Diffracted transition radiation of a relativistic electron in the artificial periodic multilayer medium

S Blazhevich[1], I Kolosova[2], A Noskov[2]
[1] National Research University Belgorod State University, Belgorod, Russia
[2] Belgorod University of Cooperation, Economics and Low, Belgorod, Russia

E-mail: blazh@bsu.edu.ru

Abstract. A theory of coherent X-ray radiation of a relativistic electron crossing the artificial periodic medium in the Laue scattering geometry is constructed. The expressions describing the spectral and angular characteristics of radiation in the direction of Bragg scattering are obtained and investigated. By analogy with the radiation emission in a crystalline medium this radiation is considered as the result of coherent summation of the contributions of two radiation mechanisms: parametric (PXR) and diffracted transition (DTR). It is shown that the yield of DTR from layered target can be more than one order higher than the yield in single crystal radiator, under similar conditions. The manifestations of the Borrmann effect for DTR in the artificial multilayer environment are demonstrated for a Laue scattering geometry.

1. Introduction

When a charged particle crosses the entrance surface of the crystal plate the transition radiation arises (TR) [1], which then is diffracted by a system of parallel atomic planes of the crystal, forming the diffracted transition radiation DTR [2–4]. At the same time a charged particle Coulomb field is scattered by a system of parallel atomic planes of the crystal, creating a parametric X-ray radiation (PXR) [5–7]. In the scheme of the symmetric reflection when the system of diffracting atomic planes is perpendicular (in the case of Laue scattering geometry) or parallel (in the case of Bragg scattering) to the surface of the crystal plate, the radiation mechanisms in the two-wave approximation of dynamic diffraction theory were considered in [8–11].

In the general case of asymmetric reflection of the radiation from the plate when the diffracted atomic planes make an arbitrary angle with the surface of the plate, the dynamic effects of PXR and DTR are considered in [12–15], where it was shown that by changing the asymmetry of reflection, we can significantly increase the radiation yield. Traditionally, the radiation of a relativistic particle in a periodically layered structure was considered in the Bragg scattering geometry for the case where the reflecting layers are parallel to the entrance surface, i.e. for the case of symmetric reflection. The radiation in a periodic layered structure is usually viewed as resonant transition radiation [5, 16]. In the works [17], the radiation from an artificial periodic structure was represented as the sum of diffracted transition radiation (DTR) and parametric X-ray radiation (PXR).

In the cited works the radiation of relativistic particles in an artificial periodic structure was considered only in the Bragg scattering geometry for the special case of symmetric reflection of the particle field with respect to the target surface, when the diffracted layers are parallel to the target surface.

In the present paper we consider the coherent X-ray radiation scattering in the Bragg direction generated by relativistic electron crossing the artificial periodic structure in the Laue scattering geometry. By analogy with the crystalline environment the coherent radiation is considered as the sum of PXR and DTR contributions. On the basis of two-wave approximation of dynamic diffraction theory [18] the expressions describing the spectral and angular characteristics of radiation are derived.

2. Amplitude of the radiation

We analyze the radiation emitted by a relativistic electron passing through a multilayer structure (Figure 1) consisting of periodically arranged amorphous layer with thickness a and b respectively ($T = a + b$ is the structure period) with the dielectric susceptibility χ_a and χ_b respectively.

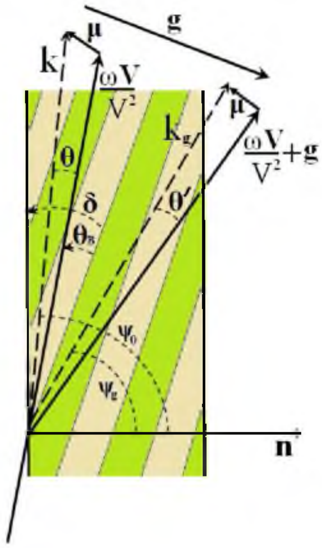

Figure 1. Geometry of the radiation process and the system of the using parameters notations, θ and θ' are the radiation angles, θ_B is Bragg angle, $\mathbf{k}$ and $\mathbf{k}_g$ are wave vectors of incident and diffracted photons.

We consider the equation for the Fourier transform of the electromagnetic field

$$\mathbf{E}(\mathbf{k},\omega) = \int dt\, d^3\mathbf{r}\, \mathbf{E}(\mathbf{r},t)\exp(i\omega t - i\mathbf{kr}). \tag{1}$$

We use the two-wave approximation of dynamic diffraction theory, in which the incident and diffracted wave are considered on equal grounds. Since the electromagnetic field associated with a relativistic particle can accurately be considered as transverse both incident $\mathbf{E}_0(\mathbf{k},\omega)$ and diffracted $\mathbf{E}_\mathbf{g}(\mathbf{k},\omega)$ electromagnetic waves are determined by pairs of transverse polarization amplitudes:

$$\begin{aligned} \mathbf{E}_0(\mathbf{k},\omega) &= E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)}, \\ \mathbf{E}_\mathbf{g}(\mathbf{k},\omega) &= E_\mathbf{g}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)} + E_\mathbf{g}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)}, \end{aligned} \tag{2}$$

where the vector $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to the vector $\mathbf{k}$ and the vectors $\mathbf{e}_1^{(1)}$ and $\mathbf{e}_1^{(2)}$ are perpendicular to the vector $\mathbf{k}_\mathbf{g} = \mathbf{k} + \mathbf{g}$. Vectors $\mathbf{e}_0^{(2)}$ and $\mathbf{e}_1^{(2)}$ lie in the plane of the vectors $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ (π-polarization), and vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ are perpendicular to this plane (σ-polarization). The vector $\mathbf{g}$ is defined similarly to the reciprocal lattice vector in the crystal - it is perpendicular to the layers of protection, and its length is equal to $g = \frac{2\pi}{T}n$, $n = 0, \pm 1, \pm 2, \ldots$

The equations for the Fourier transform of the electromagnetic field in a two-wave approximation of dynamical diffraction theory have the form [19]:

$$\begin{cases}(\omega^2(1+\chi_0)-k^2)E_0^{(s)}+\omega^2\chi_{-\mathbf{g}}C^{(s)}E_{\mathbf{g}}^{(s)}=8\pi^2 ie\omega\theta VP^{(s)}\delta(\omega-\mathbf{kV}),\\ \omega^2\chi_{\mathbf{g}}C^{(s)}E_0^{(s)}+(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)}=0,\end{cases} \tag{3}$$

where $\chi_{\mathbf{g}}$, $\chi_{-\mathbf{g}}$ are coefficients of the Fourier expansion of the periodic structure dielectric susceptibility over the reciprocal vectors $\mathbf{g}$:

$$\chi(\omega,\mathbf{r})=\sum_{\mathbf{g}}\chi_{\mathbf{g}}(\omega)\exp(i\mathbf{gr})=\sum_{\mathbf{g}}\left(\chi'_{\mathbf{g}}(\omega)+i\chi''_{\mathbf{g}}(\omega)\right)\exp(i\mathbf{gr}). \tag{4}$$

The values $C^{(s)}$ and $P^{(s)}$ in the system (3) are defined as follows

$$C^{(s)}=\mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)},\ C^{(1)}=1,\ C^{(2)}=\cos 2\theta_B,\qquad P^{(s)}=\mathbf{e}_0^{(s)}(\boldsymbol{\mu}/\mu),\ P^{(1)}=\sin\varphi,\ P^{(2)}=\cos\varphi, \tag{5}$$

where $\boldsymbol{\mu}=\mathbf{k}-\omega\mathbf{V}/V^2$ is the component of the virtual photon momentum perpendicular to the particle velocity $\mathbf{V}$, $\mu=\omega\theta/V$, $\theta<<1$ is the angle between vectors $\mathbf{k}$ and $\mathbf{V}$, θ_B is Bragg angle, φ is the radiation azimuth angle measured from the plane formed by the velocity vector $\mathbf{V}$ and $\mathbf{g}$.

The vector $\mathbf{g}$ length can be also expressed through the Bragg angle θ_B and the Bragg frequency ω_B: $g=2\omega_B\sin\theta_B/V$. The angle between the vector $\frac{\omega\mathbf{V}}{V^2}$ and the wave vector $\mathbf{k}$ of the incident wave is marked θ and the angle between the vector $\frac{\omega\mathbf{V}}{V^2}+\mathbf{g}$ and the diffracted wave vector $\mathbf{k}_{\mathbf{g}}$ is indicated as θ'. The system (3) under $s=1$ describes the fields of σ - polarization, and under $s=2$ the fields of π -polarization.

The values χ_0 and $\chi_{\mathbf{g}}$ are defined as follows:

$$\chi_0(\omega)=\frac{a}{T}\chi_a+\frac{b}{T}\chi_b,\ \chi_{\mathbf{g}}(\omega)=\frac{\exp(-iga)-1}{igT}\left(\chi_b-\chi_a\right),\ \chi'_0=\frac{a}{T}\chi'_a+\frac{b}{T}\chi'_b,\qquad \chi''_0=\frac{a}{T}\chi''_a+\frac{b}{T}\chi''_b,$$

$$\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}=\frac{2\sin\left(\frac{ga}{2}\right)}{gT}\left(\chi'_b-\chi'_a\right),\quad \mathrm{Im}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}=\frac{2\sin\left(\frac{ga}{2}\right)}{gT}\left(\chi''_b-\chi''_a\right) \tag{6}$$

By solving the dispersion equation following from the system (3)

$$(\omega^2(1+\chi_0)-k^2)(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)-\omega^4\chi_{-\mathbf{g}}\chi_{\mathbf{g}}C^{(s)^2}=0 \tag{7}$$

with the use of standard methods of dynamical theory[18], we find the expression for k and $k_{\mathbf{g}}$:

$$k=\omega\sqrt{1+\chi_0}+\lambda_0,\qquad k_{\mathbf{g}}=\omega\sqrt{1+\chi_0}+\lambda_{\mathbf{g}}. \tag{8}$$

$$\lambda_{\mathbf{g}}^{(1,2)}=\frac{\omega}{4}\left(\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right),\ \lambda_0^{(1,2)}=\omega\frac{\gamma_0}{4\gamma_{\mathbf{g}}}\left(-\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right), \tag{9}$$

where $\beta=\alpha-\chi_0\left(1-\frac{\gamma_{\mathbf{g}}}{\gamma_0}\right)$, $\alpha=\frac{1}{\omega^2}(k_{\mathbf{g}}^2-k^2)$, $\gamma_0=\cos\psi_0$, $\gamma_{\mathbf{g}}=\cos\psi_{\mathbf{g}}$, ψ_0 – the angle between the wave vector of the incident wave $\mathbf{k}$ and vector normal to the surface of the plate $\mathbf{n}$, $\psi_{\mathbf{g}}$ is the angle between the wave vector $\mathbf{k}_{\mathbf{g}}$ and the vector $\mathbf{n}$ (see figure 1). Dynamic additions λ_0 and $\lambda_{\mathbf{g}}$ for X-ray wave vectors are related by formula

$$\lambda_{\mathbf{g}}=\frac{\omega\beta}{2}+\lambda_0\frac{\gamma_{\mathbf{g}}}{\gamma_0} \tag{10}$$

Since the dynamic additions are small: $|\lambda_0|<<\omega$, $|\lambda_{\mathbf{g}}|<<\omega$, one can show that $\theta\approx\theta'$ (see figure 1), therefore further we will use the notation θ for both of these angles.

We represent the solution of the system of equations (3) for the diffracted field in a periodic structure in such a form:

$$E_{\mathbf{g}}^{(s)medium} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)}\right)\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}\right)}\delta\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{*}\right)+E_{\mathbf{g}}^{(s)^{(1)}}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)})+E_{\mathbf{g}}^{(s)^{(2)}}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}), \quad (11a)$$

where $\lambda_0^* = \omega\left(\frac{\gamma^{-2}+\theta^2-\chi_0}{2}\right)$, $\lambda_{\mathbf{g}}^* = \frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*$, $\gamma = 1/\sqrt{1-V^2}$ – Lorentz factor of the particle, $E_{\mathbf{g}}^{(s)^{(1)}}$ and $E_{\mathbf{g}}^{(s)^{(2)}}$ are free diffracted fields in the multilayer target.

For the field in vacuum in front of the radiator the solution of (3) has the form:

$$E_0^{(s)vacI} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{-\chi_0-\frac{2}{\omega}\lambda_0}\delta\left(\lambda_0-\lambda_0^*\right) = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*\right), \quad (11b)$$

where we use the relation $\delta\left(\lambda_0-\lambda_0^*\right) = \frac{\gamma_{\mathbf{g}}}{\gamma_0}\delta\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^*\right)$.

The diffracted field behind the radiator in vacuum is as follows:

$$E_{\mathbf{g}}^{(s)vac} = E_{\mathbf{g}}^{(s)Rad}\delta\left(\lambda_{\mathbf{g}}+\frac{\omega\chi_0}{2}\right), \quad (12)$$

where $E_{\mathbf{g}}^{(s)Rad}$ is the field of coherent radiation in the direction close to the Bragg direction. From the second equation of the system (3) we can derive the expression relating the incident and diffracted fields in the medium:

$$E_0^{(s)medium} = \frac{2\omega\lambda_{\mathbf{g}}}{\omega^2\chi_g C^{(s)}}E_{\mathbf{g}}^{(s)medium}. \quad (13)$$

To determine the amplitude of the field $E_{\mathbf{g}}^{(s)Rad}$, we use the boundary conditions at the entrance and exit surfaces of the multilayer plate:

$$\int E_0^{(s)vacI}d\lambda_0 = \int E_0^{(s)medium}d\lambda_0, \quad \int E_{\mathbf{g}}^{(s)medium}d\lambda_0 = 0,$$
$$\int E_{\mathbf{g}}^{(s)medium}\exp\left(i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L\right)d\lambda_{\mathbf{g}} = \int E_{\mathbf{g}}^{(s)vac}\exp\left(i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L\right)d\lambda_{\mathbf{g}}. \quad (14)$$

We will present the radiation field in the form of two terms:

$$E_{\mathbf{g}}^{(s)Rad} = E_{PXR}^{(s)} + E_{DTR}^{(s)}, \quad (15a)$$

$$E_{PXR}^{(s)} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{8\frac{\gamma_0}{\gamma_{\mathbf{g}}}\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\cdot\frac{1}{\lambda_0^*}\times\left[\left(\beta+\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right)\left(\frac{1-\exp\left(-i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)}{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}}\right)\right.$$
$$\left.-\left(\beta-\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right)\left(\frac{1-\exp\left(-i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)}{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}}\right)\right]\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{L}{\gamma_g}\right], \quad (15b)$$

$$E_{DTR}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\chi_{\mathbf{g}} C^{(s)}}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}} \left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2\lambda_0^*}\right)$$

$$\times \left[\exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right) - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right]\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right]. \qquad (15c)$$

The expression (15b) and (15c) represent the amplitudes of the radiation fields, similar to the amplitudes of PXR and DTR in a crystal. The DTR is the result of diffraction by a periodically layered artificial structure of the transition radiation, which is generated on the front surface of the target.
For further analysis of the radiation, the dynamic addition (9) can be represented as follows:

$$\lambda_g^{(1,2)} = \frac{\omega|\chi'_{\mathbf{g}}|C^{(s)}}{2}\left(\xi^{(s)} - \frac{i\rho^{(s)}(1-\varepsilon)}{2} \pm \sqrt{\xi^{(s)2} + \varepsilon - 2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)} + \kappa^{(s)}\varepsilon\right) - \rho^{(s)^2}\left(\frac{(1-\varepsilon)^2}{4} + \kappa^{(s)^2}\varepsilon\right)}\right), \qquad (16)$$

where

$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1-\varepsilon}{2\nu^{(s)}},\quad \eta^{(s)}(\omega) = \frac{\alpha}{2\left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right|C^{(s)}} \equiv \frac{\sin^2\theta_B}{V^2 C^{(s)}} \frac{gT}{|\chi'_b - \chi'_a|\left|\sin\left(\frac{ga}{2}\right)\right|}\left(1 - \frac{\omega(1-\theta\cos\varphi\cot\theta_B)}{\omega_B}\right),$$

$$\nu^{(s)} = \frac{C^{(s)}\,\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}}{\chi'_0} \equiv \frac{2C^{(s)}\left|\sin\left(\frac{ga}{2}\right)\right|}{g}\left|\frac{\chi'_b - \chi'_a}{a\chi'_a + b\chi'_b}\right|,\quad \rho^{(s)} = \frac{\chi''_0}{\left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right|C^{(s)}} \equiv \frac{a\chi''_a + b\chi''_b}{|\chi'_b - \chi'_a|C^{(s)}} \frac{g}{2\left|\sin\left(\frac{ga}{2}\right)\right|},$$

$$\kappa^{(s)} = \frac{\chi''_{\mathbf{g}}C^{(s)}}{\chi''_0} \equiv \frac{2C^{(s)}\left|\sin\left(\frac{ga}{2}\right)\right|}{g}\left|\frac{\chi''_b - \chi''_a}{a\chi''_a + b\chi''_b}\right|,\quad \varepsilon = \frac{\gamma_{\mathbf{g}}}{\gamma_0}. \qquad (17)$$

An important parameter in (17) is the parameter ε that determines the degree of the field reflection asymmetry relative to the target surface, which can be represented as

$$\varepsilon = \frac{\sin(\delta + \theta_B)}{\sin(\delta - \theta_B)}, \qquad (18)$$

where θ_B is the angle between the electron velocity and reflective layers, δ – the angle between the target surface and reflective layers. Note that the angle of electron incidence on the target surface increases when the parameter ε decreases, and vice versa (see figure 2). But in the approach used in this work the calculations will be correct only when the condition $\lambda_g^{(1,2)}(\varepsilon) << \omega\sqrt{1+\chi_0}$ (see (8)) is provided, i.e. the parameter ε must be limited in accordance with this condition and with formula (16).

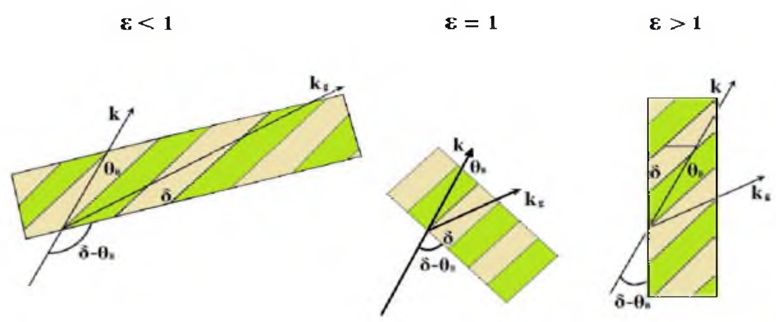

Figure 2. Schema of the asymmetric ($\varepsilon > 1$ and $\varepsilon < 1$ cases) reflection of the radiation from the multilayered plate. The case ($\varepsilon = 1$) corresponds to the symmetric reflection.

3. Spectral-angular radiation density

Substituting (16) for $\lambda_g^{(1,2)}$ into (15b) and (15c), then substituting (15b) for $E_{PXR}^{(s)}$ and (15c) for $E_{DTR}^{(s)}$ in the well-known [19] expression for the spectral-angular density of X-rays

$$\omega \frac{d^2 N}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E_{\mathbf{g}}^{(s)Rad} \right|^2 , \tag{19}$$

we will obtain the expression for summands, describing the contributions to the spectral-angular density of the radiation of the mechanisms PXR, DTR and of the summand, which is the result of the interference of these radiation mechanisms.

$$\omega \frac{d^2 N_{\mathrm{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} P^{(s)^2} \frac{\theta^2}{(\theta^2 + \gamma^{-2} - \chi_0')^2} R_{\mathrm{PXR}}^{(s)} , \tag{20a}$$

$$R_{\mathrm{PXR}}^{(s)} = \left(1 - \frac{\xi}{\sqrt{\xi^2 + \varepsilon}}\right)^2 \frac{1 + \exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(1)}\right) - 2\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(1)}\right)\cos\left(b^{(s)}\left(\sigma^{(s)} + \frac{\xi - \sqrt{\xi^2 + \varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)} + \frac{\xi - \sqrt{\xi^2 + \varepsilon}}{\varepsilon}\right)^2 + \rho^{(s)^2}\Delta^{(1)^2}} . \tag{20б}$$

$$\omega \frac{d^2 N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} P^{(s)^2} \theta^2 \left(\frac{1}{\theta^2 + \gamma^{-2}} - \frac{1}{\theta^2 + \gamma^{-2} - \chi_0'}\right)^2 R_{DTR}^{(s)} , \tag{21a}$$

$$R_{DTR}^{(s)} = \frac{4\varepsilon^2}{\xi^2 + \varepsilon} \exp\left(-b^{(s)}\rho^{(s)} \frac{1+\varepsilon}{\varepsilon}\right)\left[\sin^2\left(b^{(s)} \frac{\left(\sqrt{\xi^2 + \varepsilon}\right)}{\varepsilon}\right) + sh^2\left(b^{(s)}\rho^{(s)} \frac{(1-\varepsilon)\xi^{(s)} + 2\varepsilon\kappa^{(s)}}{2\varepsilon\sqrt{\xi^2 + \varepsilon}}\right)\right], \tag{21б}$$

where

$$\Delta^{(1)} = \frac{\varepsilon + 1}{2\varepsilon} - \frac{1 - \varepsilon}{2\varepsilon} \frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2} + \varepsilon}} - \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2} + \varepsilon}} , \quad \sigma^{(s)} = \frac{1}{|\chi_{\mathbf{g}}'| C^{(s)}} \left(\theta^2 + \gamma^{-2} - \chi_0'\right) \equiv \frac{1}{\nu^{(s)}} \left(\frac{\theta^2}{|\chi_0'|} + \frac{1}{\gamma^2 |\chi_0'|} + 1\right),$$

$$b^{(s)} = \frac{\omega \left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right| C^{(s)}}{2} \frac{L}{\gamma_0} . \tag{22}$$

The expressions (20)–(21) constitute the main result of this work. They are obtained in two-wave approximation of dynamic diffraction theory, taking into account the absorption of radiation in the layered plate substances and the orientation of the diffracting layers relative to the surface of the plate. These expressions allow us to investigate the spectral and angular characteristics of radiation depending on the energy of relativistic electrons and on the parameters of the artificial periodic structure of the target

4. Bormann effect in DTR

Since two X-ray waves determine the DTR yield, for the analysis of their contributions to the radiation spectral density it is convenient to represent the expression (21b) in such a form:

$$R_{DTR}^{(s)} = \frac{\varepsilon^2}{\xi(\omega)^2 + \varepsilon} \left[e^{-b^{(s)}\rho^{(s)}\left[\frac{1+\varepsilon}{\varepsilon} - \frac{(1-\varepsilon)\xi^{(s)} + 2\varepsilon\kappa^{(s)}}{\varepsilon\sqrt{\xi^2+\varepsilon}}\right]} + e^{-b^{(s)}\rho^{(s)}\left[\frac{1+\varepsilon}{\varepsilon} + \frac{(1-\varepsilon)\xi^{(s)} + 2\varepsilon\kappa^{(s)}}{\varepsilon\sqrt{\xi^2+\varepsilon}}\right]} - 2e^{-b^{(s)}\rho^{(s)}\frac{1+\varepsilon}{\varepsilon}} \cdot cos\left(\frac{2b^{(s)}\sqrt{\xi^2 + \varepsilon}}{\varepsilon}\right)\right] \tag{23}$$

When consider the expression (23), one can see that the terms in brackets successively describe the waves belonging to the first and second fields, and their interference. Next we write the expression (23) in a more demonstrable form of

$$R_{DTR}^{(s)} = \frac{\varepsilon^2}{\xi(\omega)^2+\varepsilon}\left[e^{-L_f\mu_1^{(s)}} + e^{-L_f\mu_2^{(s)}} - 2\cdot e^{-L_f\mu_0\left(\frac{1+\varepsilon}{2}\right)}\cdot cos\left(\frac{L_f}{L_{ext}^{(s)}}\sqrt{\xi^2+\varepsilon}\right)\right], \quad (24)$$

where

$$\mu_1^{(s)} = \mu_0\left[\frac{1+\varepsilon}{2} - \frac{(1-\varepsilon)\xi^{(s)}+2\varepsilon\kappa^{(s)}}{2\sqrt{\xi^2+\varepsilon}}\right], \qquad \mu_2^{(s)} = \mu_0\left[\frac{1+\varepsilon}{2} + \frac{(1-\varepsilon)\xi^{(s)}+2\varepsilon\kappa^{(s)}}{2\sqrt{\xi^2+\varepsilon}}\right], \quad (25)$$

L_f is the path of a photon in a crystal, $\mu_0 = \omega\chi_0''$ – the linear coefficient of X-waves absorption in the averaged amorphous medium, $L_{ext}^{(s)} = \frac{1}{\omega\left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right|C}$ – the length of the X-waves extinction in a periodic medium.

The formula (24) clearly demonstrates the dynamic Borrmann effect arising during the passage of x-rays DTR through a periodic medium. Namely, in the X-ray scattering in a periodical medium the abnormal weak absorption is observed for the first wave field $\mu_1^{(s)} << \mu_0$ (i.e. anomalous transmission of the first field X-rays) and abnormal strong absorption for the second one $\mu_2^{(s)} > \mu_0$. By this reason, for the sufficiently large photon path in the substance of the plate the DTR only by one of the fields in a periodic structure will be formed, namely, by the field with effective absorption coefficient $\mu_1^{(s)}$.

Physics of the Borrmann effect [20] consists in the formation of the standing waves from the incident and scattered waves, whose antinodes are localized in the regions of space with a lower electron density for one of the waves (first term in (23) and (24)) and in the regions of space with a higher electron density for second wave (second term in (23) and (24)). Parameter $\kappa^{(s)}$ appearing in (25) determines the degree of manifestation of the Borrmann effect in the anomalous X-ray waves passing through a periodic structure. As in the case of free X-ray waves in crystals, a prerequisite for manifestation of the effect of DTR in layered medium is the condition $\kappa^{(s)} \approx 1$, that corresponds to the minimal value of the linear absorption coefficient $\mu_1^{(s)}$.

Next, we will carry out a numerical analysis for each of the waves and of their interference term separately. For this purpose the expression (23) we write in the following form

$$R_{DTR}^{(s)} = R_1^{(s)} + R_2^{(s)} + R_{\mathrm{int}}^{(s)}, \quad (26a)$$

$$R_1^{(s)} = \frac{\varepsilon^2}{\xi(\omega)^2+\varepsilon} e^{-b^{(s)}\rho^{(s)}\left[\frac{1+\varepsilon}{\varepsilon} - \frac{(1-\varepsilon)\xi^{(s)}+2\varepsilon\kappa^{(s)}}{\varepsilon\sqrt{\xi^2+\varepsilon}}\right]}, \quad (26b)$$

$$R_2^{(s)} = \frac{\varepsilon^2}{\xi(\omega)^2+\varepsilon} e^{-b^{(s)}\rho^{(s)}\left[\frac{1+\varepsilon}{\varepsilon} + \frac{(1-\varepsilon)\xi^{(s)}+2\varepsilon\kappa^{(s)}}{\varepsilon\sqrt{\xi^2+\varepsilon}}\right]}, \quad (26c)$$

$$R_{\mathrm{int}}^{(s)} = -\frac{2\varepsilon^2}{\xi(\omega)^2+\varepsilon} e^{-b^{(s)}\rho^{(s)}\frac{1+\varepsilon}{\varepsilon}}\cdot cos\left(\frac{2b^{(s)}\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right). \quad (26c)$$

We will carry out the calculations for σ – polarized waves, i.e. for $s=1$. In order to get demonstrable results, we will consider the case when the layers are of equal thickness $a = b = T/2$. We will consider the reflections, that correspond to $g = \frac{2\pi}{T}$. In this case, the parameters in the expressions (26) will take the following values:

$$\xi(\omega)=\frac{2\pi\sin^2(\theta_B)}{|\chi'_b-\chi'_a|}\cdot\left(1-\frac{\omega}{\omega_B}\right)+\frac{1-\varepsilon}{2\nu^{(1)}},\ \kappa^{(1)}=\frac{2}{\pi}\cdot\left|\frac{\chi''_b-\chi''_a}{\chi''_b+\chi''_a}\right|,\quad \rho^{(1)}=\frac{\pi}{2}\cdot\left|\frac{\chi''_b+\chi''_a}{\chi'_b-\chi'_a}\right|,\ \nu^{(1)}=\frac{2}{\pi}\cdot\left|\frac{\chi'_b-\chi'_a}{\chi'_b+\chi'_a}\right|,$$

$$b^{(1)}=\frac{\omega_B|\chi'_b-\chi'_a|}{2\pi\sin(\delta-\theta_B)}L\,. \tag{27}$$

For a thin target ($b^{(1)}=5$), the curves drawn by (26), are shown in figure 3 describing the spectral density of the DTR (for $\omega_B=8\ \kappa eV$) in the artificial periodic structure consisting of amorphous layers of beryllium (Be) and tungsten (W). We see in this case, that the DTR is formed by the fields of two waves in a periodic structure, whose contributions in the spectral distribution are of comparable magnitude which will cause a strong interference of these waves. The interference term brings oscillations in the spectral density.

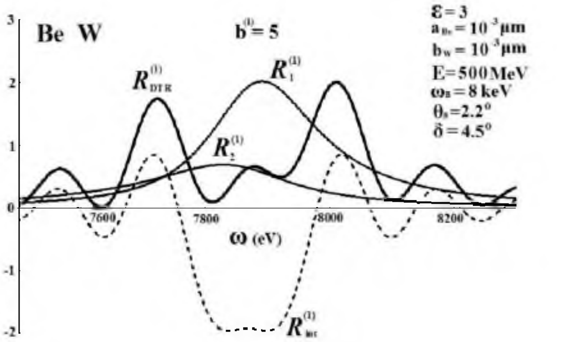

Figure 3. The contributions of the two fields, $R_1^{(1)}$ and $R_2^{(1)}$, and of their interference term $R_{\text{int}}^{(1)}$ into the total spectral density of DTR $R_{DTR}^{(1)}=R_1^{(1)}+R_2^{(1)}+R_{\text{int}}^{(1)}$

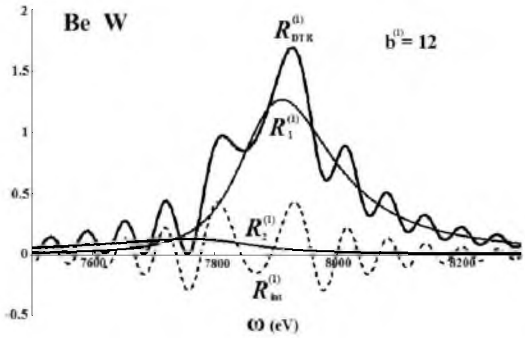

Figure 4. The same as figure 3 for bigger target thickness.

With increase of the target thickness one of the waves decays rapidly (figure 4 and figure 5), while the other one traverses the target without a significant decrease in amplitude. Under these conditions the contribution of the interference term markedly decreases and the spurious peaks in the spectrum are attenuated and then completely disappear (figure 6).

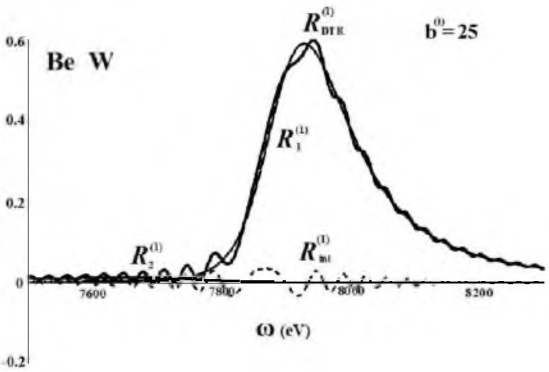

Figure 5. The same as in figure 4 for bigger target thickness.

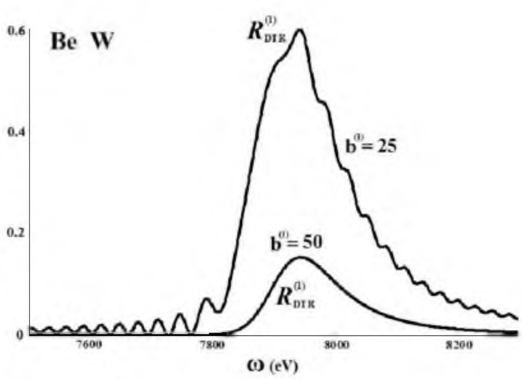

Figure 6. The spectral density of the relativistic electron DTR for different values of the target thickness.

It should be noted that the spectral curves in figure 5 and Figure 6 are constructed for a large target thickness, when the photon path length is longer than the average photo-absorption in an amorphous medium $l_{abs} = \frac{1}{\mu_0}$, which corresponds to the conditions of the Borrmann effect manifestation in an artificial periodic structure.

5. Conclusion

A theory for the coherent radiation of the relativistic electron crossing an artificial periodic structure is constructed for the case of Laue scattering geometry. The expressions for spectral-angular characteristics of the radiation in Bragg direction are derived and investigated.

The contributions to the DTR yield of two X-ray waves, which are responsible for DTR formation, are studied. It is shown that with increase of the target thickness, one of the waves is absorbed anomalously strongly and the other wave abnormally weakly, i.e. the Borrmann effect is manifested in DTR in an artificial periodic structure in the Laue geometry.

Based on these expressions it is shown that the angular density of diffracted transition radiation in layered target is more than one order higher than the density for a single crystal radiator under similar conditions.

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