# Analyzing The Classical and Extended Bouc-Wen Model Parameters

Tatiana Yu. Zablotskaya
Belgorod State University (Stary Oskol branch)
Stary Oskol, Russia
E-mail: zablotskaja_t@mail.ru

*Abstract* — **In this paper, the phenomenological Bouc-Wen model is analyzed as it has found wide application for functional description of nonlinear hysteretic systems and phenomena which are common for different engineering fields. Being formulated as a system of differential equations, the model has proven its versatility and feasibility across a broad range of theoretical and engineering problems and can be used as a hysteresis quantizer within a more complex system. The model allows it to be retuned and adjusted to new operation mode barely due to parameter setting with no amendments to its structure. The parameter values effect on the hysteresis loop significantly influencing its shape and size, and the model response to the input disturbance as well. Thus, the model parameter identification becomes an urgent problem and solving it, the best performance of the model can be achieved, which results in accurate and adequate behavior of the model. Therefore, the paper considers standard and extended modifications of the Bouc-Wen model, where the hysteresis is described without regard for energy dissipation or with degradation and pinching effects factored in correspondingly. Essential parameters of the model are studied in detail and classified depending on their impact on the shape and size of the hysteretic curve and response of the model as a consequence. Also, according to the research results, the efficient parameter ranges are recommended to provide the adequate model response to the input excitation.**

***Keywords: hysteresis, the Bouc-Wen model, parameter identification, modeling.***

## I. Introduction

Hysteresis is a widely observed phenomenon, both in natural and designed nonlinear systems, being an intrinsic property of such a system or a consequence of degradation and imperfections in it. It reveals itself in number of fields, including fundamental physical mechanisms [1], solid-state theory [2], economic systems [3], biological systems [4-5], and others. Also, it can be intentionally built into a system to monitor its behavior [6]. Regardless to its origin, the hysteresis effect has two crucial features: lagging and rate independence [7]. The first of them, the lagging effect, can be effectively described by knowing that a system undergoing hysteresis contains a slippage or retarding effect when the operating force reverses its sign. The second particularity, rate independence, implies that a system's response to the inputs depends on the input value, rather than the input's velocity variations.

From a mathematical point of view, all the hysteresis models can generally be divided into two classes according to their hysteresis nonlinearity types: differential models that have local memories and a Preisach model type with nonlocal memories. The latter uses the concept of operators [7], formulated and elaborated by Krasnoselskii, Pokrovskii, and Mayergoyz to become a classical Preisach model [8]. Hysteresis with local memories can be distinguished from other models by the property of being based on differential equations. One of the most utilized and widely accepted differential model is the Bouc-Wen model [9]. This one opts for using first-order nonlinear differential equations that relate input displacement to output restoring force in a hysteretic way and has been proven as an incredibly versatile tool in hysteresis study.

One of the advantages of the differential models is their possibility to be embodied directly into the differential equations that guide the motion of a particular system while assigning values to the parameters that define the system's behavior. There are no hysteretic operators in this prospect. Thus, the Bouc-Wen model can often be simpler to perform versus the non-differential Preisach model due to reliance of the former on differential equations [10-11].

The Bouc-Wen model deals with first-order differential equations that comprise a number of parameters to describe the hysteresis phenomenon. The influence of the parameters on the shape and size of the hysteresis loop is highly nonlinear and is difficult to assess. The parameters relationship and their joint effect on the model behavior are the research point of this paper. The Bouc-Wen model has been widely spread for its application to inverse problems, where evaluation of the model parameters is required to produce a curve which follows the experimental data with maximum accuracy [12-13]. Previously, this model was proposed as a hysteresis quantizer for biological artificial neural network with nonlinear activation function as a trigger-control mechanism designed to accurate and instant signal switching within the bioANN, as cellular systems show enhanced hysteretic switching in noisy environments, which can provide a reliable background for decision making and calculation process [14]. Hence, the parameter identification problem is of high urgency for the Bouc-Wen modeling, though, it will be shown that it could be efficiently performed for only certain types of hysteresis loops, while there are other loop shapes where it is less optimal.

## II. Physical background of the Bouc-Wen model

The Bouc-Wen model can be presented in the abstract form. The modeled object should be simplified and given as an equivalent system that consists of a mass $m$ paralleled to a linear spring $k$, a linear viscous damper $c$, and a hysteretic element $z(t)$ (Figure 1). The characteristics of such simplified system elements may be set empirically ad hoc depending on the problem to be solved. Nevertheless, the Bouc-Wen model is not to be deemed as entirely empirical and escapes weaknesses that empirical models have, when the data obtained are only valid for the case studied and may not be extrapolated out-of-case. Instead, including such physical entities as damper, spring, and mass, the Bouc-Wen model is valid for a diverse range of inputs due to its adjustable

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parameters and has been actively used in engineering mechanics and other fields.

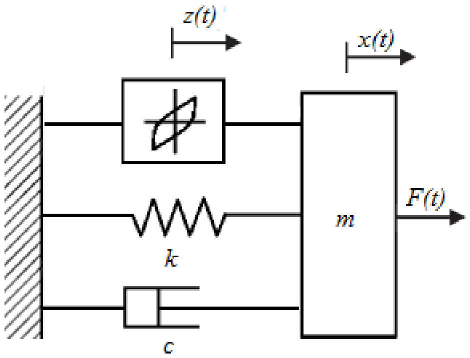



Fig. 1. Mechanical interpretation of a hysteretic single degree of freedom system

Here, the model comprises linear and nonlinear components within. The nonlinear behavior is performed via the hysteretic element, e.g. a nonlinear spring (Fig. 1). The hysteretic force is a function of hysteretic variable $z = z(t)$ (or hysteretic displacement), which is a function of the total displacement, $x$. The vise-versa scheme where hysteretic displacement is a function of the hysteretic force is also appropriate. The nonlinear response of the model is controlled by the energy absorbed by the hysteretic element and the hysteretic displacement $z$. Thereat, the Bouc-Wen model allows describing a nonlinear hysteretic system in coordinate representation $x(t) \to F_R(x)(t)$:

$$F_R(\mathrm{x},\,\mathrm{t}) = \alpha k x(t) + (1-\alpha) D k z(t)\,. \tag{1}$$

Initially this model was conforming to mechanical systems, where $x$ is a displacement, $F_R$ is restoring force, and elastic hysteresis $F_R\ (x)(t)$ is a superposition of elastic component $\alpha kx$ and hysteresis component $(1 - \alpha)Dkz(t)$. In (1), $D$ stands for plastic yield displacement, $D > 0$, and $\alpha$ defines the ratio of final stiffness $k_f$ to initial stiffness $k_i$ stiffness, $0 < \alpha < 1$ (Fig. 2):

$$\alpha = \frac{k_f}{k_i}\,. \tag{2}$$

According to (1), $x$ represents the input time history and $F_R$ is the hysteretic output time history. If both variables are bounded, the bounded input – bounded output (BIBO) model property leads to its stability, other options are unbounded input or unbounded response, which cause unstable behavior of the model [9].

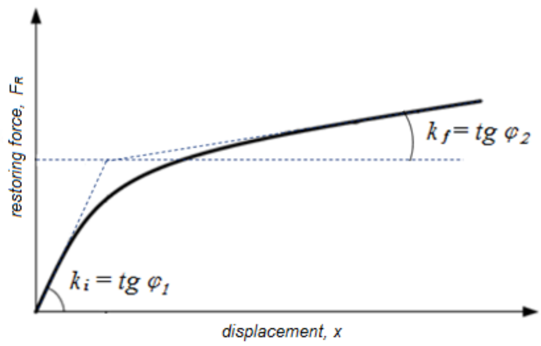



Fig. 2. Defining initial and final stiffness via the hysteresis loop

## III. The Bouc-Wen model modifications

### A. Classical model

The hysteretic displacement $z$ is a function of the hysteretic force. Thus, a hysteresis curve can be obtained as a rate-type dependency between the hysteretic displacement $z$ with $x$ ($x = x(t)$), as $z$ varies with $x$ at different rates depending on the movement phase (loading or unloading) and displacement level. Therefore, an elliptic type of hysteresis can be observed when $z$ increases more rapidly at small negative $x$ under loading and decreases at large positive $x$ under unloading. At larger displacements, the Bouc-Wen model expresses $z$ in form of the differential equation

$$\dot{z} = D^{-1}(A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \gamma\dot{x}|z|^{n})\,, \tag{3}$$

where $x$ is total displacement of the mass; $z$ is hysteretic displacement, a phase dimensionless parameter of hysteresis, $z = z\ (t)$; $A$ is a parameter controlling hysteresis amplitude; $\beta$, $\gamma$, $n$ are the parameters describing shape and amplitude of hysteresis, $n$ defines smoothness of elastic-non-elastic transition, whereas the larger the integer values of $n$, the more abrupt the transition is, $n \geq 1$, $D > 0$. Equation (3) describes a non-pinching and non-degrading type of a nonlinear hysteretic system, a standard Bouc-Wen model with five key parameters. It can be written in extended form with more parameters to include dissipative effects (7)-(14).

As it follows from (3), the parameter $A$ influences on $z$ varying with time and regulates the hysteretic displacement amplitude $z_{max}$. Thus, $A$ controls hysteretic stiffness: when describing the hysteresis by a continuous function, then the hysteretic stiffness is equal to zero at the local maximum or minimum, which are the points on the hysteresis loop where the movement direction shifts. There, at an infinitely small distance $dz$ away from $z_{max}$, where the velocity is close but not equal to zero, the hysteretic displacement is as follows

$$\dot{z}_{\max} = A\dot{x} - \beta|\dot{x}||z_{\max}|^{n-1}z_{\max} - \gamma\dot{x}|z_{\max}|^{n})\ ; \tag{4}$$

$$z_{\max} = \pm\sqrt[n]{\frac{A}{\beta+\gamma}}\,. \tag{5}$$

To enhance versatility of the model, the parameter $A$ may be varied with dissipated energy. Still, this parameter is a slightly redundant regarding the fact that both, hysteretic stiffness and hysteretic force, being a function of the hysteretic displacement range, can be varied by the parameters $\beta$, $\gamma$, $n$, the rigidity ratio $\alpha$, and degradation parameters in case of the extended model (7)-(14). Hence, if the parameter $A$ is set equal to 1.0, a non-degrading non-pinching hysteretic system with no energy dissipation can thereby be described as

$$\dot{z}(t) = \dot{x}(t) - (\beta|\dot{x}(t)||z(t)|^{n-1}z(t) + \gamma\dot{x}(t)|z(t)|^{n})\,. \tag{6}$$

### B. Extended model

In standard form, the Bouc-Wen model is formulated as (3) or (6). The model parameters defining the shape and size of the hysteresis loop are $\beta$, $\gamma$, $n$, $\alpha$, and $A$. Regarding to the energy dissipation which takes place in most of the hysteretic systems and including the pinching and degradation effects, the Bouc-Wen model can be formulated as follows [15]:

$$\dot{z} = h \cdot \frac{1}{\eta}\left(A\dot{x} - v\left(\beta|\dot{x}||z|^{n-1}z - \gamma\dot{x}|z|^{n}\right)\right). \qquad (7).$$

Nonlinear pinching function can be stated as:

$$h = 1 - \xi_1 e^{\frac{-(z\,\mathrm{sgn}(x) - q z_{max})^2}{\xi_2^2}}, \qquad (8)$$

where $\xi_1$ defines the pinching rate and slope, $0 \le \xi_1 < 1$; $\xi_2$ characterizes the pinching spread:

$$\xi_1 = \xi_s\left(1 - e^{-p\varepsilon}\right); \qquad (9)$$

$$\xi_2 = \left(\psi + \delta_\psi \varepsilon\right)\left(\lambda + \xi_1\right). \qquad (10)$$

Parameters $\lambda$, $p$, $q$, $\psi$, $\delta_\psi$, $\xi_S$ are the loop pinching parameters: $q$ and $p$ dictate the pinching level and the initial drop of the pinching area's slope respectively; $\xi_S$ defines displacement (the total slip), its larger values correspond to stronger pinching effect; $\psi$ contributes to amount of pinching and $\delta_\psi$ is the spread of it within the loop, $\lambda$ is a pinching ratio that defines connection between $\xi_1$ and $\xi_2$.

The response history dependency of the model is formulated as follows:

$$v(\varepsilon) = v_0 + \delta_v\varepsilon; \qquad (11)$$

$$\eta(\varepsilon) = \eta_0 + \delta_\eta\varepsilon; \qquad (12)$$

$$A(\varepsilon) = A_0 - \delta_A\varepsilon, \qquad (13)$$

where $\delta_v$ and $\delta_\eta$ define strength and stiffness degradation correspondingly, and $\delta_A$ – dependency of $A$ to the absorbed hysteretic energy $\varepsilon = \varepsilon(t)$; $v_0 = \eta_0 = A_0 = 1$. Dissipated energy:

$$\varepsilon = (1-\alpha)\cdot\omega_n^2\int_0^t z\dot{x}\,dt, \qquad (14)$$

where $\omega_n$ is pseudo-natural frequency of a nonlinear system. The value of (14) controls system dissipativity – both strength and stiffness degradation and pinching The term "strength degradation" is not precise though as far as strength degradation can only be modeled when displacement to be the input. Also, other extensions of the Bouc-Wen model are applicable depending on the problem context [11, 13].

## IV. The Model Parameters and Their Effect on the Hysteresis Loop

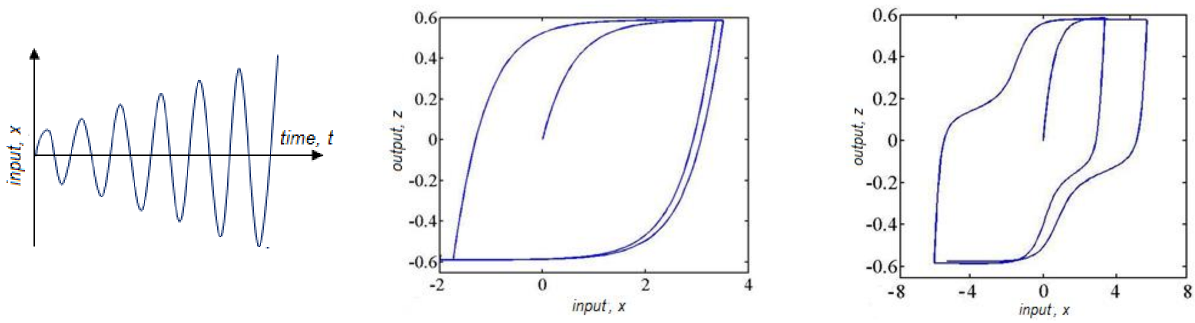
According to (7)-(14) for the extended type of the Bouc-Wen model, we have the following set of parameters (Table 1).

The differences between the analytical expressions of the Bouc-Wen model types result in different response to the input disturbance as well (Fig. 3). If the pinching function (8) and response history dependencies (11)-(12) are set to 1.0 ($h(z) = v = \eta = 1$), both the pinching effect and degradation disappear from the hysteresis loop, which then follows standard non-pinching shape (3).

Table 1. Parameters of the Bouc-Wen model

| № | Parameter | Description |
|---|---|---|
| | | ***hysteresis parameters*** |
| 1. | $\beta$ | describes shape and size of hysteresis loop |
| 2. | $\gamma$ | describes shape and size of hysteresis loop |
| 3. | $n$ | describes shape and size of hysteresis loop |
| 4. | $\alpha$ | rigidity ratio |
| 5. | $A$ | controls hysteresis amplitude |
| | | ***pinching parameters*** |
| 6. | $\lambda$ | pinching parameter |
| 7. | $\xi$ | defines the severity and rate of pinching |
| 8. | $p$ | controls the rate of initial drop in slope |
| 9. | $\psi$ | pinching parameter |
| 10. | $\delta_\psi$ | specifies the rate of pinching with $z$ |
| | | ***degradation parameters*** |
| 11. | $\delta_A$ | specifies the dependency of $A$ to dissipated energy |
| 12. | $\delta_v$ | degradation parameter |
| 13. | $\delta_\eta$ | degradation parameter |

As long as the input is induced by a steadily increasing periodic force (Fig. 3, a), and nor pinching neither degradation is factored in, the model (3) describes an elliptic curve depending on the values of the parameters $n$, $\beta$, and $\gamma$ (Fig. 3, b). Once energy dissipation parameters are included, the hysteresis loop is no more elliptic and becomes pinched (Fig. 3, c). From the standard model type (3), the key parameters associated with the Bouc-Wen model are: the rigidity ratio $\alpha$, the hysteresis amplitude controlling parameter $A$, and the hysteresis loop shape controlling parameters $\beta$, $\gamma$, and $n$, which are the hysteresis parameters in Table 1.




a) b) c)

Fig. 3. A hysteresis loop sample for standard (b) and extended (c) Bouc-Wen models induced by a steadily increasing periodic force (a)

There are some inherent difficulties when it comes to fitting the Bouc-Wen model to a wide range of hysteretic data, partially because of the simultaneous error minimization with these parameters. This often leads to local minimums in the least squares approach, which may or may not be in a good overall fit [16]. Figures from 4 till 9 show how each of the key parameters effects on the overall shape of the standard Bouc-Wen model hysteresis loop. In each scenario, there is only one parameter changing, while the others hold the following values, chosen partially arbitrarily for creating a medium-sized loop: $\alpha = 0.5$, $\beta = 4$, $\gamma = 2$, $A = 0.8$, $n = 1$.

Numerous research results state that values of three parameters, namely β, γ, and $n$, and their interrelation determine the basic shape and size of the hysteresis loop. For the amplitude controlling parameter $A$ and loop controlling parameter $n$, the following trends can be noticed: as $A$ increases, the loop grows wider and slope becomes sharper (Fig. 4); as $n$ increases, the transition between elastic and post-elastic areas of the loop becomes more abrupt (Fig. 5), for large values, the hysteretic curve approaches that of the bilinear model [11]. There is a strong linear correlation between β and γ, thus, changing setting for β and γ proportionally reveals relatively low sensitivity of the Bouc-Wen model to the absolute values of the parameters except for their excessive values (more than 50.0), as this causes significant distortion. Hysteretic stiffness and strength correspond inversely to the absolute values of β and γ, as well as smoothness of the curve, still, their influence is not significant (Fig. 6, 7). Also, the model shapes up to softening or hardening hysteretic behavior if $\beta < 0$ and $\beta > 0$ correspondingly, and to linear hysteretic behavior, if $\beta = 0$ (Fig. 8).

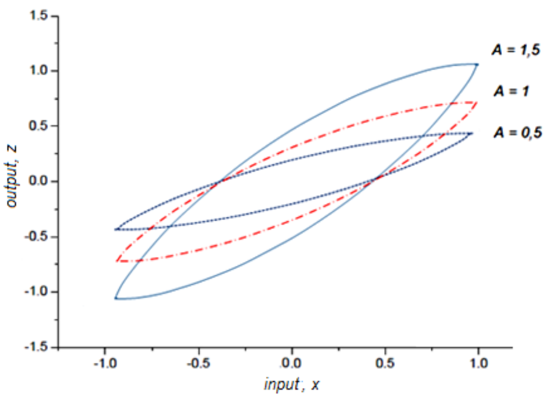



Fig. 4. Hysteresis loop shape for different values of $A$

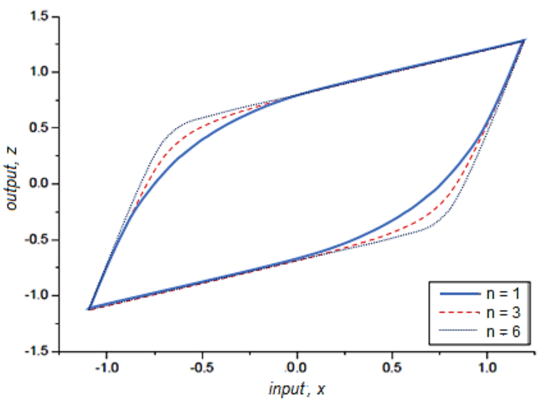



Fig. 5. Hysteresis loop shape for different values of $n$

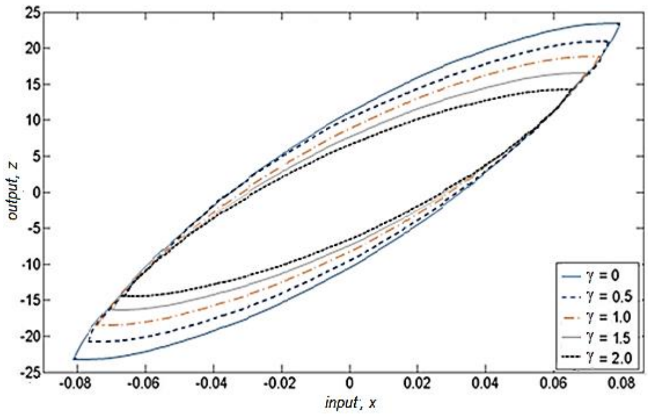



Fig. 6. Hysteresis loop shape for different values of γ

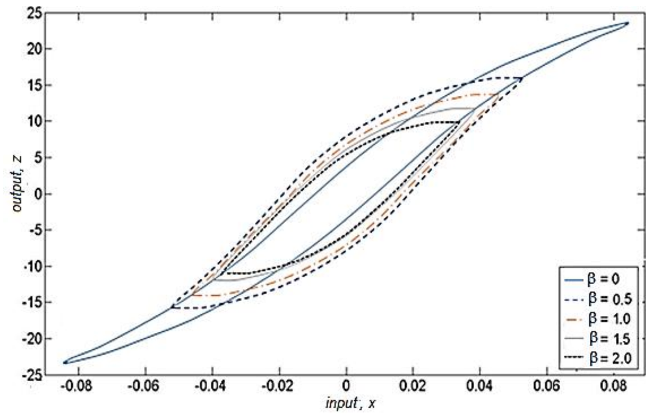



Fig. 7. Hysteresis loop shape for different values of β

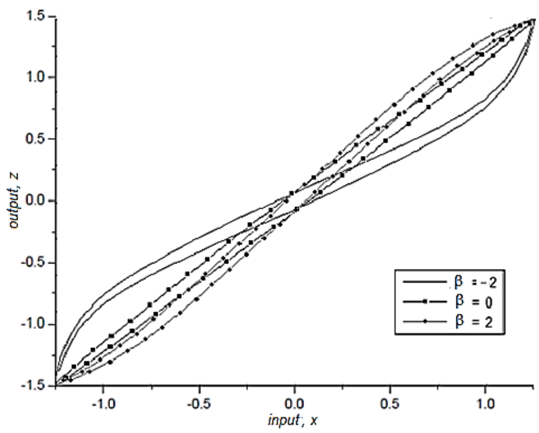



Fig. 8. Hysteresis loop shape for different signs of β

Much more sensitivity the model demonstrates to the relative value of β with respect to $\gamma$ or vice versa. The balance between these two parameters specifies whether the model corresponds to hardening or softening input-output dependency (Fig. 9). Setting $n = 1.0$, the following relationships between β and γ affecting the hysteresis loop can be observed:

1) $\beta + \gamma > 0$; $\gamma - \beta < 0$ stands for weak softening and for loading mode, if second condition is replaced by equation $(\gamma - \beta = 0)$;
2) $\beta + \gamma > \beta - \gamma$ defines strong softening both for loading and unloading, which narrows the hysteresis loop;
3) $\beta + \gamma = 0$; $\gamma - \beta < 0$ describes weak hardening;
4) $\beta + \gamma < 0$; $\beta + \gamma > \gamma - \beta$ describes strong hardening.

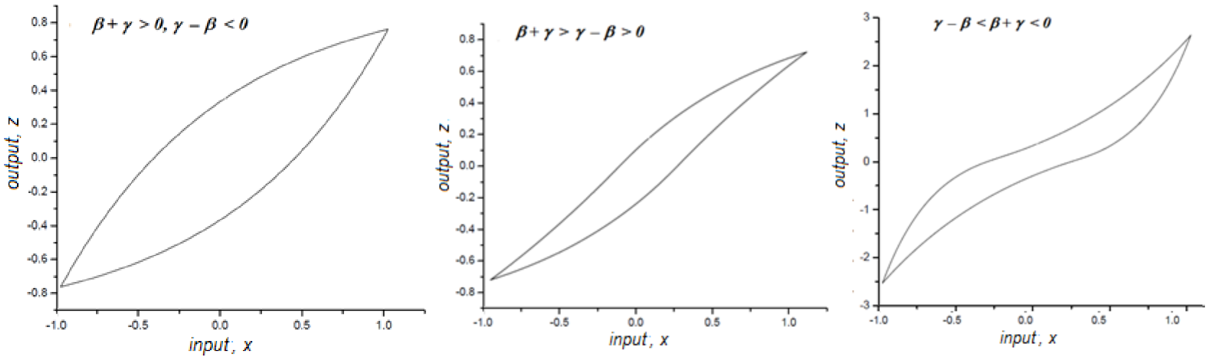



Fig. 9. Hysteresis loop shape for various combinations of β and γ

The Bouc-Wen model is highly dependent on its parameter values, and there are plenty of parameter combinations that will lead to unusable results. The results obtained and research review of parameter identification [14-17] allows some bottom-lining as for parameter setting that proves to be reasonable bounds for numerous cases of the hysteresis modeling. The parameter ranges given in Table 2 below provides stable and complete hysteresis loops of standard type of the model.

TABLE 2. A SAMPLE OF THE OPTIMAL BOUC-WEN PARAMETER RANGES

| **Parameter** | **Values** | |
|---|---|---|
| | ***min*** | ***max*** |
| $\alpha$ | 0.2 | 0.8 |
| $\beta$ | 3.0 | 5.0 |
| $\gamma$ | 1.1 | 3.0 |
| $A$ | 0.4 | 1.1 |
| $n$ | 1.0 | 2.0 |

The parameter bounds specified above provide an adequate response of the model for certain types of hysteresis loops. Nevertheless, for the loops with significant shape or size distortions – large width, extremely skinny area, etc. – the parameter ranges need to be revisited. In the generation of the Bouc-Wen hysteresis curve, the differential equations draw the loop from quiescent conditions. The Bouc-Wen loop is also prone to have variability in its amplitude before setting in a more consistent pattern after several iterations. Thus, the Bouc-Wen model is allowed to run for many cycles, with the last complete cycle being used as a baseline loop.

## V. CONCLUSIONS

Given that all the key parameters of the standard Bouc-Wen model have been detailed, the model can be solved for bounded input – bounded output case using appropriate software. Extended modification of the model to be simulated according to its parameters. Depending on the case, the identification problem may have unique or more than one solution. Nevertheless, such a solution is highly sensitive to the model parameter values and their interrelation, especially when restoring force is deemed as independent variable. This handicaps the parameters accurate identifying manually by trial and error and demands for sound identification approach to be found.

Despite its versatility and ability to perform smoothly pinched hysteresis curves varying in their sizes and shapes, the Bouc-Wen model is not fully appropriate for any hysteretic system. Thus, the Bouc-Wen model may be malfunctioned for slack systems, where initial stiffness is zero or next to zero until the slack is passed, or has some limitations [18]. Other limitations of the model come as natural sequences of the following aspects: 1) being highly dependent on dissipated energy, the model describes the hysteresis behavior of a system with similar energy characteristics as the one the model was adjusted to; 2) versatility of the model becomes moderate once experimental data are not noise-free, and here, the fitting problem arises.

Thus, while the Bouc-Wen model may give a good approximation of a true hysteresis loop for a specific input excitation used with parametric identification or tuning purposes, it may not be appropriate to represent the behavior of a true hysteretic system under general input excitations without proper parameter adjusting. Therefore, finding an approach to the model parameter identification seems to be the focus point for further research.

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