Magnetic soliton motion in a nonuniform magnetic field

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We discuss the dynamics of a magnetic soliton in a one-dimensional ferromagnet placed in a weakly nonuniform magnetic field. In the presence of a constant weak magnetic-field gradient the soliton quasimomentum is a linear function of time, which induces oscillatory motion of the soliton with a frequency determined by the magnetic-field gradient; the phenomenon is similar to Bloch oscillations of an electron in a weak electric field. An explicit description of soliton oscillations in the presence of a weak magnetic-field gradient is given in the adiabatic approximation. Two turning points are found in the motion of the soliton and the varieties of bounded and unbounded soliton motion are discussed. The Landau–Lifshitz equations are solved numerically for the case of a soliton moving in a weakly nonuniform magnetic field. The soliton is shown to emit a low-intensity spin wave near one of the turning points due to violation of the adiabatic approximation, and the necessary conditions for such an approximation to hold are established.

1. INTRODUCTION

A remarkable feature of the nonlinear dynamics of the magnetization of ferromagnets and antiferromagnets is the presence of dynamic magnetic solitons. By a dynamic soliton we mean a spatially localized perturbation in the magnetization field whose stability is ensured by the presence of certain integrals of motion for the dynamical equations of this field. In one-dimensional ferromagnets, where the magnetization dynamics is described by Landau–Lifshitz equations, a complete description of all types of nonlinear excitations is possible; in particular, there exists an exact analytical description of dynamic solitons in uniaxial and biaxial magnetic materials in a uniform magnetic field in the absence of dissipation.[1,2]

The phenomenological Landau–Lifshitz equations for ferromagnets have a quantum mechanical basis and provide an accurate explanation of the dynamical properties of magnetically ordered media. This suggests the possibility of broadening the range of physical phenomena to which the Landau–Lifshitz equations can be applied. In particular, we will describe the important effect of nonuniformity of the magnetic field on the motion of a magnetic soliton. To formulate the problem within a general setting, we start by recalling the basic principles of the nonlinear dynamics of the magnetization of ferromagnets at low temperatures.

The instantaneous state of a ferromagnet is determined by the magnetization vector as a function of position and time, $\mathbf{M}(\mathbf{r},t)$. According to current ideas about the exchange spin nature of ferromagnetism, the magnitude M_0 of the magnetization vector remains unchanged, so that the magnetization dynamics reduces to the precessional motion of this vector.[3] In other words, if we introduce the polar angles θ and φ, the magnetization vector $\mathbf{M}$ of the ferromagnet can be written as

$$M_x + iM_y = M_0 \sin\theta e^{i\varphi}, \quad M_z = M_0 \cos\theta. \tag{1}$$

In terms of the angular variables θ and φ, the Landau–Lifshitz equations

$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{2\mu_0}{\hbar}\mathbf{M}\times\frac{\delta E}{\delta \mathbf{M}}$$

take the form

$$\sin\theta\frac{\partial\theta}{\partial t} = -\frac{2\mu_0}{\hbar M_0}\frac{\delta E}{\delta\varphi}, \quad \sin\theta\frac{\partial\varphi}{\partial t} = \frac{2\mu_0}{\hbar M_0}\frac{\delta E}{\delta\theta}, \tag{2}$$

where the right-hand sides of the equations contain variational derivatives of the total magnetic energy of the magnetic material, E, with respect to the magnetization and the angular variables, and μ_0 is the Bohr magneton. The total energy E can be written as

$$E = \int w\{\theta,\varphi\}d^3x, \tag{3}$$

where the magnetic energy density w depends on the angular variables θ and φ and their gradients.

We limit our discussion to the case of a ferromagnet with uniaxial magnetic anisotropy placed in an external magnetic field **H** that is directed along the anisotropy axis **n**. We identify the z axis with this axis. Then the magnetic energy density can be written as[3]

$$w\{\theta,\varphi\}=w_0(\theta,\nabla\theta,\nabla\varphi)+M_0(1-\cos\,\theta)H, \tag{4}$$

where

$$w_0=\frac{1}{2}\,\alpha\left(\frac{\partial\mathbf{M}}{\partial x_i}\right)^2+\frac{1}{2}\beta M_0^2\sin^2\,\theta,$$

with α being the exchange constant and β the anisotropy constant. The function $w_0(\theta,\nabla\theta,\nabla\varphi)$ depends on the gradients of the angular variables but does not depend explicitly on the angular variable φ (the phase). Some statements referring to a magnetic soliton are unrelated to the specific form of the function w_0 if the latter depends on the specified arguments.

The Landau–Lifshitz equations for a uniaxial ferromagnet always has two constants of motion: the total magnetic-excitation energy E, and the projection of the total magnetic moment on the anisotropy axis. The second constant of motion, related to the presence of the cyclic coordinate φ, can be conveniently written as

$$N=\frac{1}{2\mu_0}\int[M_0-M_z(\theta)]d^3x$$
$$=\frac{M_0}{2\mu_0}\int(1-\cos\,\theta)d^3x. \tag{5}$$

The normalization (5) makes it possible to assume that N is the number of magnons whose bound state forms the soliton.[1,2]

If the external magnetic field is uniform, the total excitation field momentum (total quasimomentum)

$$\mathbf{P}=-\frac{\hbar M_0}{2\mu_0}\int(1-\cos\,\theta)\nabla\varphi d^3x \tag{6}$$

is also conserved (in addition to E and N).

A dynamic magnetic soliton is a solution of Eqs. (2) localized in space, moving with a constant velocity, and corresponding to finite values of the constants of motions E, N, and **P**. Such a solution has the form

$$\theta=\theta(\mathbf{r}-\mathbf{V}t),\quad \varphi=\Omega t+\psi(\mathbf{r}-\mathbf{V}t), \tag{7}$$

where **V** is the soliton velocity, Ω is the frequency of the soliton's internal precession, and the functions $\theta(\xi)$ and $\varphi(\xi)$ possess the following properties:

$$\theta(\xi)=0,\quad |\nabla\varphi|<\infty \quad \text{as}\quad \xi\to\pm\infty. \tag{8}$$

Hence a magnetic soliton is a two-parameter excitation, with **V** and Ω being the parameters.

The constants of motion E, N, and **P** are connected by a remarkable relationship, which is independent of the type of the functions (7); namely, under small variations of the functions θ and φ the variation of the total energy is[1,2]

$$\delta E=\mathbf{V}\cdot\delta\mathbf{P}+\hbar\Omega\,\delta N. \tag{9}$$

This yields two equations of motion for the soliton:

$$\mathbf{V}=\left(\frac{\partial E}{\partial\mathbf{P}}\right)_N,\quad \hbar\Omega=\left(\frac{\partial E}{\partial N}\right)_{\mathbf{P}}, \tag{10}$$

where the first determines the rate of variation of the position of the soliton's center of gravity, and the second the rate of variation of its phase. In a uniaxial ferromagnet in the presence of a uniform magnetic field, the position of the soliton's center of gravity and its phase are cyclic variables, which ensure the validity of the following conservation laws:

$$\mathbf{P}=\text{const},\quad N=\text{const}. \tag{11}$$

In this paper we study the dynamics of a magnetic soliton in a uniaxial ferromagnet in a weakly nonuniform magnetic field; in particular, we investigate the case of a constant magnetic-field gradient:

$$H=H_0+\eta x,\quad \eta=\frac{dH}{dx}. \tag{12}$$

In Sec. 2 we show that in this case the soliton position ceases to be a cyclic variable, with the result that the quasimomentum **P** ceases to be a constant of motion. When η is small, the quasimomentum **P** is a linear function of time, which changes the soliton dynamics dramatically.

For an object for which the calculations can be carried out analytically we take a one-dimensional easy-axis ferromagnet. The magnetic energy density w_0 of such a ferromagnet in terms of the angular variables θ and φ is

$$w_0=\frac{1}{2}\,\alpha M_0^2\left[\left(\frac{\partial\theta}{\partial x}\right)^2+\sin^2\,\theta\left(\frac{\partial\varphi}{\partial x}\right)^2\right]+\frac{1}{2}\beta M_0^2\sin^2\,\theta. \tag{13}$$

We discuss the dynamics of a magnetic soliton with the magnetic energy (13) in a one-dimensional uniaxial ferromagnet. In a uniform magnetic field, the energy of such a soliton is a periodic function of P (Refs. 1 and 2):

$$E=E_0(P,N)+2\mu_0NH_0,\quad E_0(P,N)=2W_0l_0\kappa(P,N),$$
$$l_0\kappa(P,N)=\tanh\frac{N}{N_1}\left[1+\frac{\sin^2(\pi P/2P_0)}{\sinh^2(N/N_1)}\right], \tag{14}$$

where $W_0=2M_0^2\sqrt{\alpha\beta}a^2$ is the surface energy of the domain boundary; $l_0=\sqrt{\alpha/\beta}\gg a$, with l_0 the characteristic magnetic length and a the interatomic separation; $P_0=\pi\hbar a^3M_0/\mu_0$; and $N_1=2a^2l_0M_0/\mu_0$ (here N_1 coincides in order of magnitude with the maximum number of spin deviations that can occur over the length l_0, and $P_0=2\pi s\hbar/a$, where s is the atomic spin, which determines the magnetism of the material).

If the magnetic-field gradient is so weak that $\eta l_0\ll H$, then in the presence of a small η the dependence of the energy E on P can still be described by Eq. (14) but the field momentum P must be assumed to be a linear function of time (this corresponds to what is known as the adiabatic approximation).

If the momentum P is a linear function of time, Eqs. (14) and (10) imply that the soliton oscillates with a frequency determined by the magnetic-field gradient. Since such motion is similar to the oscillations of a Bloch electron in a uniform electric field, we call it Bloch oscillations. The phenomenon of Bloch oscillations of a magnetic soliton in a magnetic field with a weak gradient was noted recently by one of the present authors in Ref. 4.

In Sec. 3 we study the dynamics of a one-dimensional soliton in the more general case of a weakly nonuniform magnetic field. If the nonuniformity of the magnetic field is located in a bounded interval of the one-dimensional magnetic material (the x axis), it creates an effective potential barrier for the soliton's motion. We discuss the various variants of bounded and unbounded soliton motion in the presence of different potential barriers.

Finally, in Sec. 4 we present the results of a numerical solution of the one-dimensional Landau–Lifshitz equations for the case of a soliton moving in a weakly nonuniform magnetic field. We find that when a soliton is in oscillatory motion, near one of the turning point a low-intensity spin wave with a frequency Ω is emitted. Such emission of a spin wave is due to the violation of the condition of validity of the adiabatic approximation. The criteria for the applicability of such an approximation are discussed.

2. BLOCH OSCILLATIONS OF A SOLITON IN A MAGNETIC FIELD WITH A CONSTANT GRADIENT

Let us study the soliton dynamics of a one-dimensional uniaxial ferromagnet with uniaxial anisotropy placed in a nonuniform magnetic with a weak gradient of type (12). As noted earlier, in a nonuniform magnetic field, P ceases to be a constant of motion. Let us examine the emerging time dependence of the quasimomentum defined according to the definition (6) and the property (8):

$$\frac{dP}{dt}=\frac{\hbar M_0}{2\mu_0}\int\left[\sin\theta\frac{\partial\varphi}{\partial t}\frac{\partial\theta}{\partial x}-\sin\theta\frac{\partial\varphi}{\partial x}\frac{\partial\theta}{\partial t}\right]dx. \tag{15}$$

We now use the equations of motion (2):

$$\frac{dP}{dt}=\int\left[\frac{\delta E}{\delta\theta}\frac{\partial\theta}{\partial x}+\frac{\delta E}{\delta\varphi}\frac{\partial\varphi}{\partial x}\right]dx. \tag{16}$$

Since the magnetic field depends on the x coordinate, the magnetic energy density also depends on x explicitly: $w\{\theta,\varphi;x\}$. This means that

$$\frac{dP}{dt}=\int\left[dw\{\theta,\varphi;x\}-\frac{\partial w}{\partial x}dx\right]$$
$$=-\int\frac{\partial w}{\partial x}dx=-\eta M_0\int(1-\cos\theta)dx=-2\eta\mu_0 N. \tag{17}$$

Thus, the quasimomentum is a linear function of time:

$$P(t)=P(0)-2\eta\mu_0 Nt,\quad P(0)=\text{const}. \tag{18}$$

Let us now calculate the total energy E of a soliton moving in the field of the weak gradient η. A weak gradient of the magnetic field can be interpreted as a weak perturbation of the soliton's motion in a uniform field. Then, in the adiabatic approximation,[5–7] the soliton retains its shape and the distribution of magnetization in it remains the same function of x given by Eq. (7):

$$\theta=\theta(x-X(t)),\quad \varphi=\varphi_0(t)+\psi(x-X(t)), \tag{19}$$

where the coordinate $X(t)$ of the soliton's center of gravity and its phase $\varphi_0(t)$ are functions of time to be determined. The main dynamical parameters of the soliton, the velocity V of the center of gravity and the frequency Ω, are given by obvious relationships:

$$V=\frac{dX}{dt},\quad \Omega=\frac{d\varphi_0}{dt}. \tag{20}$$

The total soliton energy can be written as

$$E=E_0(P,N)+2\mu_0 NH_0+\eta M_0\int(1-\cos\theta)x\,dx, \tag{21}$$

where $E_0(P,N)$ is defined in (14), and in the third term on the right-hand side we must bear in mind that in the adiabatic approximation $\theta(\xi)=\theta(-\xi)$:

$$\int(1-\cos\theta)x\,dx=\int[1-\cos\theta(x-X(t))]x\,dx=X(t)$$
$$\times\int(1-\cos\theta(\xi))d\xi=2\eta\mu_0 NX. \tag{22}$$

Combining (21) and (22), we get

$$E=E_0(P,N)+2\mu_0 NH_0+2\eta\mu_0 NX. \tag{23}$$

We see that the energy E is a function of three dynamical variables, P, X, and N, and that the expression (17) for the time derivative of the momentum serves as one of the canonical Hamiltonian equations:

$$\frac{dP}{dt}=-\frac{\partial E}{\partial X},\quad \frac{dX}{dt}=\frac{\partial E}{\partial P}. \tag{24}$$

On the other hand, selecting the initial coordinate of the soliton appropriately, we can use (23) and (14) to find the explicit time dependence of the coordinate of the soliton's center of gravity:

$$X(t)=X(0)+\frac{W_0[\cos(\pi P(t)/P_0)-\cos(\pi P(0)/P_0)]}{\eta\mu_0 N\sinh(2N/N_1)}, \tag{25}$$

where $P(t)$ is given in (18). If $P(0)\neq 0$ holds for short times, as long as $\eta\mu_0 Nt\ll P_0$ is satisfied, the soliton is in uniform motion:

$$X(t)=X(0)+\frac{2\pi W_0\sin(\pi P(0)/P_0)}{P_0\sinh(2N/N_1)}t. \tag{26}$$

For long times ($\eta\mu_0 Nt>P_0$), the soliton is in oscillatory motion. As Eq. (25) implies, the amplitude of the spatial oscillations is

$$\Delta X=\frac{W_0}{\eta\mu_0 N\sinh(2N/N_1)}. \tag{27}$$

Naturally, it is inversely proportional to the magnetic- field gradient and drops off rapidly as N increases, i.e., as the soliton grows in size:

$$V(t)\equiv\frac{dX}{dt}=V_m\frac{\sin(\pi P(t)/P_0)}{\sinh(2N/N_1)},\qquad(28)$$

where $V_m=2gM_0\sqrt{\alpha\beta}$ is the minimum phase velocity of the spin waves ($g=2\mu_0/\hbar$).

The oscillations of the soliton precession frequency can be found from the second formula in (10) and the definition (23):

$$\Omega(t)=\frac{1}{\hbar}\frac{\partial E_0(P,N)}{\partial N}+gH_0+g\eta X(t),\qquad(29)$$

with the first term on the right-hand side specified as

$$\frac{\partial E_0(P,N)}{\partial N}=\hbar\omega_0\left\{\frac{\cos^2(\pi P(t)/2P_0)}{\cosh^2(N/N_1)}-\frac{\sin^2(\pi P(t)/2P_0)}{\sinh^2(N/N_1)}\right\},\qquad(30)$$

where $\omega_0=g\beta M_0$ is the frequency of the homogeneous ferromagnetic resonance.

3. SOLITON MOTION IN A NONUNIFORM MAGNETIC FIELD

We now turn to the more general problem of the dynamics of a soliton moving in a one-dimensional uniaxial ferromagnet placed in a nonuniform magnetic field $\mathbf{H}(x)$ that is parallel to the anisotropy axis. We assume that the function $H(x)$ varies in an arbitrary manner as a function of x but that the characteristic spatial scale over which $H(x)$ varies is much larger than the soliton width. Since the magnetic energy density of a uniaxial ferromagnet is independent of the phase φ, the energy E and the number N of magnons remain constants of motion. The dependence of the soliton energy on the three dynamical variables $\mathbf{P}$, X, and N in the adiabatic approximation is an obvious generalization of (23), i.e.,

$$E=E_0(P,N)+2\mu_0NH(X),\qquad(31)$$

where $E_0(P,N)$ is still given by (14) and corresponds to the soliton energy in the absence of a magnetic field.

The time dependence of the quasimomentum P is given by an expression that is an obvious generalization of (17):

$$\frac{dP}{dt}=-2\mu_0N\frac{dH(X)}{dX}.\qquad(32)$$

Equation (32) together with (31) and (14) solve the problem of the time dependence of P. However, it is more convenient to study the motion of the center of gravity directly by using the explicit form of single-soliton solutions of the Landau–Lifshitz equations.

In the reference frame attached to the center of gravity ($\xi=x-Vt$), the single-soliton solution of Eqs. (2) for a uniform magnetic field has the form[1]

$$\tan^2\frac{\theta(\xi)}{2}=\frac{2\kappa}{(\kappa_M-\kappa_m)\cosh(2\kappa\xi)+\kappa_m+\kappa_M-2\kappa},\qquad(33)$$

where $\kappa=(E-2\mu_0NH)/2W_0l_0$ is the inverse of the soliton width, $\kappa_m=l_0^{-1}\tanh(N/N_1)$, $\kappa_M=1/\kappa_m l_0^2$, and the soliton velocity V and the angular precession frequency Ω are related to the parameter κ as follows:

$$V=V_ml_0\sqrt{(\kappa-\kappa_m)(\kappa_M-\kappa)},\qquad(34)$$

$$\Omega=gH+gM_0\beta[1-(\kappa l_0)^2-(V/V_m)^2].\qquad(35)$$

Since the parameter κ depends on the soliton's total energy and coordinate of the center of gravity, it is convenient to take it as the dynamical characteristic of the soliton. In particular, Eq. (34) shows that the possible movements of the soliton are limited to the range of values of κ for a fixed N: $\kappa_m\leqslant\kappa\leqslant\kappa_M$.

The motion of a soliton in a slowly varying magnetic field can be described in the adiabatic approximation by the same Eqs. (33)–(35) if we put $\xi=x-X(t)$ in them. Here the position of the center of gravity $X(t)$ of the soliton at time t is uniquely determined by the laws of conservation of the energy and the projection of the total magnetic moment on the z axis. In a nonuniform magnetic field, the parameters E and N, which enter into Eqs. (33)–(35), are still constants of motion, and the velocity V, the precession frequency Ω, and the inverse of the soliton width, κ, become slowly varying functions of time, and functionally the quantities V, Ω, and κ are determined by the running values of the magnetic field H at the soliton's center of gravity $X(t)$:

$$\kappa(X)=\frac{E-2\mu_0NH(X)}{2W_0l_0},$$

$$\Omega(X)=gH(X)+gM_0\times\left\{2-l_0(\kappa_M+\kappa_m)\frac{E-2\mu_0NH(X)}{2W_0}\right\},\qquad(36)$$

$$V(X)=\frac{2g\mu_0N}{M_0}\{[H(X)-H_m][H_M-H(X)]\}^{1/2},\qquad(37)$$

where $H_m=(E-2W_0l_0\kappa_M)(2\mu_0N)^{-1}$ and $H_M=(E-2W_0l_0\kappa_m)(2\mu_0N)^{-1}$ are the values of the magnetic field at the turning points x_m and x_M of the soliton (Fig. 1).

Integrating the equation $dX/dt=V(X)$ together with Eq. (37), we obtain the dependence of the coordinate X of the center of gravity on time t in the implicit form

$$t=\frac{N_1}{gl_0N}[F(X)-F(X(0))],\qquad(38)$$

where

$$F(X)=\int\frac{dX}{\{[H_M-H(X)][H(X)-H_m]\}^{1/2}}.$$

If the magnetic field $H(x)$ reaches both characteristic values H_m and H_M for finite x, the soliton is in a state of bounded motion between the turning points x_m and x_M (Fig. 1a) with a period

$$T=\frac{N_1}{gl_0N}[F(x_M)-F(x_m)].\qquad(39)$$

In a field with a constant magnetic-field gradient $\eta=dH/dx$, the soliton is in a state of harmonic oscillatory motion, where neither the frequency $\Omega_0=2gl_0(N/N_0)|\eta|$ nor the oscillation amplitude $\Delta X=|x_M-x_m|/2=(N_1\beta M_0/N|\eta|)/\sinh(2N/N_1)$ depends on the soliton energy E; they depend only on the number N of magnons. If

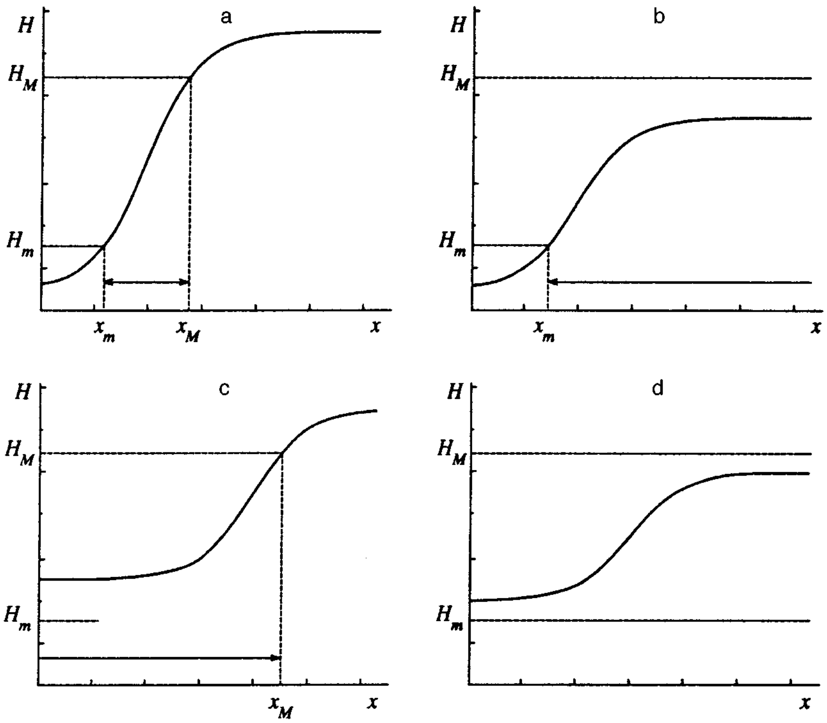

FIG. 1. Soliton motion in a nonuniform magnetic field: (a) bounded motion, (b) inbounded motion in the direction $x=\infty$, (c) unbounded motion in the direction $x=-\infty$, and (d) unbounded above-the-barrier motion.

$H(x)$ reaches only one value, H_m or H_M, the soliton is in unbounded motion, the reflection of the magnetic soliton from a "magnetic potential barrier" (Figs. 1b and 1c). If $H_m < H(x) < H_M$, the unbounded motion takes place above the barrier (Fig. 1d).

4. COMPUTER SIMULATION OF SOLITON OSCILLATIONS IN A NONUNIFORM FIELD

To verify our results we used a computer to numerically solve the one-dimensional Landau–Lifshitz equations

$$\frac{\partial \mathbf{M}}{\partial t} = g\mathbf{M} \times \left[\beta \mathbf{n}(\mathbf{M}\cdot\mathbf{n}) - \alpha \frac{\partial^2 \mathbf{M}}{\partial x^2} + \mathbf{H}(x) \right], \qquad (40)$$

where the unit vector $\mathbf{n}$ is directed along the anisotropy axis (the z axis), and the nonuniformity of the magnetic field is characterized by a constant value of the field's gradient $\eta = dH/dx$. Equations (40) were solved by a standard fourth-order Runge–Kutta method with automatic selection of the timestep, and the spatial derivatives $\partial^2\mathbf{M}/\partial x^2$ were calculated via a five-point finite-difference approximation; the number of grip points in the variable x was $n=400$. At first glance it seems that Eqs. (40) are entirely useless, since in addition we must know the soliton's initial state in the nonuniform magnetic field, and this is unknown. Hence we employed the following method of "preparing" a soliton with fixed values of the parameters E and N in the nonuniform magnetic field. First we constructed an effective magnetic field $H(x)$ by joining a horizontal section $H(x) = \mathrm{const}$ with a slanted section with the given gradient dH/dx (Fig. 2a). Then a soliton was placed in the horizontal section (curve *1*), for which Eqs. (33)–(35) were used. Finally the procedure of numerical solution of Eqs. (40) was initiated. As the soliton moves it enters the slanted section, where the field is nonuniform (curve *2*). At the moment when the soliton is entirely in the slanted section the horizontal section is replaced by a slanted one (Fig. 2b). Since the amplitude of magnetization oscillations in this section is infinitesimal, the soliton is "unaware" of such a substitution (curve *2′*).

Figure 3 depicts the results of computer simulation of the soliton motion with the following values of the parameters:

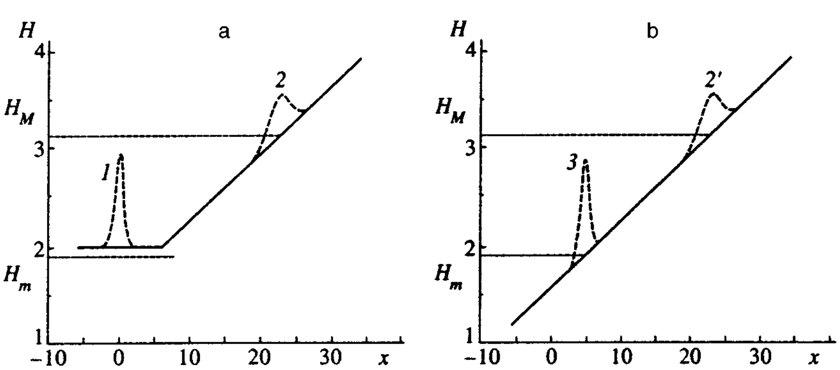

FIG. 2. "Preparation" of a soliton with given values of the parameters E and N: (a) the soliton is in an effective field, and (b) the soliton is in a field with a constant gradient.

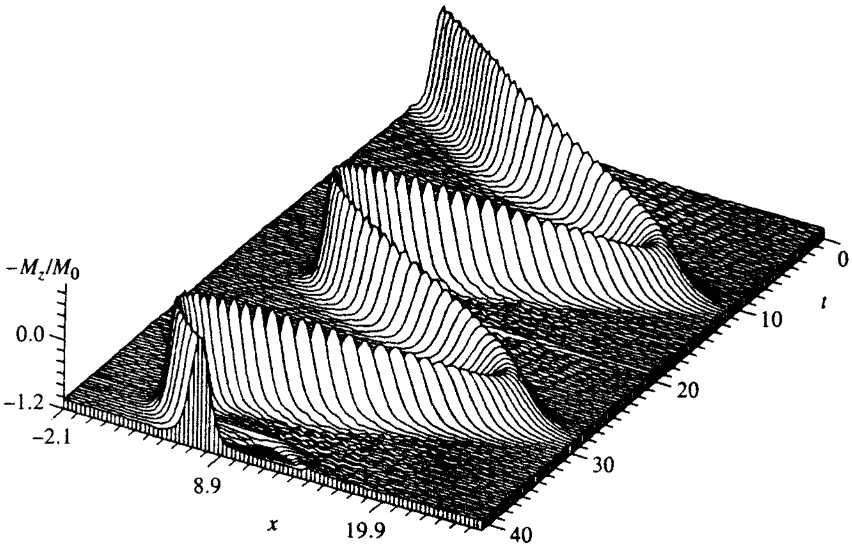

FIG. 3. Oscillatory motion of a soliton in a nonuniform magnetic field.

$$\frac{N}{N_1}=0.75, \quad \frac{E}{W_0}=6, \quad \frac{l_0|\eta|}{\beta M_0}=0.07.$$

The results of our calculations agree fairly well with those found from the formulas of the adiabatic theory. For instance, for the given values of the parameters, the theoretical values of the period and amplitude of the oscillations are $T=59.8/gM_0\beta$ and $\Delta X=8.95l_0$. The numerical calculations yield $T=60.4/gM_0\beta$ and $\Delta X=8.7l_0$, which are close to the theoretical values. However, more accurate calculations show that near the right turning point x_M the soliton emits a small-amplitude spin wave whose frequency corresponds to the frequency of magnetization precession in the soliton, Ω (Fig. 4). This phenomenon is related to the violation of adiabaticity and is described by higher-order corrections in the magnetic-field gradient.

The violation of the adiabatic approximation results from the interaction of the soliton and the spin waves, where the spin waves with the frequency Ω play the major role. The domain of existence of the waves, $x<x_s$, is bounded by the turning point x_s for these spin waves, where x_s can be found from the equation $gH(x_s)+gM_0\beta=\Omega(X)$.

Using the formulas in (36), we can calculate the distance between the soliton's center of gravity and the boundary of the spin-wave range:

$$|X-x_s|=\left[(1+\kappa_m^2 l_0^2)\frac{\kappa(X)}{\kappa_m}-1\right]\frac{\beta M_0}{|\eta|}. \tag{41}$$

We see that the distance is inversely proportional to the field gradient η and depends on X, the coordinate of the center of gravity.

According to (33), the amplitude of magnetization oscillations in the soliton falls off at large distance as $e^{-\kappa|\xi|}$. Thus, the interaction of the soliton and the spin waves is characterized by the small parameter $\exp(-\kappa|X-x_s|)$. Correspondingly, the adiabatic approximation holds if the distance from the center of gravity X to the spin-wave turning point x_s is much larger than the soliton size κ^{-1}:

$$|X-x_s|\kappa(X)\gg 1. \tag{42}$$

We note that the adiabatic approximation may be valid in one spatial region and invalid in another. According to (41), the distance $|X-x_s|$ reaches its minimum value $\beta M_0 l_0^2\kappa_m^2/|\eta|$ near the right turning point. At this point the adiabaticity condition (42) amounts to the requirement that the magnetic-field gradient be small:

$$\left|\frac{dH}{dx_M}\right|\ll\beta M_0 l_0^2\kappa_m^3. \tag{43}$$

Hence, the adiabatic theory that we developed to describe the soliton motion in a weakly nonuniform magnetic field is valid if

$$\frac{l_0}{\beta M_0}\left|\frac{dH}{dx_M}\right| \Big/ \tanh^3\frac{N}{N_1}\ll 1. \tag{44}$$

As the magnetic-field gradient gets stronger, the adiabatic approximation is violated first of all near the turning point that corresponds to the maximum admissible value of the magnetic field (which in our case is the right turning point).

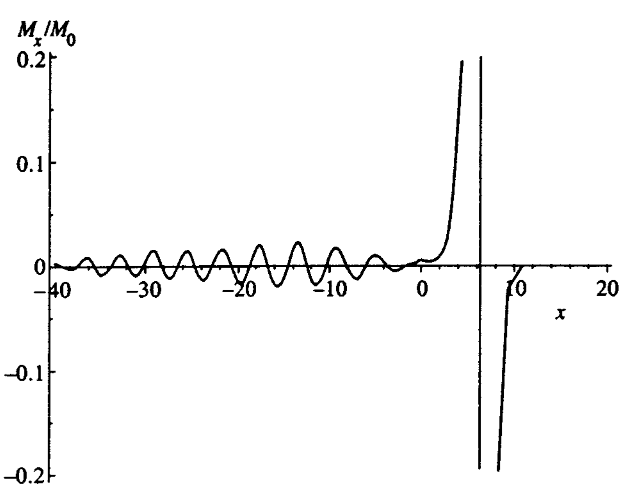

FIG. 4. Emission of a spin wave by a magnetic soliton.

For the values of the parameters used in the computer simulation of soliton oscillations, the left-hand side of (44) is $\varepsilon = 0.3$, which is the limit of the applicability of the adiabatic approximation. Near the left turning point the condition (44) is replaced by a less stringent condition,

$$\frac{I_0}{\beta M_0}\left|\frac{dH}{dx_m}\right|\tanh^3\frac{N}{N_1} \ll 1, \qquad (45)$$

whose left-hand side in our example is $\varepsilon = 0.017$. Note that $\exp(-1/\varepsilon)$ is the small parameter in the adiabatic approximation.

Thus, a magnetic soliton placed in a nonuniform magnetic field can be in a state of periodic motion with a frequency Ω_0 of order 10^9 Hz.

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