \omega^2\sum_{\boldsymbol{g}}{}'\chi_{-\boldsymbol{g}}\boldsymbol{E}_{\omega\boldsymbol{k}+\boldsymbol{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\boldsymbol{v}\delta(\omega - \boldsymbol{k}\boldsymbol{v}).\end{aligned} \tag{1}$$

Within the frame of two-wave approximation of the X-ray dynamical diffraction theory [7] the total field $\boldsymbol{E}_{\omega\boldsymbol{k}}$ is defined by the components $\boldsymbol{E}^{\mathrm{tr}}_{\omega\boldsymbol{k}} = \sum_{\lambda=1}^{2}\boldsymbol{e}_{\lambda0}E_{\lambda0}$ and $\boldsymbol{E}_{\omega\boldsymbol{k}+\boldsymbol{g}} = \sum_{\lambda=1}^{2}\boldsymbol{e}_{\lambda g}E_{\lambda g}$ only (here $\boldsymbol{e}_{\lambda 0,g}$ are the polarization vectors, $\boldsymbol{k}\boldsymbol{e}_{\lambda0} = (\boldsymbol{k}+\boldsymbol{g})\boldsymbol{e}_{\lambda g} = 0$). The system (1) is reduced to the well-known equations of the dynamical diffraction theory,

$$\begin{aligned}&\left(k^2 - \omega^2(1+\chi_0)\right)E_{\lambda0} - \omega^2\chi_{-\boldsymbol{g}}\alpha_\lambda E_{\lambda g}\\&\quad = \frac{\mathrm{i}\omega e}{2\pi^2}\boldsymbol{e}_{\lambda0}\boldsymbol{v}\delta(\omega - \boldsymbol{k}\boldsymbol{v}),\end{aligned} \tag{2}$$

$$\left((\boldsymbol{k}+\boldsymbol{g})^2 - \omega^2(1+\chi_0)\right)E_{\lambda g} - \omega^2\chi_{\boldsymbol{g}}\alpha_\lambda E_{\lambda0} = 0,$$

where $\alpha_1 = 1$, $\alpha_2 = \boldsymbol{k}(\boldsymbol{k}+\boldsymbol{g})/k|\boldsymbol{k}+\boldsymbol{g}|$.

The solution of the system (2) is given by

$$\begin{aligned}E_{\lambda0} &= b_{\lambda\boldsymbol{k}_\parallel}\delta(k_x - k_1) + c_{\lambda\boldsymbol{k}_\parallel}\delta(k_x - k_2)\\&\quad + \frac{\mathrm{i}\omega e}{2\pi^2}\boldsymbol{e}_{\lambda0}\boldsymbol{v}\frac{k_x^2 - \omega^2(1+\chi_0) + k_\parallel^2 + 2k_y\Delta}{(k_x^2 - k_1^2)(k_x^2 - k_2^2)}\\&\quad \times\delta(\omega - \boldsymbol{k}\boldsymbol{v}),\end{aligned}$$

$$E_{\lambda g} = \frac{\omega^2\chi_{\boldsymbol{g}}\alpha_\lambda}{k_x^2 - \omega^2(1+\chi_0) + k_\parallel^2 + 2k_y\Delta}E_{\lambda0}, \tag{3}$$

$$k_{1,2}^2 = \omega^2(1+\chi_0) - k_\parallel^2 - k_y\left(\Delta \pm \sqrt{\Delta^2 + \beta_\lambda^2}\right),$$

$$\Delta = g\left(\frac{g}{2k_y} - 1\right), \quad \beta_\lambda^2 = \frac{\omega^4}{k_y^2}\chi_{\boldsymbol{g}}\chi_{-\boldsymbol{g}}\alpha_\lambda^2,$$

where $\boldsymbol{k}_\parallel = \boldsymbol{k}_y + \boldsymbol{k}_z$.

Determining the solution of equations for the fields $E_{\lambda0,g}$ outside the target (these equations follow from (2) in the limit $\chi_0 = \chi_{\boldsymbol{g}} = 0$), one can find the unknown coefficients by the use of ordinary boundary conditions. The final expression for the field $E_{\lambda0}$ in a vacuum behind the crystal is given by

$$E^{v}_{\lambda 0} = a_{\lambda \boldsymbol{k}_\parallel} \delta\left(k_x - \sqrt{\omega^2 - k_\parallel^2}\right) + \frac{\mathrm{i}\omega e}{2\pi^2 v_x} \boldsymbol{e}_{\lambda o} \boldsymbol{v} \frac{\delta(k_x - k_*)}{k_x^2 - \omega^2 + k_\parallel^2},$$

$$a_{\lambda \boldsymbol{k}_\parallel} = \frac{\mathrm{i}\omega e}{2\pi^2 v_x} \boldsymbol{e}_{\lambda o} \boldsymbol{v} \exp\left(\mathrm{i}\left(k_* - \sqrt{\omega^2 - k_\parallel^2}\right)L\right) \times \left\{ \frac{\Delta - \sqrt{\Delta^2 + \beta_\lambda^2}}{\sqrt{\Delta^2 + \beta_\lambda^2}} \left(\frac{1}{k_*^2 - \omega^2 + k_\parallel^2} - \frac{1}{k_*^2 - k_1^2} \right) \times \left(1 - \mathrm{e}^{-\mathrm{i}(k_* - k_1)L}\right) - \frac{\Delta + \sqrt{\Delta^2 + \beta_\lambda^2}}{\sqrt{\Delta^2 + \beta_\lambda^2}} \times \left(\frac{1}{k_*^2 - \omega^2 + k_\parallel^2} - \frac{1}{k_*^2 - k_2^2} \right) \left(1 - \mathrm{e}^{-\mathrm{i}(k_* - k_2)L}\right) \right\}, \tag{4}$$

where $k_* = (1/v_x)(\omega - \boldsymbol{k}_\parallel \boldsymbol{v}_\parallel)$. It is easy to see that the first term in the expression for $E^{v}_{\lambda 0}$ in (4) describes an emission field. The second one corresponds to a particle Coulomb field. To find an emission amplitude it is necessary to perform the integration

$$E^{\mathrm{Rad}}_{\lambda} = \int \mathrm{d}^3 k a_{\lambda \boldsymbol{k}_\parallel} \delta\left(k_x - \sqrt{\omega^2 - k_\parallel^2}\right) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{n}r} \rightarrow A_\lambda \frac{\mathrm{e}^{\mathrm{i}\omega r}}{r}, \tag{5}$$

where $\boldsymbol{n}$ is the unit vector in the direction of an emitted photon observation. The result of the integration (5) may be obtained in wave zone by the stationary phase method. The emission amplitude is described by the simple formula

$$A_\lambda = -2\pi \mathrm{i}\omega n_x a_{\lambda \omega \boldsymbol{n}_\parallel}, \tag{6}$$

determining the spectral–angular and polarization characteristics of emitted photons.

3. TR spectral–angular distribution in the case of a thin non-absorbing crystal

Using the obtained general results (4) and (6), let us consider the emission spectral–angular distribution in the case of non-absorbing crystal with dielectric susceptibilities $\chi_0 = -\omega_0^2/\omega^2$ and $\chi_g = -\omega_g^2/\omega^2$, where ω_0 is the plasma frequency of a medium, $\omega_g^2 = \omega_0^2 F(\boldsymbol{g}) \mathrm{e}^{-(1/2)g^2u^2}/Z$ for the simplest crystal with one atom in the elementary cell ($F(\boldsymbol{g})$ is the atom form factor, Z is the number of electrons in an atom, $\mathrm{e}^{-(1/2)g^2u^2}$ is the Debye–Waller factor). Definition of the angular variable $\boldsymbol{\Theta}$ by the formula

$$\boldsymbol{n} = \frac{\boldsymbol{v}}{v}\left(1 - \frac{1}{2}\boldsymbol{\Theta}^2\right) + \boldsymbol{\Theta}, \quad \boldsymbol{v}\boldsymbol{\Theta} = 0, \quad \Theta \ll 1, \tag{7}$$

allows us to simplify the important expressions $k_*^2 - \omega^2 + k_\parallel^2$ and $k_*^2 - k_{1,2}^2$ in (4),

$$k_*^2 - \omega^2 + k_\parallel^2 \approx \omega^2(\gamma^{-2} + \Theta^2),$$

$$k_*^2 - k_{1,2}^2 \approx \omega^2 \left(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + \Theta^2 + \chi_\lambda\left(\xi_\lambda \pm \sqrt{\xi_\lambda^2 + 1}\right) \right), \tag{8}$$

$$\xi_\lambda = \frac{g^2}{2\omega_g^2 |\alpha_\lambda|} \left(\frac{\omega_b'}{\omega} - 1 \right), \quad \chi_\lambda = \frac{\omega_g^2}{\omega^2}|\alpha_\lambda|,$$

where $\omega_b' = \omega_b \cdot (1 + \Theta_\parallel \cot \varphi)$, $\omega_b = g/2\sin\varphi$ is the Bragg frequency, $\gamma = (1 - v^2)^{-1/2}$.

Taking into account the formulae (4), (6) and (8), it is easy to obtain the final expression for the total emission spectral–angular distribution in the form

$$\omega \frac{\mathrm{d}N_\lambda}{\mathrm{d}\omega\, \mathrm{d}^2\Theta} = |A_\lambda|^2 = |A_\lambda^{\mathrm{TR}} + A_\lambda^{\mathrm{PXR}}|^2,$$

$$A_\lambda^{\mathrm{TR}} = \frac{e}{2\pi}\Theta_\lambda \left(\frac{1}{\gamma^{-2} + \Theta^2} - \frac{1}{\gamma^{-2} + (\omega_0^2/\omega^2) + \Theta^2} \right) \times \left\{ \left(1 + \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2 + 1}} \right) \left(1 - \mathrm{e}^{-\mathrm{i}\sigma_-}\right) + \left(1 - \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2 + 1}} \right) \left(1 - \mathrm{e}^{-\mathrm{i}\sigma_+}\right) \right\},$$

$$A_\lambda^{\rm PXR} = \frac{e}{2\pi}\frac{\Theta_\lambda \chi_\lambda}{\gamma^{-2}+(\omega_0^2/\omega^2)+\Theta^2}\frac{\omega L}{2\cos\varphi\sqrt{\xi_\lambda^2+1}} \times \left[\frac{1}{\sigma_+}\left(1-e^{-i\sigma_+}\right) - \frac{1}{\sigma_-}\left(1-e^{-i\sigma_-}\right)\right], \quad (9)$$

where $\Theta_1 = \Theta_\perp$, $\Theta_2 = \Theta_\parallel$,

$$\sigma_\pm = \frac{\omega L}{2\cos\varphi}\left(\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+\Theta^2+\chi_\lambda\left(\xi_\lambda \pm \sqrt{\xi_\lambda^2+1}\right)\right).$$

The total emission amplitude A_λ consists of two components. The amplitude $A_\lambda^{\rm TR}$ describes TR with taking into account the dynamical diffraction effects. It is easy to see that in the limit $|\xi_\lambda| \gg 1$ the amplitude $A_\lambda^{\rm TR}$ takes a simple form

$$A_\lambda^{\rm TR} \to \frac{e}{\pi}\Theta_\lambda\left(\frac{1}{\gamma^{-2}+\Theta^2} - \frac{1}{\gamma^{-2}+(\omega_0^2/\omega^2)+\Theta^2}\right) \times \left\{1-\exp\left[-\frac{i\omega L}{2\cos\varphi}\left(\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+\Theta^2\right)\right]\right\} \quad (10)$$

coinciding with the well-known emission amplitude for the ordinary TR emitted by a relativistic particle crossing a layer of amorphous medium [8]. The dynamical diffraction effects manifest in the close vicinity to the Bragg frequency ω_b only.

The second amplitude $A_\lambda^{\rm PXR}$ describes the PXR containing the contribution of two branches of the solution of wave equations. Only the quantity σ_- may be equal to zero considered as a function of ξ_λ. Therefore the corresponding item in the expression for $A_\lambda^{\rm PXR}$ makes the essential contribution to the PXR yield.

The width of TR angular distribution has a value of the order of γ^{-1} in accordance with the expression (9) for $A_\lambda^{\rm TR}$. On the other hand the performed analysis (see [9] as well) shows that PXR angular distribution has a dip gap in the range $\Theta < \sqrt{\gamma^{-2}+(\omega_0^2/\omega^2)}$. This circumstance allows to neglect the contribution of PXR to emission yield within the angular range $\Theta \approx \gamma^{-1}$ in the case of high enough emitting electron's energy,

$$\gamma^2\omega_0^2/\omega_b^2 \gg 1. \quad (11)$$

The condition (11) allows to single out the TR contribution by the use of a photon collimator with angular size $\delta\Theta \approx \gamma^{-1}$. We assume the condition (11) to be valid and consider TR properties, described by the formula

$$\omega\frac{dN_\lambda^{\rm TR}}{d\omega\, d^2\Theta} = \frac{e^2}{\pi^2}\frac{\Theta_\lambda^2}{(\gamma^{-2}+\Theta^2)^2}T_0(\xi_\lambda,\tau,\tau_\lambda), \quad (12)$$

where

$$T_0 = \left[\cos(\tau+\tau_\lambda\xi_\lambda) - \cos\left(\tau_\lambda\sqrt{\xi_\lambda^2+1}\right)\right]^2 + \left[\sin(\tau+\tau_\lambda\xi_\lambda) - \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\sin\left(\tau_\lambda\sqrt{\xi_\lambda^2+1}\right)\right]^2$$

following from (9). Here

$$\tau = \frac{\omega_0^2 L}{2\omega_b\cos\varphi}, \quad \tau_\lambda = \frac{\omega_g^2 L|\alpha_\lambda|}{2\omega_b\cos\varphi},$$

$$\alpha_1 = 1, \quad \alpha_2 = \cos 2\varphi.$$

The quantity T_0 in (12) is considered as a function of the fast variable ξ_λ. This function describes TR correctly in the close vicinity near Bragg frequency. The very important parameter τ describes the interference between the ordinary TR waves emitted from *in* and *out* surface of a target. It is easy to see that in the limit $|\xi_\lambda| \gg 1$ (far from the vicinity of Bragg frequency) the function T_0 is described by

$$T_0 \to 2(1-\cos\tau) \quad (13)$$

in accordance with the general result (10).

The result (13) shows interesting possibility to obtain a very narrow TR peak using interference properties of the ordinary TR. Choosing the crystal thickness L satisfying the condition

$$\tau = \frac{\omega_0^2}{g}\tan\phi = 2\pi, \quad (14)$$

one can suppress the ordinary TR contribution in the vicinity of Bragg frequency ω_b due to the negative interference between TR waves emitted from *in* and *out* surface of the target. The possibility to use an intrafoil resonance in TR has been recently demonstrated experimentally [10].

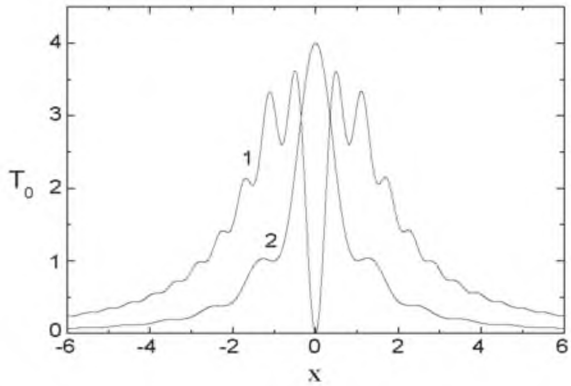

Fig. 1. Dependence of the reflection coefficient T_0 on the photon energy ($x = \xi_\lambda$): (1) $\tau = 2\pi,\ \tau_\lambda = 6$; (2) $\tau = 2\pi,\ \tau_\lambda = 3.14$.

The function $T_0(\xi_\lambda, 2\pi, \tau_\lambda)$ calculated for different values of the parameter τ_λ is illustrated in Fig. 1. This function characterises the natural spectral width of the predicted dynamical TR peak,

$$\Delta\omega \approx \omega \frac{2\omega_g^2|\alpha_\lambda|}{g^2},$$

observed at the fixed angle. The total spectral width of the considered peak $\Delta\omega \approx \omega_b\gamma^{-1}$ determined by the dependence of emitted photon's energy on an observation angle and the total angular spread $\Delta\Theta \approx \gamma^{-1}$ following from (12) is much less than the spectral width of the minimum in ordinary TR yield $\Delta\omega \approx \omega_b$ defined by the formulae (13) and (14). Therefore the dynamical TR peak may be observed as isolated one.

4. TR in a thick crystal. The anomalous Borrmann absorption

Assuming the inequality (11) to be valid let us consider the TR yield separately with taking into account a photoabsorption in the simplest form,

$$\chi_0 = -\frac{\omega_0^2}{\omega^2} + \mathrm{i}\chi_0'', \quad \chi_g = \chi_{-g} = -\frac{\omega_g^2}{\omega^2} + \mathrm{i}\chi_g''. \tag{15}$$

In the conditions under consideration the TR amplitude is given by

$$A_\lambda^{\mathrm{TR}} \approx \frac{e}{2\pi}\frac{\Theta_\lambda}{\gamma^{-2}+\Theta^2} \times \left[\left(1+\frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right)\left(1-\mathrm{e}^{-\sigma_-''-\mathrm{i}\sigma_-'}\right) + \left(1-\frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right)\left(1-\mathrm{e}^{-\sigma_+''-\mathrm{i}\sigma_+'}\right)\right], \tag{16}$$

where the expressions for $\sigma_\pm'$ coincide with that determined in the formula (9). The very important quantities $\sigma_\pm''$ are described by the expressions

$$\sigma_\pm'' = \frac{\omega L}{2\cos\varphi}\left(\chi_0'' \pm \frac{|\alpha_\lambda|}{\sqrt{\xi_\lambda^2+1}}\chi_g''\right). \tag{17}$$

It is easy to see that in the frequency range defined by inequality $\xi_\lambda^2 \gg 1$ (far from the vicinity of dynamical diffraction) the absorption coefficients $\sigma_\pm''$ coincide with the analogous coefficient realising in the case of an amorphous medium

$$\sigma_+'' \approx \sigma_-'' = \sigma_0 = \frac{\omega L}{2\cos\varphi}\chi_0''. \tag{18}$$

But within the narrow frequency range defined by inequality $\xi_\lambda^2 < 1$ the influence of a photoabsorption depends strongly on an emitted photon polarization and may be very different for different branches of a wave equation's solution. As it follows from (17) the absorption for emitted photon corresponding to sign $(-)$ in (17) becomes very small in case $\chi_g'' \approx \chi_0''$ and $\alpha_\lambda = 1$ ($\lambda = 1$ or $\varphi \ll 1$). On the other hand the absorption is larger than usual for the photon corresponding to sign $(+)$ in (17).

To study the influence of this effect known in the X-ray diffraction theory as anomalous Borrmann absorption [11] on TR properties let us consider the TR spectral–angular distribution that is given by the expression

$$\omega\frac{\mathrm{d}N_\lambda^{TR}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta} = \frac{e^2}{\pi^2}\frac{\Theta_\lambda^2}{(\gamma^{-2}+\Theta^2)^2}T_1,$$

$$T_1 = 1 + \frac{1}{4}\left(1 + \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right)^2 e^{-2\sigma''_-} + \frac{1}{4}\left(1 - \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right)^2 e^{-2\sigma''_+} + \frac{1}{2}\frac{1}{\xi_\lambda^2+1} e^{-\sigma''_- - \sigma''_+} \cos 2\tau_\lambda \sqrt{\xi_\lambda^2+1} - \left[e^{-\sigma''_-} + e^{-\sigma''_+} + \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\left(e^{-\sigma''_-} - e^{-\sigma''_+}\right)\right] \times \cos \tau_\lambda \sqrt{\xi_\lambda^2+1} \times \cos(\tau + \tau_\lambda \xi_\lambda) - \left[e^{-\sigma''_-} - e^{-\sigma''_+} + \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\left(e^{-\sigma''_-} + e^{-\sigma''_+}\right)\right] \times \sin \tau_\lambda \sqrt{\xi_\lambda^2+1} \sin(\tau + \tau_\lambda \xi_\lambda), \quad (19)$$

following from (16). It is easy to show that more general result (19) coincides with more simple result (12) in the limit $\sigma''_\pm \to 0$.

Photoabsorption may change essentially TR properties. For example in the frequency range $\xi_\lambda^2 \gg 1$ the function T_1 is reduced to

$$T_1 \to 1 + e^{-2\sigma_0} - 2e^{-\sigma_0} \cos \tau. \quad (20)$$

The obtained expression (20) coincides with (13) in the case of thin enough crystal when $\sigma_0 \ll 1$, but in the opposite case $\sigma_0 \gg 1$ the function $T_1 = 1$ in contrast with (13). In the conditions under consideration TR wave emitted from the *in* surface of the target is absorbed completely and the total TR yield is determined by TR wave emitted from *out* surface only.

Let us consider TR yield in the conditions of anomalous Borrmann absorption $\sigma''_- \leqslant 1 \ll \sigma_0$ when only one of the two waves emitted from *in* surface of a target contributes to the emission yield behind a target. The function T_1 has the following form:

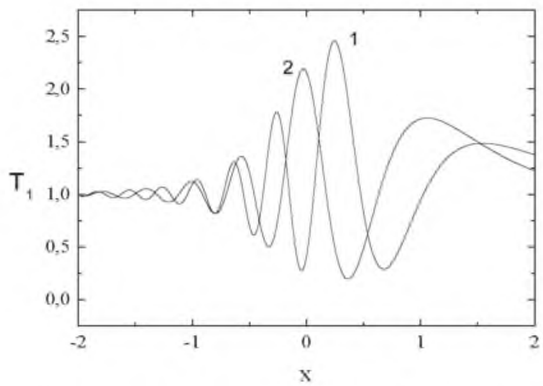

Fig. 2. Influence of the Borrmann effect on TR spectrum in the vicinity of the Bragg frequency ω_b ($x = \xi_\lambda$). The curves are calculated on conditions $(\omega L \chi''_0)/(2\cos\varphi) = 3$, $\alpha_\lambda = 1$, $\chi''_g/\chi''_0 = 1$, $\tau = 10\pi$: (1) $\tau_\lambda = 12$; (2) $\tau_\lambda = 9$.

$$T_1 = 1 + \frac{1}{4}\left(1 + \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right)^2 e^{-2\sigma''_-} - \left(1 + \frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right) \times e^{-\sigma''_-} \cos\left(\tau + \tau_\lambda\left(\xi_\lambda - \sqrt{\xi_\lambda^2+1}\right)\right) \quad (21)$$

on this condition.

In the conditions $\sigma''_0 \gg 1$ under consideration the function $\sigma''_-(\xi_\lambda)$ may be small within the narrow range $|\xi_\lambda| \ll 1$ only. As a consequence a difference between the function $T_1(\xi_\lambda)$ (21) and unity may be manifested in the same range as well. The function $T_1(\xi_\lambda)$ calculated by the formula (21) is presented in Fig. 2.

5. PXR contribution. Interference between TR and PXR

Let us return to the general formula (9) to analyse an influence of PXR contribution on a total emission properties. It is easy to see that only item in the amplitude A_λ^{PXR} containing the quantity σ_- makes an essential contribution to PXR yield as it has been mentioned above. The PXR spectral–angular distribution is given by

$$\omega \frac{\mathrm{d}N_\lambda^{\mathrm{PXR}}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta} = \frac{e^2}{\pi^2} \frac{\Theta_\lambda^2}{(\gamma^{-2} + (\omega_0^2/\omega^2) + \Theta^2)^2} P(\tau_\lambda, \xi_\lambda),$$

$$P = \frac{1}{\xi_\lambda^2 + 1} \frac{\sin^2\left[\left(\frac{1}{2}\right)\tau_\lambda\left(\xi_\lambda - \sqrt{\xi_\lambda^2 + 1} + \delta_0/\chi_\lambda\right)\right]}{\left[\xi_\lambda - \sqrt{\xi_\lambda^2 + 1} + \delta_0/\chi_\lambda\right]^2}, \tag{22}$$

where $\delta_0 = \gamma^{-2} + (\omega_0^2/\omega^2) + \Theta^2$. The function $P(\xi_\lambda)$ illustrating the "natural" PXR spectral distribution which may be observed at the fixed angle Θ is presented in Fig. 3. On the condition $\tau_\lambda \gg 1$, the more traditional expression for PXR spectral–angular distribution (see [9]) follows from (22),

$$\omega \frac{\mathrm{d}N_\lambda^{\mathrm{PXR}}}{\mathrm{d}\omega\,\mathrm{d}^2\Theta} = \frac{e^2\omega_g^4}{\pi g^2} \frac{\chi_\lambda^2}{\delta_0^2} \frac{\Theta_\lambda^2\alpha_\lambda^2}{\delta_0^2 + \chi_\lambda^2} \frac{L}{\cos\varphi} \delta(\omega - \omega_b'). \tag{23}$$

It should be noted that the obtained result (23) differs by a factor $\chi_\lambda^2/\delta_0^2$ from that for PXR emitted in the Bragg scattering direction. The value of this factor is about unity in the emitting particle energy range (11). But this factor becomes very small if $\gamma \ll \omega_b/\omega_0$. Therefore the measurement condition of the last experiment [12] devoted to PXR in the direction of emitting particle propagation where the parameter $\gamma\omega_0/\omega$ values were 3/4 and 3/5 were not optimal.

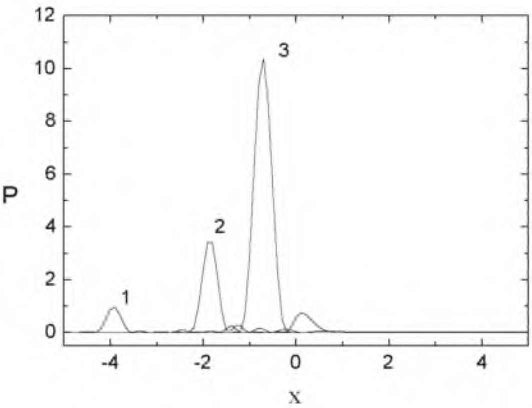

Fig. 3. PXR spectral–angular distribution ($x = \xi_\lambda$): (1) $\delta_0/\chi_\lambda = 8$, $\tau_\lambda = 8$; (2) $\delta_0/\chi_\lambda = 4$, $\tau_\lambda = 8$; (3) $\delta_0/\chi_\lambda = 2$, $\tau_\lambda = 8$.

To determine the optimal conditions for observation of PXR to forward direction one should take into account that in the conditions of high enough electron energies (11) the angular distributions of PXR and TR do not overlap. Therefore taking into account the existence of a sharp maximum in the PXR angular distribution at the observation angle $\Theta \approx \omega_0/\omega_b$ one can obtain in an experiment a high relative contribution of PXR to total emission yield by the use of a photon collimator with small angular sizes $\Delta\Theta_\perp \ll \omega_0/\omega_b$, $\Delta\Theta_\parallel \ll \omega_0/\omega_b$ located along the direction $\Theta_\perp = 0$, $\Theta_\parallel \approx \omega_0/\omega_b$. In the conditions under consideration the number of photons N^{PXR} and the spectral width $\Delta\omega$ are given by

$$N^{\mathrm{PXR}} \approx \frac{e^2\omega_g^8 L \sin\varphi\cos^3\varphi}{8\pi g^3\omega_0^4} \times \frac{(\omega_b/\omega_0)\Delta\Theta_\perp(\omega_b/\omega_0)\Delta\Theta_\parallel}{1 + \omega_g^4\cos^2\varphi/4\omega_0^4}, \tag{24a}$$

$$\Delta\omega \approx \omega_b\Delta\Theta_\parallel \cot\varphi. \tag{24b}$$

It is necessary to compare the number of photons N^{PXR} with analogous value for the ordinary bremsstrahlung measured on the same conditions,

$$N^{\mathrm{Br}} = \frac{\mathrm{d}N^{\mathrm{Br}}}{\mathrm{d}\omega}\Delta\omega \approx \frac{Ze^4\ln(mR)\omega_0^3 L}{2\pi^2 mg} \frac{\omega_b^2}{\gamma^2\omega_0^2} \frac{\omega_b}{\omega_0}\Delta\Theta_\perp\left(\frac{\omega_b}{\omega_0}\Delta\Theta_\parallel\right)^2, \tag{25}$$

where R is scrining radius in the Fermi–Thomas model of atom.

For example, the properties of an emission by electrons with the energy $\varepsilon = m\gamma = 1$ GeV crossing the crystal of Si(220) with the thickness $L = 20$ μm on conditions $\Delta\Theta_\perp \approx \Delta\Theta_\parallel \approx \frac{1}{3}(\omega_0/\omega_b)$, $\varphi = \pi/10$ are described by estimations $N^{\mathrm{PXR}} \approx 5 \times 10^{-9}$, $\Delta\omega \approx 30$ eV, $N^{\mathrm{Br}} \ll N^{\mathrm{PXR}}$. Thus on condition under consideration PXR makes the main contribution to a measured emission yield but X-ray detector with very high energy resolution $\delta\omega$ of about 10 eV is needed in an experiment because of very small spectral width of PXR.

Let us consider now an influence of PXR contribution to total emission yield in the emitting particle energy range $\gamma \sim \omega_b/\omega_0$ where an interference between PXR and TR may be essential. The main part of interference item follows from the general formula (9) in the form

$$\omega \frac{dN_\lambda^{INT}}{d\omega\, d^2\Theta} \approx -\frac{e^2}{2\pi^2}\frac{\Theta_\lambda^2}{\delta_0^2} Q(\xi_\lambda, \tau_\lambda),$$

$$Q = \frac{1}{\xi_\lambda^2+1}\frac{\sin\tau_\lambda\left(\xi_\lambda - \sqrt{\xi_\lambda^2+1}+\delta_0/\chi_\lambda\right)}{\xi_\lambda - \sqrt{\xi_\lambda^2+1}+\delta_0/\chi_\lambda} \times \sin\tau_\lambda\left(\xi_\lambda + \sqrt{\xi_\lambda^2+1}+\delta_0/\chi_\lambda\right) \times \left[\frac{\omega_0^2/\omega_b^2}{\gamma^{-2}+\Theta^2}\left(\sqrt{\xi_\lambda^2+1}-\xi_\lambda\right) + \frac{1}{\xi_\lambda + \sqrt{\xi_\lambda^2+1}+\delta_0/\chi_\lambda}\right], \quad (26)$$

where the first item in the brackets describes an interference between TR and PXR, the second one corresponds to an interference between different branches of a wave equation solution for PXR. It should be noted that the expression (26) takes into account the terms proportional to $(\omega L/2\cos\varphi)$ $(\sin(\sigma_-)/\sigma_-)$ only because these terms dominately contributes into the most experimentally interesting case $\tau_\lambda \gg 1$. The spectral–angular distribution of a total emission is described by the sum of expressions (22) and (26) and more general than (12) expression for TR distribution following from (9):

$$\omega \frac{dN_\lambda^{TR}}{d\omega\, d^2\Theta} = \frac{e^2}{\pi^2}\Theta_\lambda^2\left(\frac{1}{\gamma^{-2}+\Theta^2}-\frac{1}{\delta_0}\right)^2\left[\left(1+\frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right)^2 \times \sin^2\left(\frac{1}{2}\sigma_-\right) + \left(1-\frac{\xi_\lambda}{\sqrt{\xi_\lambda^2+1}}\right)^2 \times \sin^2\left(\frac{1}{2}\sigma_+\right) + \frac{2}{\xi_\lambda^2+1}\sin\left(\frac{1}{2}\sigma_-\right) \times \sin\left(\frac{1}{2}\sigma_+\right)\cos\frac{1}{2}(\sigma_- - \sigma_+)\right]. \quad (27)$$

Returning to the formula (26) it should be noted that an interference between TR and PXR dominates in the emitting particle energy range (11). The function $Q(\xi_\lambda)$ is presented in Fig. 4. In the conditions $\tau_\lambda \gg 1$ the interference item (26) may be presented in the form

$$\omega \frac{dN_\lambda^{INT}}{d\omega\, d^2\Theta} \approx -\frac{e^2\omega_g^4}{\pi g^2}\frac{\Theta_\lambda^2\alpha_\lambda^2}{\delta_0^2+\chi_\lambda^2}\left[\frac{\omega_0^2/\omega_b^2}{\gamma^{-2}+\Theta^2}+\frac{\chi_\lambda^2}{\delta_0^2+\chi_\lambda^2}\right] \times \frac{2}{\omega_b\delta_0}\sin\left(\frac{\omega_b L}{2\cos\varphi}\frac{\delta_0^2+\chi_\lambda^2}{\delta_0}\right) \times \delta(\omega-\omega_b'). \quad (28)$$

It is interesting to compare the expressions (23) and (28). In accordance with (23) PXR yield is proportional to the target thickness L. In contrast with this behaviour the interference item oscillates as a function of L (it is easy to show that the ar-

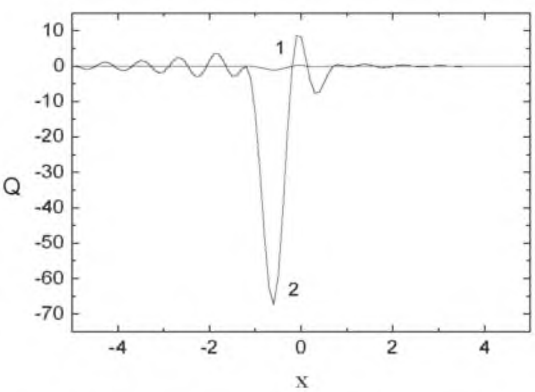

Fig. 4. The interference between PXR and transition radiation ($x = \xi_\lambda$): (1) $\tau_\lambda = 4,\ \delta_0/\chi_\lambda = 10,\ (\omega_0^2/\omega_b^2)/(\gamma^2+\Theta^2) = 0,1$; (2) $\tau_\lambda = 4,\ \delta_0/\chi_\lambda = 2,\ (\omega_0^2/\omega_b^2)/(\gamma^2+\Theta^2) = 20$.

gument of the function sin in (28) is large on condition $\tau_\lambda \gg 1$). The most essential contribution of an interference may be observed on condition (11) in the range of observation angles $\Theta \sim \gamma^{-1}$.

6. Analysis of a possible experiment

Let us consider the possibility of an experimental verification of the predicted dynamical diffraction effects. The conditions of a narrow TR peak observation in the process of a relativistic electron beam interaction with (2 2 0) plane of Si crystal are analysed in this work.

Choosing the orientational angle φ between an emitting particle velocity and (2 2 0) crystallographic plane to be equal to 24° one can obtain for the Bragg frequency ω_b the value $\omega_b \simeq 8$ keV. In the conditions under consideration a photoabsorption is small enough ($\chi_0'' \simeq 0.351 \times 10^{-6}$, $\chi_g'' \simeq 0.34 \times 10^{-6}$ in accordance with [7]) and the inequality $2\sigma_0 \ll 1$ (see (18)), which allows to neglect photoabsorption, is valid if the crystal thickness $L \ll 70$ μm.

The target thickness L must be chosen in accordance with the condition (14) permitting to suppress an ordinary TR contribution in the vicinity of the Bragg frequency ω_b. The suitable value of L is $L \simeq 20$ μm.

It should take into account the condition (11). This condition allows to separate TR contribution by the use of a photon collimator. In case the beam of KEK electron accelerator with accelerated particle's energy $\varepsilon \simeq 1$ GeV is used the ratio $\gamma\omega_0/\omega_b$ has the great enough value of about 7.5.

Three quantities (the emission spectrum $\omega dN^{TR}/d\omega$, the angular distribution $dN^{TR}/d^2\Theta$ and the number of emitted photons N^{TR}) are of interest for an experiment. The shape of an emission spectrum depends strongly on the collimator's angular size Θ_d. The formula for $\omega(dN^{TR}/d\omega)$ calculated by the use of (12) where quantities τ_λ have the values $\tau_1 = 4,19$ and $\tau_2 = 2.94$ is given by

$$\omega \frac{dN^{TR}}{d\omega} = \frac{e^2}{\pi^2} \sum_{\lambda=1}^{2} \delta_\lambda G_\lambda(y, \gamma\Theta_d),$$

$$G_1 \simeq \frac{1}{\sqrt{1+y^2}} \left[\arctan(\Phi) - \frac{\Phi}{1+\Phi^2} \right] \times \int_{\xi_1^-}^{\xi_1^+} dt\, T_0(t, 2\pi, \tau_1), \qquad (29)$$

$$G_2 \simeq \frac{|y|}{(1+y^2)^{\frac{3}{2}}} \left[\arctan(\Phi) + \frac{\Phi}{1+\Phi^2} \right] \times \int_{\xi_2^-}^{\xi_2^+} dt\, T_0(t, 2\pi, \tau_2),$$

where

$$\Phi = \frac{\gamma\Theta_d}{\sqrt{1+y^2}}, \qquad \delta_\lambda = 2\frac{\omega_g^2}{g^2}\gamma \tan(\varphi)|\alpha_\lambda| \ll 1,$$

$$\xi_\lambda^\pm = \frac{y \pm \gamma\Theta_d}{\delta_\lambda}, \qquad y = \gamma \tan\varphi \left(\frac{\omega}{\omega_b} - 1 \right), \qquad (30)$$

fast variable $T_0(\xi_\lambda)$ is defined by expression (12).

It should be noted that only fast variable $T_0(\xi_\lambda)$ has been integrated when deriving the formula (29) due to used condition $\delta_\lambda \ll 1$. The functions $G_1(y)$ and $G_2(y)$ are presented in Figs. 5 and 6.

It is easy to see that the spectral width $\Delta\omega$ of TR peak (29) is very small. The maximum $\Delta\omega$ is achieved on condition $\gamma\Theta_d \gg 1$ and has the value $\Delta\omega \approx \omega_b \gamma \tan\varphi \approx 4$ eV. Therefore X-ray detector with very high energy resolution must be used in the experiment devoted to narrow TR peak observation.

Angular distribution of a dynamical TR peak following from (12) has the usual form for TR,

$$\frac{dN_\lambda^{TR}}{d^2\Theta} = \frac{2e^2}{\pi^2} \frac{\omega_g^2}{g^2} \int_{-\infty}^{\infty} dt\, T_0(t, 2\pi, \tau_\lambda)|\alpha_\lambda| \frac{\Theta_\lambda^2}{(\gamma^{-2} + \Theta^2)^2}. \qquad (31)$$

In accordance with (31) the angular spread of an emitted photon flux has the value of about γ^{-1}.

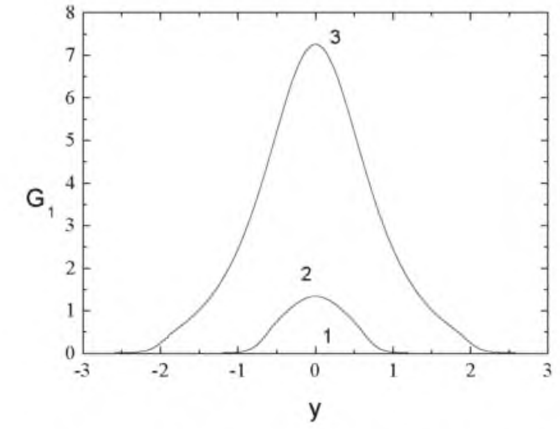

Fig. 5. Influence of the collimator site Θ_d on the dynamical TR peak spectrum for σ polarization: (1) $\delta_\lambda = 0,1$, $\tau_\lambda = 8$, $\gamma\Theta_d = 0,1$; (2) $\delta_\lambda = 0,1$, $\tau_\lambda = 8$, $\gamma\Theta_d = 0,7$; (3) $\delta_\lambda = 0,1$, $\tau_\lambda = 8$, $\gamma\Theta_d = 2$.

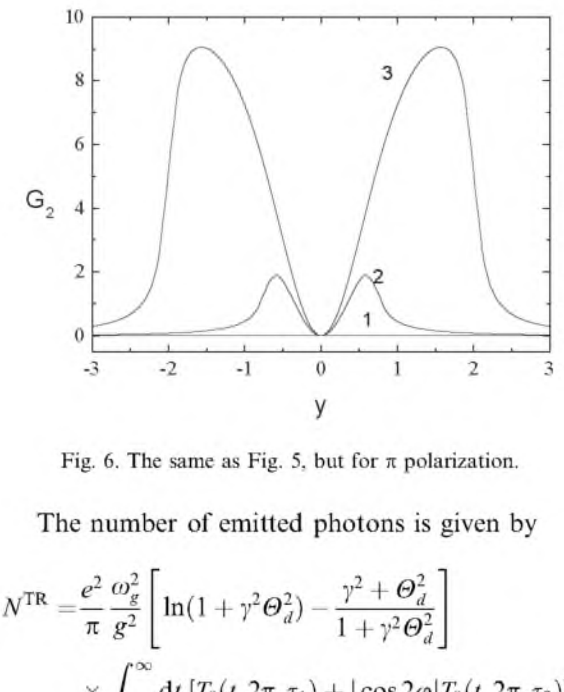

Fig. 6. The same as Fig. 5, but for π polarization.

The number of emitted photons is given by

$$N^{\mathrm{TR}} = \frac{e^2}{\pi} \frac{\omega_g^2}{g^2} \left[\ln(1+\gamma^2\Theta_d^2) - \frac{\gamma^2+\Theta_d^2}{1+\gamma^2\Theta_d^2} \right] \times \int_{-\infty}^{\infty} \mathrm{d}t\, [T_0(t, 2\pi, \tau_1) + |\cos 2\varphi| T_0(t, 2\pi, \tau_2)]. \tag{32}$$

The obtained formula (32) predicts for N^{TR} the value of the order of 10^{-6} quanta. This value must be compared with the ordinary bremsstrahlung yield

$$N^{\mathrm{Br}} = \frac{16 Z^2 e^6 n_0 L \ln(mR)}{3m^2} \frac{\omega_b^2}{\gamma^2 \omega_0^2} \frac{\delta\omega}{\omega_b}, \tag{33}$$

where $\delta\omega$ is the detector's energy resolution. On condition under consideration the number of bremsstrahlung quanta N^{Br} is about $3 \times 10^{-10} \delta\omega$ (eV). This value is much smaller than N^{TR} from (32) for the detector's energy resolution $\delta\omega \approx 200$ eV. But it should be noted that the estimation (33) takes into account the bremsstrahlung suppression due to Ter-Mikaelian effect [13], predicted for the case of a relativistic particle bremsstrahlung in an unbounded medium. The theoretical and experimental study of Ter-Mikaelian effect performed in the case of a thin target [14] has shown the absence of Ter-Mikaelian effect. Without taking into account Ter-Mikaelian effect the formula (33) predicts $N^{\mathrm{Br}} \approx 2 \times 10^{-8} \delta\omega$ (eV). Therefore the predicted dynamical TR peak can be observed on condition $\delta\omega \ll 50$ eV only.

7. Conclusion

Thus, the main results of the performed study of X-ray emission by relativistic particles crossing a crystal are as follows.

1. The dynamical diffraction of TR wave emitted by a fast particle from *in* surface of a target may change essentially an interference between this wave and that emitted from *out* surface of a crystalline target. This effect causes the manifestation of a dynamical TR peak with very narrow angular and spectral width. Characteristics of such peak have been studied in this work in detail.
2. The effect of anomalous Borrmann absorption can be observed in a relativistic particle TR from a crystal. The conditions of this predicted effect manifestation have been considered in this work.
3. The optimal conditions of a relativistic particle PXR emitted along the emitting particle velocity have been discussed as well as the conditions of a dynamical TR peak observation.

It should be noted that all the effects considered in this work may be used in the field of quasimonochromatic, tunable X-ray sources creation.

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References

[1] V. Ginzburg, I. Frank, J. Phys. (USSR) g (1945) 353.
[2] G. Garibian, C. Yang, Sov. Phys. JEPT 34 (1972) 445.
[3] V. Baryshevskii, I. Feranchuk, Phys. Lett. A 57 (1976) 183.
[4] A. Caticha, Phys. Rev. A 40 (1989) 4322.
[5] X. Artru, P. Rullhusen, Nucl. Instr. and Meth. B 145 (1998) 1.
[6] N. Nasonov, Phys. Lett. A 246 (1998) 148.
[7] Z. Pinsker, Dynamical Scattering of X-rays in Crystals, Springer, Berlin, 1984.
[8] G. Garibian, C. Yang, X-ray Transition Radiation, as of Armenian SSR, Erevan, 1983.
[9] V. Baryshevsky, Nucl. Instr. and Meth. B 122 (1997) 13.
[10] T. Amata, K. Yajima, T. Tanaka, M. Imai, A. Itoh, N. Imanishi, M. Oyamada, S. Urasawa, T. Nakasato, K. Yoshida, K. Nakayama, A. Potylitsin, Radiat. Phys. Chem. 50 (1997) 207.
[11] G. Borrmann, Z. Phys. 42 (1941) 157.
[12] I. Vnukov, B. Kalinin, G. Naumenko, D. Padalko, A. Potylitsin, in: Abstract 29, International Conference Physics of Fast Particle Interaction with Crystals, Moscow, Russia, 31 May–2 June 1999, p. 50.
[13] M. Ter-Mikaelian, High Energy Electromagnetic Processes in Condensed Media, Wiley, New York, 1972.
[14] U. Arkatov, S. Blazhevich, G. Bochek, E. Gavrilichev, A. Grinenko, V. Kulibaba, N. Maslov, N. Nasonov, V. Pirogov, Y. Virchenko, Phys. Lett. A 219 (1996) 355.