On the coherent radiation of relativistic electrons and positrons in crystal in the range of high energies of gamma-quanta

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Abstract

The formulae for the cross-section of coherent radiation of high energy gamma-quanta by relativistic electrons and positrons in crystal taking into account the contribution of the second Born approximation into radiation are obtained. The dependence of the radiation cross-section in the field of atomic plane on the sign of charge of the particle is considered.

Keywords: Quantum electrodynamics; Second Born approximation; Coherent radiation

1. Introduction

The coherent effect in radiation takes the place under motion of relativistic electrons in crystal under small angle to any crystallographic axis or plane [1–3]. Because of this effect the radiation cross-section of electrons in crystal can substantially exceed the corresponding cross-section for the particles in amorphous medium. The range of frequencies in which this effect takes place rapidly increases with the particle energy growth, and the account of recoil effect in radiation is necessary if the energies are large enough. There exist different methods of description of the radiation process by relativistic electrons with account of recoil effect. These methods are based on the Born approximation and on different variants of quasiclassical approximation of quantum electrodynamics (see, for instance [3–8] and references in them). The special interest has the consideration of the dependence of the radiation cross-section on the sign of the particle's charge. This dependence in the radiation process arises under account of fine effects connected with the highest orders of perturbation theory on the interaction of the particle with an external field. Hence the analysis of this process is rather important in the determination of the applicability range of different approximated methods used for the description of radiation by fast particles in an external field.

The dependence of the radiation cross-section on the particle's charge sign in the case of radiation of high energy electrons and positrons in the

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field of single atom is rather small [9]. Different situation arises for coherent interaction of relativistic particles with atoms of crystal lattice. In this case, due to the coherent effect the dependence of the radiation cross-section on the particle's charge sign can be substantially amplified in comparison with analogous dependence of the radiation cross-section in an amorphous medium. The attention to this fact was paid in [10] during consideration of contribution of the second Born approximation into coherent radiation cross-section of relativistic electrons in the field of atomic plane of the crystal. It was demonstrated that in considered case the relative contribution of the second Born approximation into coherent radiation cross-section is determined by the parameter

$$\alpha_{\mathrm{p}} = \frac{Ze^2R}{\varepsilon a^2\theta^2}, \tag{1}$$

where $Z|e|$ is the charge of the nucleus of crystal lattice atom, R is the screening radius of the atomic potential, a is the average distance between atoms in the crystal plane, ε is the energy of the particle and θ is the angle of the electron beam incidence to one of the atomic planes in the crystal. In this case the Born expansion of the radiation cross-section is valid if $\alpha_{\mathrm{p}} \ll 1$. The parameter α_{p} represents by the order of value the ratio of the squared critical angle of plane channeling $\theta_{\mathrm{c}} = \sqrt{Ze^2R/\varepsilon a^2}$ [11] to the squared angle of incidence θ of the beam to the plane

$$\alpha_{\mathrm{p}} \sim \frac{\theta_{\mathrm{c}}^2}{\theta^2}. \tag{2}$$

This parameter determines the dependence of the coherent radiation cross-section on the particle's charge sign. The parameter α_{p} rapidly increases with θ decrease. Under $\alpha_{\mathrm{p}} \sim 1$ the account of effects of channeling and above-barrier motion of particles in respect to the crystal atomic plane is necessary [3,11,12].

The results obtained in [10] were related to the range of small frequencies of radiated photons, for which the recoil effect under radiation is negligible. In this paper some results of investigation of the second Born approximation contribution into the cross-section of the coherent radiation of relativistic electrons and positrons in crystal with account of the recoil effect under radiation are presented. The corresponding formulae for the cross-section of radiation of electrons in non-uniform external field of arbitrary structure are obtained in the paper. Dependence of the cross-section of coherent radiation on the sign of the particle charge under interaction with the atomic plane is considered on the base of these formulae.

2. Differential cross-section of the radiation process

The cross-section of the bremsstrahlung of electrons and positrons in an external field is determined by the relation [9]

$$\mathrm{d}\sigma = \frac{e^2}{4(2\pi)^4\omega\varepsilon\varepsilon'}\delta(\varepsilon - \varepsilon' - \omega)\overline{|M|^2}\,\mathrm{d}^3p'\,\mathrm{d}^3\vec{k}, \tag{3}$$

where e is the electron charge, $\varepsilon, \vec{p}$ and $\varepsilon', \vec{p}'$ are the energy and the momentum of the initial and final particles, ω and $\vec{k}$ are the frequency and the wave vector of the radiated wave, $\delta(\varepsilon - \varepsilon' - \omega)$ is the delta-function that determines the energy conservation under radiation, and M is the matrix element of the radiation process. The line above $|M|^2$ means the averaging over polarization of initial particles and summation over polarization of final particles. According to the rules of diagram technique [9] the squared matrix element in (3) can be written with the account of the contribution of the second Born approximation in the form

$$|M|^2 = |M_1|^2U_g^2 - 2U_g\mathrm{Re}\int M_1M_2^*U_qU_{g-q}\frac{\mathrm{d}^3q}{(2\pi)^3}, \tag{4}$$

where U_g is the Fourier component of the potential energy of the electron (positron) in an external field, $g_\mu = (0, \vec{g})$ is the four-momentum transferred to the external field (it is assumed that the external field is stationary), $g_\mu = p_\mu - p'_\mu - k_\mu$, p_μ, p'_μ, k_μ are the four-momenta of initial and final electrons and the photon, M_1 and M_2 are the matrix elements which determine contributions of the first and the second Born approximations. Discriminating in the propagators in M_1 and M_2 the dependence on longitudinal and transverse components of transferred momenta $\vec{g}$ and $\vec{q}$ in an explicit form, one can write M_1 and M_2 in the form

$$M_1 = \bar{u}'\left[b\hat{e} - \frac{\hat{e}\hat{g}\gamma_0}{2\varepsilon\sigma_g} - \frac{\gamma_0\hat{g}\hat{e}}{2\varepsilon'\tau_g}\right]u, \tag{5}$$

$$M_2 = \bar{u}'\left\{\hat{e}\frac{\hat{p}+m-\hat{g}}{2p\sigma_g}\gamma_0\frac{1-\frac{\hat{q}\gamma_0}{2\varepsilon}}{v\sigma_q} - \frac{1+\frac{\gamma_0\hat{q}'}{2\varepsilon'}}{v'\tau_{q'}}\hat{e}\frac{1-\frac{\hat{q}\gamma_0}{2\varepsilon}}{v\sigma_q} + \frac{1+\frac{\gamma_0\hat{q}'}{2\varepsilon'}}{v'\tau_{q'}}\gamma_0\frac{\hat{p}'+m+\hat{g}}{2p'\tau_g}\hat{e}\right\}u, \tag{6}$$

where e_μ is the photon polarization vector, $\hat{p} = p_\mu\gamma^\mu$, γ_μ are the Dirac matrices, v and v' are the initial and final velocities of the electron, $q'_\mu = g_\mu - q_\mu$. The values b, σ_g and τ_g in M_1 and M_2 are determined by the relations

$$b = \frac{1}{\sigma_g} - \frac{1}{\tau_g}, \quad \sigma_g = g_\parallel - \frac{\vec{g}^2}{2p},$$
$$\tau_g = g_\parallel + \vec{n}_\perp\vec{g} + \frac{\vec{g}^2}{2p'}, \tag{7}$$

where $\vec{n} = \vec{p}'/p'$ is the unit vector along the momentum $\vec{p}'$ direction, and $\vec{n}_\perp$ are the components of this vector orthogonal to the $\vec{p}$.

The matrix element of the radiation process depends on the momentum transferred to the external field $\vec{g}$ in an explicit form. The cross-section itself can be also expressed directly through the transferred momentum (and also through the angle ϑ between the vectors $\vec{k}$ and $\vec{p}$). Such presentation is especially convenient in the range of small values of the transferred momentum $g_\perp \ll m$, because it is possible to make an expansion in the matrix element by the powers of $g_\perp$ in this case. Transformation to the new variables is described in [1,3]. The differential cross-section in this case takes the form

$$d\sigma = \frac{e^4}{(2\pi)^4}\frac{\varepsilon'}{\varepsilon}\overline{|M|^2}\frac{\delta}{m^2}\frac{d\omega}{\omega}\frac{dy}{\sqrt{1-y^2}}d^3g, \tag{8}$$

where $\delta = \omega m^2/2\varepsilon\varepsilon'$. The variable y is connected to ϑ by the relation

$$(\varepsilon\vartheta/m)^2 = f + y\sqrt{a}, \quad -1 \leqslant y \leqslant 1, \tag{9}$$

where

$$a = \frac{4g_\perp^2}{m^2\delta}\left(g_\parallel - \delta - \frac{g_\perp^2}{2\varepsilon}\right),$$
$$f = \frac{1}{\delta}\left(g_\parallel - \delta - \frac{g_\perp^2}{2\varepsilon} + \frac{g_\perp^2\delta}{m^2}\right),$$

$g_\parallel$ and $\vec{g}_\perp$ are the components of $\vec{g}$, parallel and orthogonal to the momentum $\vec{p}$ of the incident particle. From the fact that the value a in the radical in (9) must be positive one can conclude that

$$g_\parallel \geqslant \delta + g_\perp^2/2\varepsilon. \tag{10}$$

Note that Eq. (9) determines the possible values of the radiation angle ϑ under given values of $g_\parallel$ and $g_\perp$.

There exists the following relation for the values $\tau_{q'}$, σ_q and τ_g in (6):

$$\tau_{q'} + \sigma_q = \tau_g + \Delta_q, \tag{11}$$

where $\Delta_q = (\omega/2\varepsilon\varepsilon')\vec{q}_\perp^2 + (\vec{k}_\perp\vec{q}/\varepsilon')$. It is also easy to see that

$$\frac{\sigma_g}{\tau_g} = \left(1 - \frac{\Delta_g}{\sigma_g}\right)^{-1}, \quad \frac{\sigma_{q'}}{\tau_{q'}} = \left(1 - \frac{\Delta_{q'}}{\sigma_{q'}}\right)^{-1}. \tag{12}$$

Substituting these relations into (6), we obtain after expanding of M_2 by the parameters Δ_g/σ_g and $\Delta_{q'}/\sigma_{q'}$ the following expression for M_2:

$$M_2 = \bar{u}'\left\{\left[Q_1 - \frac{\omega}{\varepsilon\varepsilon'\tau_g}\left(\hat{e} + \frac{\gamma_0\hat{g}\hat{e}}{2\varepsilon'}\right)\right]\frac{\vec{q}_\perp\vec{q}'_\perp}{2\varepsilon\sigma_q\sigma_{q'}} + Q_2\right\}u, \tag{13}$$

where Q_1 is the spinor structure of M_1,

$$Q_1 = \hat{e}b - \frac{\hat{e}\hat{g}\gamma_0}{2\varepsilon\sigma_g} - \frac{\gamma_0\hat{g}\hat{e}}{2\varepsilon'\tau_g},$$

and

$$Q_2 = -\frac{\hat{e}\hat{g}\hat{q}_\perp}{4\varepsilon^2\sigma_g\sigma_q} - \frac{\hat{q}'_\perp\hat{g}\hat{e}}{4\varepsilon'^2\tau_g\tau_{q'}} + \frac{\gamma_0\hat{q}'_\perp\hat{e}\hat{q}_\perp\gamma_0}{4\varepsilon\varepsilon'\tau_{q'}\sigma_q}.$$

In deriving Eq. (13) we have used the symmetry of M_2 relatively to the exchange $\vec{q} \leftrightarrow \vec{q}'$ and we have neglected the terms of the order of m^2/ε^2.

After summing over polarizations of final particles and averaging over polarization of initial particle we obtain with accuracy to terms of order of m^2/ε^2 the following equations for the values $\overline{|M_1|^2}$, and $\overline{M_1M_2^*}$ in (3):

$$\overline{|M_1|^2} = \frac{2}{g_\parallel^2}\left[\left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right)g_\perp^2 - 2m^2b^2\right], \tag{14}$$

$$\overline{M_1 M_2^*} = \frac{\vec{q}_\perp \vec{q}'_\perp}{2\varepsilon\sigma_q\sigma_{q'}} \left\{ \overline{|M_1|^2} - \frac{2\omega}{\varepsilon'\tau_g} \left[2(p'\cdot p - 2m^2)b - \frac{\varepsilon}{\varepsilon'}\frac{\vec{g}^2}{\tau_g} + \vec{p}'_\perp \cdot \vec{g}\left(\frac{\varepsilon+\varepsilon'}{\varepsilon'\sigma_g} - \frac{2\varepsilon}{\varepsilon'\tau_g}\right)\right] - 8\frac{\varepsilon}{\varepsilon'}b \right\}, \tag{15}$$

where $p'\cdot p = \varepsilon'\varepsilon - \vec{p}'\vec{p}$.

Substituting these equations into (3), we obtain after the integration over y and expansion on $g_\perp/m$ the following expression for the cross-section of the radiation with account of the second Born approximation:

$$\mathrm{d}\sigma = \frac{e^2}{4\pi^3}\frac{\varepsilon'}{\varepsilon}\frac{\delta}{m^2}\frac{\mathrm{d}\omega}{\omega}\frac{g_\perp}{g_\parallel}\mathrm{d}^3 g\left\{ F|U_g|^2 + \frac{1}{(2\pi)^3\varepsilon} \times \left[F + \frac{\omega}{\varepsilon'}\left(1 - 4\frac{\delta}{g_\parallel}\left(1-\frac{\delta}{g_\parallel}\right) + \frac{\omega^2}{2\varepsilon\varepsilon'}\left(1-\frac{\delta}{g_\parallel}\right)\right)\right] U_g \mathrm{Re} \times \int \mathrm{d}^3 q \frac{(\vec{g}_\perp - \vec{q}_\perp)\vec{q}_\perp}{(g_\parallel - q_\parallel + \mathrm{i}0)(q_\parallel + \mathrm{i}0)} U_q U_{g-q} \right\}, \tag{16}$$

where

$$F = 1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{g_\parallel}\left(1 - \frac{\delta}{g_\parallel}\right).$$

Let us consider some particular cases of the Eq. (16). If the condition $\delta \ll q_{\parallel_{\mathrm{eff}}}$ is satisfied, where $q_{\parallel_{\mathrm{eff}}}$ are the characteristic values of the longitudinal component of the momentum $\vec{q}$ in (16), we can neglect the dependence of U_g and U_{g-q} on $g_\parallel$ in (16). After integration over $g_\parallel$ we obtain that

$$\mathrm{d}\sigma(g_\perp) = \mathrm{d}w(g_\perp)\left(1 + \frac{3}{4}\frac{\omega^2}{\varepsilon\varepsilon'}\right)\frac{\mathrm{d}^2 g_\perp}{(2\pi)^2}\left\{ |U_g|^2 - \frac{\varepsilon+\varepsilon'}{2\varepsilon\varepsilon'} U_{g_\perp}\mathrm{Re}\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\frac{\vec{q}'_\perp \vec{q}_\perp}{(q_\parallel - \mathrm{i}0)^2} U_g U_{q'} \right\} \times \left(1 + \mathrm{O}\left(\frac{g_{\parallel_{\mathrm{eff}}}}{q_{\parallel_{\mathrm{eff}}}}\right)\right), \tag{17}$$

where

$$\mathrm{d}w(g_\perp) = \frac{2e^2}{3\pi}\frac{\varepsilon'}{\varepsilon}\frac{g_\perp^2}{m^2}\frac{\mathrm{d}\omega}{\omega}. \tag{18}$$

For $\omega \ll \varepsilon$ Eq. (17) corresponds to the product of the radiation probability $\mathrm{d}w/\mathrm{d}\omega$ and the cross-section of elastic scattering of the particle in the external field $\mathrm{d}\sigma_{\mathrm{el}}$ with account of contribution of the second Born approximation,

$$\mathrm{d}\sigma_{\mathrm{el}}(g_\perp) = \frac{\mathrm{d}^2 g_\perp}{4\pi^2}\left\{ |U_g|^2 - \frac{1}{\varepsilon}U_g \mathrm{Re}\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\frac{\vec{q}_\perp \vec{q}'_\perp}{(q_\parallel - \mathrm{i}0)^2} U_q U_{q'} \right\}.$$

For the Coulomb field of the nucleus with charge $Z|e|$ last equation transforms to the form

$$\mathrm{d}\sigma_{\mathrm{el}}(g_\perp) = \frac{4Z^2 e^4\,\mathrm{d}\Omega}{\varepsilon^2\vartheta^4}\left\{1 - \frac{e}{|e|}\frac{\pi Z e^2}{2}\vartheta\right\},$$

where the scattering angle $\vartheta \approx g_\perp/p$. The last result coincides with the corresponding result of the paper [13] obtained by different method. For arbitrary external field the formula for $\mathrm{d}\sigma_{\mathrm{el}}$ was obtained in [14,15].

So in the range of frequencies $\omega \sim \varepsilon$ the theorem about factorization of the radiation cross-section, according to which

$$\mathrm{d}\sigma \approx \mathrm{d}w(g_\perp)\,\mathrm{d}\sigma_{\mathrm{el}}(g_\perp), \tag{19}$$

is justified with an accuracy to the correction which determines the contribution of the second Born approximation.

3. The cross-section for radiation of relativistic electrons and positrons in the field of atomic plane in a crystal

Now consider the coherent radiation of electrons and positrons in the field of continuous potential of one of the atomic planes in a crystal under incidence of the beam under small angle θ to this plane. The continuous potential of the plane is determined by the equation [11,12]

$$U(x) = \frac{1}{L_y L_z}\int \mathrm{d}y\,\mathrm{d}z \sum_{n=1}^{N} u(\vec{r} - \vec{r}_n), \tag{20}$$

where $u(\vec{r} - \vec{r}_n)$ is the potential of the single atom of the crystal plane located in the point $\vec{r}_n$, L_y and L_z are the linear dimensions of the plane and x is the coordinate, orthogonal to the atomic plane of the crystal (summation in (20) is made over all atoms of the crystal plane). Taking the atomic potential in the form of the screened Coulomb potential

$$u(r) = \frac{Z|e|}{r} e^{-r/R},$$

we find the expression for the Fourier component of the continuous potential of the plane,

$$U_g = (2\pi)^2 \delta(g_z)\delta(g_y) \frac{1}{a_y a_z} u_g, \tag{21}$$

where a_y and a_z are the distances between atoms in the plane along the axes y and z, and

$$u_g = \frac{4\pi Z|e|}{\vec{g}^2 + R^{-2}}.$$

Substituting the Fourier component (21) into (16), we obtain the following expression for the radiation cross-section:

$$\begin{aligned} \mathrm{d}\sigma = {} & Z^2 e^6 16\pi \frac{N}{a_y a_z} \frac{\varepsilon'}{\varepsilon} \frac{\delta}{m^2} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}g_x}{\theta^2} \\ & \times \left\{ \left[1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{g_x\theta}\left(1 - \frac{\delta}{g_x\theta}\right)\right] \frac{1}{(g_x^2 + R^{-2})^2} \right. \\ & + \frac{e}{|e|} \frac{2Ze^2}{\varepsilon a_y a_z} \frac{1}{g_x^2 + R^{-2}} \left[1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{g_x\theta}\left(1 - \frac{\delta}{g_x\theta}\right) \right. \\ & + \frac{\omega}{\varepsilon'}\left(1 - 4\frac{\delta}{g_x\theta}\left(1 - \frac{\delta}{g_x\theta}\right) \right. \\ & \left.\left.\left. + \frac{\omega^2}{2\varepsilon\varepsilon'}\left(1 - \frac{\delta}{g_x\theta}\right)\right)\right] \frac{1}{\theta^2} \frac{2\pi R}{g_x^2 + 4R^{-2}} \right\}. \end{aligned} \tag{22}$$

Here we have used the fact that in the case under consideration $g_\parallel \approx \theta g_x$. The value g_x here covers the range $g_x \geqslant \delta/\theta$. Under $\omega \ll \varepsilon$ the Eq. (22) transforms to the corresponding result of the paper [10].

Eq. (22) demonstrates that for given value of the angle θ the radiation spectrum $\omega\,\mathrm{d}\sigma/\mathrm{d}\omega$ possesses the maximum in the range of frequencies satisfying the condition

$$\frac{2\varepsilon(\varepsilon - \omega)}{m^2\omega} \sim \frac{2R}{\theta}. \tag{23}$$

With the particle energy growth the position of this maximum moves to the region of high frequencies. For $\varepsilon \sim m^2 R/\theta$ the maximum is located in the region of frequencies for which the effect of recoil under radiation is substantial.

According to (22) the correction leading to the dependence of the cross-section of the coherent radiation on the sign of the charge of the particle is determined by the parameter (1) for $\omega \sim \varepsilon$ as well as in the case $\omega \ll \varepsilon$. For $\varepsilon \sim m^2 R/\theta$ this parameter takes the form

$$\alpha_\mathrm{p} \sim \frac{Ze^2}{m^2 a^2 \theta}. \tag{24}$$

So in the range of energies under consideration with the decrease of θ the dependence of the cross-section of the coherent radiation on the charge sign of the particle becomes substantial in the whole range of frequencies of radiated photons.

Eq. (22) demonstrates also that for all frequencies the cross-section of radiation by positrons turns out to be larger than the cross-section of radiation by electrons. This result can be explained by the following way. The electron is attracted to the plane and, in distinct to the positron, spend less time in the region with large gradient of the potential. Because of that the electron radiates weaker than the positron, and this difference increases with the decrease of the angle θ.

4. Conclusions

The results obtained demonstrate that the radiation cross-section for relativistic electrons and positrons in an external field with account of the second Born approximation is determined by Eq. (16) in whole region of frequencies of radiated photons. The dependence of the contribution of the second Born approximation into the radiation cross-section on the energy of the emitted photon for the case $\omega \sim \varepsilon$ differs from the corresponding dependence of the contribution of the first Born approximation into the cross-section. This leads, in particular, to the fact that the theorem about factorization of the radiation cross-section is valid for $\omega \sim \varepsilon$ only with accuracy to the contribution of the second Born approximation into the cross-section.

The account of the contribution of the second Born approximation into radiation cross-section leads to dependence of the cross-section on the particle charge sign in whole region of frequencies of radiated photons. For the radiation by relativistic particle in the field of single atom this dependence is rather small. Substantial increase of the dependence of radiation cross-section on the particle charge sign is possible for the radiation of the particle in crystal. Such increase takes place, for example, under incidence of the beam under small angle θ to one of crystallographic planes. The dependence of the cross-section on the charge sign in whole region of frequencies of emitted photons is determined by the parameter (2). Under decreasing the angle θ this parameter rapidly increases. Under $\theta \sim \theta_c$ the account of effects connected with the phenomenon of channeling is necessary. This phenomenon could not be described in the frameworks of Born approximation. Hence Eq. (22) is valid for $\theta_c \ll \theta \ll 1$.

Experimental studies of the radiation process by relativistic electrons and positrons in crystals were carried out in [16–20]. However, in these papers the attention was paid mainly to the study of radiation characteristics under conditions, when the channeling of particles in crystal takes place. It was demonstrated that in this case the radiation cross-sections for electrons and positrons are substantially differ from each other. Detailed experimental investigation of the dependence of radiation cross-section on the particle charge sign under conditions of applicability of Eq. (22) has not been carried out yet. There exist only some measurements of orientation dependence of radiation cross-sections for electrons and positrons under collimation of radiation [16,17]. The results of those measurements are in qualitatively agreement with basic predictions of the Eq. (22).

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