ON THE INFLUENCE OF MOTION OF THE MEDIUM ON THE PHOTOPHORESIS OF A SPHEROIDAL SOLID AEROSOL PARTICLE

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This paper considers the influence of motion of the medium (account of convective terms in the heat conduction equations) on the photophoresis of a large aerosol spheroidal particle at small relative temperature drops in its vicinity. It has been shown that at a fixed ratio of semiaxes with increasing intensity of the incident radiation the total contribution of the medium motion leads to a monotonic decrease in the rate of photophoresis, whose degree depends on the equatorial radius of the spheroid.

Introduction. It is known that an aerosol particle suspended in a thermodynamically nonequilibrium gaseous medium begins to move. Such motion may be caused, in particular, by the appearance of a temperature gradient along the particle surface. When it is due to the nonuniform heating of the particle surface because of the electromagnetic radiation absorption, the particle motion is called photophoresis [1–3]. Photophoresis can play an important role in atmospheric processes, in cleaning industrial gases from aerosol particles, and in creating devices intended for selective size separation of particles, etc.

The mechanism of photophoresis in the case of a solid particle can be described briefly as follows. When electromagnetic radiation interacts with a particle, inside it thermal energy is released with a certain volume density q_p which nonuniformly heats the particle. The gas molecules surrounding the particle upon colliding with its surface repel from the heated side of the particle with a higher velocity than from the cold side. As a result, the particle acquires an uncompensated momentum directed from the hot side of the particle to the cold one. Depending on the size and optical properties of the particle material, both the illuminated and the shadow side of the particle may turn out to be hotter and, therefore, both positive (particle motion in the direction of the radiation) and negative photophoresis can take place. Moreover, if the radiation flux is inhomogeneous in the cross-section, then transverse (with respect to the direction of electromagnetic radiation propagation) motion of the particle in the gas may arise [4].

Size classification of particles is carried out proceeding from the Knudsen criterion $\mathrm{Kn} = \lambda/L$. In so doing, particles are called large if $\mathrm{Kn} << 0.01$, moderate at $0.01 \le \mathrm{Kn} \le 0.3$, and small at $\mathrm{Kn} >> 1$. Large and moderate particles are described theoretically by means of the methods of gas dynamics, and particles with $\mathrm{Kn} >> 1$ are described by means of mathematical methods of kinetic theory of gases.

In the present work, large particles whose linear sizes are on average within the range of $5\ \mu\mathrm{m} < L < 30\ \mu\mathrm{m}$ are considered. Particles moving in the gas are subjected to the simultaneous action of forces of different nature and a situation may form where the action of the photophoretic force becomes insignificant. In particular, at large Froude numbers the photophoresis can be neglected compared to the gravitational motion.

It should be noted that up to now a sufficiently complete theoretical description of the photophoretic motion of large and moderate aerosol particles has been provided for spherical particles [3–18]. The formulas for the force and rate of photophoresis thereby have been obtained at both small [3–8, 11–17] and significant [9, 10] (when the temperature dependence of the molecular transfer coefficient and of the density coefficient of the gaseous medium were taken into account) relative temperature drops in the vicinity of the particle. For example, at small relative temperature drops the following formula for the velocity of large solid spherical particles has been obtained [6]

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$$U_{ph} = -\nu_g \frac{K_{t.s}}{2\pi R^3 T_\infty (\lambda_p + 2\lambda_g)} d_{ph}, \quad d_{ph} = \int_V \mathbf{r} q_p dV,$$

integration is performed throughout the particle volume. From this formula it follows that in those cases where $\lambda_p << \lambda_g$ the heat conduction of the particle has little effect on the velocity of motion, at $\lambda_p \approx \lambda_g$ the heat conductions of the particle and the gas produce a comparable effect on the transfer rate, and at $\lambda_p >> \lambda_g$ the photophoretic velocity is largely influenced by the heat conduction of the particle.

At $\lambda_p/\lambda_g \to \infty$ the inhomogeneity of the temperature distribution along the particle surface is smoothed out and the influence of the photophoretic force on the motion of a large particle can be neglected.

Of interest are the results of [17], where it has been shown for the first time that the contribution of the convective heat transfer due to the transverse temperature drop to the force and rate of photophoresis of moderate particles can be significant.

In recent years, interest in the theory of photophoresis of aerosol particles has grown again. This is due, in particular, to the fact that the theory of particle transfer in the stratosphere with account for the photophoretic force is still incomplete [15], and to the necessity of making more exact estimates of the photophoretic velocity of particles, which can be done knowing only the value of the absorption factor [14, 16], as well as to the possibility of using photophoresis in creating coatings from nanoparticles.

As mentioned above, most theoretical results on the photophoresis of large and moderate particles, in particular solid ones, have been obtained for spherical particles.

Many solid particles that occur in nature and are used in industrial facilities have a surface shape other than the spherical one, for example, a shape close to spheroidal. Therefore, it is interesting to derive formulas that will make it possible to evaluate the photophoretic motion of solid spheroidal particles. Under real conditions, as a result of the fluctuation of the gaseous medium density, the orientation of the spheroidal particle in the space changes with time. In so doing, the photophoretic motion is additively composed of two motions — longitudinal and transverse — about the symmetry axis of the spheroidal particle.

Mathematical methods for describing the motion of arbitrarily oriented large and moderate spheroidal particles have not yet been developed. Therefore, in choosing the formulas for the force and rate of photophoresis, we have considered one limiting case of the particle motion — the case where the symmetry axis of the particle is parallel to the direction of radiation propagation. The problem can thereby be solved analytically. In this case, the particle has a minimum photophoresis rate, whose difference from the maximum in the case of truly solid particles, as comparison with the velocity of motion of crosswise arranged cylindrical particles has shown, exceeds 40%. The photophoretic motion of longitudinally arranged particles can also be observed in practice, e.g., when photophoretic deposition of spheroidal particles occurs on charged surfaces. The problem has been solved with the use of the method of joined asymptotic expansions permitting sequential account of the influence of convective heat and mass transfer on the particle motion [19–21].

Formulation of the Problem. Consider a spheroidal solid aerosol particle suspended in a gas with temperature T_∞, density ρ_g, and viscosity μ_g. An electromagnetic radiation is incident on the particle and heats nonuniformly its surface. The gas interacting with the heated surface begins to move along the surface up in temperature. This phenomenon is called the thermal slip of gas. The mechanism of this phenomenon is analogous in its physical nature to thermophoresis [22]. The thermal slip causes the appearance of a photophoretic force. Under the action of the photophoretic force and the viscous force of the medium the particle begins to move uniformly. The velocity of uniform motion of the particle is referred to as the photophoretic velocity (U_{ph}).

In describing the process of photophoretic motion of the particle theoretically, we shall assume that by virtue of the smallness of the thermal relaxation time the process of heat transfer in the particle–gaseous medium system proceeds quasi-stationarily. The particle motion occurs at small Peclet and Reynolds number and at small relative temperature drops in its vicinity, i.e., when $(T_s - T_\infty)/T_\infty << 1$. When this condition is fulfilled, then the heat conductivity and dynamic and kinematic viscosity coefficients can be considered to be constants. The problem is solved by the hydrodynamic method, i.e., the hydrodynamics equations with corresponding boundary conditions are solved and it is assumed that the phase transition is absent and the particle is homogeneous in composition and large.

The electromagnetic radiation incident on the particle is adsorbed by the particle and is distributed throughout its volume. As a result, inside the particle sources of thermal energy with density q_p arise. It is convenient to introduce a reference system connected to the center of mass of the moving particle and orient the $0z$ axis in the direction of propagation of the uniform radiant flux (in this case the problem is reduced to the analysis of the flow past the particle of an infinite plane-parallel stream with velocity U_∞, and the gas velocity determined in such a coordinate system on infinity is equal, with the opposite sign, to the value of the photophoresis rate, $U_{ph} = -U_\infty$). We shall describe the flow past the particle in the spheroidal system of coordinates (ε, η, φ). The curvilinear coordinates ε, η, φ are related to the Cartesian coordinates by the following relations [19]:

$$x = c \cosh \varepsilon \sin \eta \cos \varphi , \quad y = c \cosh \varepsilon \sin \eta \sin \varphi , \quad z = c \sinh \varepsilon \cos \eta , \tag{1}$$

$$x = c \sinh \varepsilon \sin \eta \cos \varphi , \quad y = c \sinh \varepsilon \sin \eta \sin \varphi , \quad z = c \cosh \varepsilon \cos \eta , \tag{2}$$

where $c = \sqrt{a^2 - b^2}$ in the case of an oblate spheroid ($a > b$, formula (1)), and with $c = \sqrt{b^2 - a^2}$ in the case of a prolate spheroid ($a < b$, formula (2)). The position of the Cartesian coordinate system therewith is fixed relative to the particle so that the origin of coordinates is in the center of the spheroid and the $0z$ axis coincides with its symmetry axis.

Within the framework of the formulated assumptions the distributions of velocity U_g, pressure P_g, and temperatures T_g and T_p are described by the following system of equations [20]:

$$\nabla P_g = \mu_g \Delta U_g , \quad \text{div}\, U_g = 0 , \quad \rho_g c_{pg} (U_g \cdot \nabla)\, T_g = \lambda_g \Delta T_g , \quad \Delta T_p = - q_p / \lambda_p . \tag{3}$$

System (3) was solved with the following boundary conditions in the coordinate system of the oblate spheroid:

$$\varepsilon = \varepsilon_0 , \quad U_\varepsilon = 0 , \quad U_\eta = K_{t.s} \frac{\nu_g}{T_g} (\nabla T_g \cdot \mathbf{e}_\eta) , \quad T_g = T_p , \quad \lambda_g (\nabla T_g \cdot \mathbf{e}_\varepsilon) = \lambda_p (\nabla T_p \cdot \mathbf{e}_\varepsilon) ; \tag{4}$$

$$\varepsilon \to \infty , \quad U_\varepsilon = U_\infty \cos \eta , \quad U_\eta = - U_\infty \sin \eta , \quad T_g \to T_\infty , \quad P_g \to P_\infty ; \tag{5}$$

$$\varepsilon \to 0 , \quad T_p \neq \infty . \tag{6}$$

The boundary conditions (4) on the particle surface take into account the impenetrability condition for the normal component and the thermal slip for the tangent component of the mass velocity, the equality of temperatures, and the continuity of the heat flows. To the particle surface there corresponds the coordinate surface $\varepsilon = \varepsilon_0$. At a large distance from the particle the boundary conditions (5) are valid, and the finiteness of the physical quantities characterizing the particle at $\varepsilon \to 0$ is taken into account in (6).

Let us dedimensionalize Eq. (3) and the boundary conditions (4)–(6) by introducing the dimensionless coordinates, the temperature, and the velocity in the following way: $y_k = x_k/a$, $t = T/T_\infty$, $V = U/U_\infty$.

At $\zeta = \text{Re} = (\rho_g U_\infty a)/\mu_g \ll 1$ the incident flow produces only a disturbing effect and, therefore, the solution of the hydrodynamics equations should be sought in the form

$$V_g = V_g^{(0)} + \zeta V_g^{(1)} + \ldots , \quad P_g = P_g^{(0)} + \zeta P_g^{(1)} + \ldots . \tag{7}$$

In determining the photophoretic force and rate, we restrict ourselves to the corrections of the first order of infinitesimal in ζ.

Let us seek a solution of the equation describing the temperature distribution outside the particle by the method of joined asymptotic expansions [21, 22]. Let us write the internal and external asymptotic expansions of the dedimensionalized temperature as

$$t_g (\varepsilon, \eta) = t_{g0} (\varepsilon) + \sum_{n=1}^{\infty} f_n (\zeta)\, t_{gn} (\varepsilon, \eta) , \tag{8}$$

$$t_{\rm g}^{*}(\xi,\eta)=t_{\rm g0}^{*}(\xi)+\sum_{n=1}^{\infty}f_n^{*}(\zeta)\,t_{{\rm g}n}^{*}(\xi,\eta)\,, \tag{9}$$

where $\xi = \zeta\lambda$ is a "compressed" radial coordinate [21], $\lambda = \sinh\varepsilon$. And it is required that

$$\frac{f_{n+1}}{f_n}\to 0\,,\quad \frac{f_{n+1}^{*}}{f_n^{*}}\to 0 \quad \text{at} \quad \zeta\to 0\,.$$

The deficient boundary conditions for the internal and external expansions follow from the conditions of identity of asymptotic continuations of both into some intermediate region

$$t_{\rm g}(\varepsilon\to\infty,\eta)=t_{\rm g}^{*}(\xi\to 0,\eta)\,. \tag{10}$$

The asymptotic expansion of the solution inside the particle, as the boundary conditions on the spheroid surface (4) show, should be sought in a form analogous to (8):

$$t_{\rm p}(\varepsilon,\eta)=t_{\rm p0}(\varepsilon)+\sum_{n=1}^{\infty}f_n(\zeta)\,t_{{\rm p}n}(\varepsilon,\eta)\,. \tag{11}$$

As for the functions $f_n(\zeta)$ and $f_n^{*}(\zeta)$, it is just assumed that the order of their infinitesimal in ζ increases with increasing n. With account for the compressed radial coordinate we have the following equation for the temperature $t_{\rm g}^{*}$:

$$\frac{\rm Pr}{a}\left(V_{\rm g}^{*}\cdot\nabla^{*}\right)t_{\rm g}^{*}=\Delta^{*}t_{\rm g}^{*}\,,\quad t_{\rm g}^{*}\to 1 \quad \text{at} \quad \xi\to\infty\,,$$

$$V_{\rm g}^{*}(\xi,\eta)=\mathbf{n}_z+\zeta V_{\rm g}^{*}(\xi,\eta)+\ldots\,. \tag{12}$$

Here $\Delta = \Delta^{*}(\xi,\eta)$ is a Laplace operator; $V_{\rm g}^{*} = V_{\rm g}^{*}(\xi,\eta)$; $t_{\rm g}^{*} = t_{\rm g}^{*}(\xi,\eta)$; $\mathbf{n}_z$ is a unit vector in the direction of the $0z$ axis.

The form of the boundary conditions (5) points to the fact that the solution in the zeroth approximation for the mass velocity components should be sought in the form

$$V_{\varepsilon}(\varepsilon,\eta)=\frac{1}{c\cosh\varepsilon\,H_{\varepsilon}}\,G(\varepsilon)\cos\eta\,,\quad V_{\eta}(\varepsilon,\eta)=-\frac{1}{c\,H_{\varepsilon}}\,g(\varepsilon)\sin\eta\,, \tag{13}$$

where $G(\varepsilon)$ and $g(\varepsilon)$ are arbitrary functions depending on the dedimensionalized radial coordinate ε; $\varepsilon = c\sqrt{\cosh^2\varepsilon-\sin^2\eta}$ is the Lamé coefficient.

Temperature Distribution in the Vicinity of the Spheroidal Particle. Determining sequentially the zeroth and first expansions and taking into account the conditions of joining the internal and external expansions [21], we get

$$t_{\rm g}=t_{\rm g0}+\zeta t_{\rm g1}\,,\quad t_{\rm g}^{*}=t_{\rm g0}^{*}+\zeta t_{\rm g1}^{*}\,,\quad t_{\rm p}(\varepsilon,\eta)=t_{\rm p0}(\varepsilon)+\zeta t_{\rm p1}(\varepsilon,\eta)\,,\quad t_{\rm g0}^{*}=1\,,$$

$$t_{\rm g0}=1+\gamma\lambda_0\,\mathrm{arccot}\,\lambda\,,\quad t_{\rm p0}(\lambda)=M_0+N_0\,\mathrm{arccot}\,\lambda-\int_{\lambda_0}^{\lambda}Q_0\,\mathrm{arccot}\,\lambda d\lambda+\mathrm{arccot}\,\lambda\int_{\lambda_0}^{\lambda}Q_0 d\lambda\,,$$

$$t^*_{g1} = \frac{\gamma\lambda_0}{\xi}\exp\left\{\frac{c\mathrm{Pr}}{2a}\,\xi\,(x-1)\right\}, \quad t_{g1} = -\frac{\omega\lambda_0 c}{2a}\left(1-\frac{\mathrm{arccot}\,\lambda}{\mathrm{arccot}\,\lambda_0}\right) + \cos\eta\,\Bigg((\lambda\,\mathrm{arccot}\,\lambda - 1)\,C_3$$

$$+\beta\left\{A_2\left[\mathrm{arccot}\,\lambda - \frac{\lambda}{2}\,\mathrm{arccot}^2\,\lambda\right] + \frac{A_1}{2}\left[\mathrm{arccot}\,\lambda - \lambda\,\mathrm{arccot}^2\,\lambda\right] + \frac{c^2}{2}\,\lambda\,\mathrm{arccot}\,\lambda\right\}\Bigg),$$

$$t_{p1} = \cos\eta\left(M_1 c\lambda + N_1\,(1-\lambda\,\mathrm{arccot}\,\lambda) + \lambda\int_{\lambda_0}^{\lambda}(\lambda\,\mathrm{arccot}\,\lambda - 1)\,Q_1 d\lambda - (\lambda\,\mathrm{arccot}\,\lambda - 1)\int_{\lambda_0}^{\lambda}\lambda Q_1 d\lambda\right).$$

Here

$$\omega = \gamma\mathrm{Pr}\,; \quad \lambda_0 = \sinh\varepsilon_0\,; \quad \beta = \frac{\omega\lambda_0}{ac}\,;$$

$$Q_n = \frac{2n+1}{2\lambda_p T_\infty}\int_{-1}^{1} c^2 q_p\,(\lambda^2 + x^2)\,P_n(x)\,dx \qquad (n \ge 0)\,;$$

$$M_0 = 1 + \left(1 - \frac{\lambda_g}{\lambda_p}\right)\gamma\lambda_0\,\mathrm{arccot}\,\lambda_0\,; \quad N_0 = \frac{1}{4\pi c\lambda_p T_\infty}\int_V q_p dV\,; \quad N_1 = \frac{3}{4\pi c^2\lambda_p T_\infty}\,J\,,$$

$J = \int_V q_p z dV$ is the dipole moment of the density of heat sources, $z = c\lambda x$; $\gamma = t_s - 1$ is the dimensionless parameter characterizing the heating of the spheroid surface, and $t_s = T_s/T_\infty$, T_s is the average temperature of the spheroid surface defined by the formula

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi c\lambda_0\lambda_g T_\infty}\int_V q_p dV\,.$$

Determination of the Photophoretic Force and Rate. The general solution of the hydrodynamics equations satisfying the finiteness at $\varepsilon \to \infty$ has the form [19]

$$U_\varepsilon(\varepsilon,\eta) = \frac{U_\infty}{c\cosh\varepsilon H_\varepsilon}\cos\eta\,\left\{\lambda A_2 + \left[\lambda - (1+\lambda^2)\,\mathrm{arccot}\,\lambda\right]A_1 + c^2\,(1+\lambda^2)\right\};$$

$$U_\eta(\varepsilon,\eta) = -\frac{U_\infty}{cH_\varepsilon}\sin\eta\left[\frac{A_2}{\lambda} + (1-\lambda\,\mathrm{arccot}\,\lambda)\,A_1 + c^2\lambda\right],$$

$$P_g(\varepsilon,\eta) = P_\infty + c\,\frac{\mu_g U_\infty}{H_\varepsilon^4}\,x\,(\lambda^2 + x^2)\,A_2\,. \tag{14}$$

The integration constants A_1, A_2, and C_3 are determined from the boundary conditions on the spheroid surface, in particular,

$$A_2 = -\frac{2c^2}{\lambda_0 + (1-\lambda_0^2)\,\mathrm{arccot}\,\lambda_0} + K_{t.s}\frac{c\nu_g}{U_\infty t_s}\frac{\lambda_0 - (1+\lambda_0^2)\,\mathrm{arccot}\,\lambda_0}{\lambda_0 + (1-\lambda_0^2)\,\mathrm{arccot}\,\lambda_0}\frac{1-\lambda_0\,\mathrm{arccot}\,\lambda_0}{(1+\lambda_0^2)\,\Delta}$$

$$\times\left[\frac{6}{4\pi c^2\lambda_0\lambda_p T_\infty}\int_V q_p z dV + \frac{\Pr\,\gamma\lambda_0\delta c}{a\,(\lambda_0 + (1-\lambda_0^2)\,\mathrm{arccot}\,\lambda_0)}\left(\lambda_0\,\mathrm{arccot}\,\lambda_0 + 3 - \frac{\mathrm{arccot}^2\,\lambda_0}{1-\lambda_0\,\mathrm{arccot}\,\lambda_0}\right)\right], \tag{15}$$

$$\Delta = (1-\delta)\,\mathrm{arccot}\,\lambda_0 + \frac{\delta\lambda_0}{1+\lambda_0^2} - \frac{1}{\lambda_0}, \quad \delta = \frac{\lambda_g}{\lambda_p}.$$

The total force acting on the spheroid is determined by integrating the stress tensor over the surface of the aerosol particle [20]:

$$F_z = -4\pi\frac{\mu_g U_\infty}{c}A_2\,. \tag{16}$$

It is seen that the total force acting on a large spheroidal solid aerosol particle at small relative temperature drops in its vicinity will be additively composed of the force of the viscous resistance of the medium F_μ, the photophoretic force F_{ph} proportional to the dipole moment J, and the force F_{dh} that is due to the medium motion (i.e., with account for the convective terms in the heat conduction equation):

$$F = F_\mu + F^{(1)}, \quad F^{(1)} = F_{ph} + F_{dh}, \tag{17}$$

where

$$F_\mu = 6\pi a\mu_g U_\infty f_\mu n_z; \quad F_{ph} = -6\pi a\mu_g f_{ph} J n_z; \quad F_{dh} = -6\pi a\mu_g f_{dh} n_z.$$

The values of the coefficients f_μ, f_{dh}, and f_{ph} can be estimated from the following expressions:

$$f_\mu = \frac{4}{3\sqrt{1+\lambda_0^2}}\frac{1}{\lambda_0 + (1-\lambda_0^2)\,\mathrm{arccot}\,\lambda_0},$$

$$f_{ph} = K_{t.s}\frac{\nu_g}{\pi t_s\lambda_p T_\infty}\frac{\lambda_0 - (1+\lambda_0^2)\,\mathrm{arccot}\,\lambda_0}{\lambda_0 + (1-\lambda_0^2)\,\mathrm{arccot}\,\lambda_0}\frac{1-\lambda_0\,\mathrm{arccot}\,\lambda_0}{a^3\lambda_0\Delta},$$

$$f_{dh} = K_{t.s}\frac{\nu_g}{\pi t_s\lambda_p T_\infty}\frac{\lambda_0 - (1+\lambda_0^2)\,\mathrm{arccot}\,\lambda_0}{\lambda_0 + (1-\lambda_0^2)\,\mathrm{arccot}\,\lambda_0}\frac{1-\lambda_0\,\mathrm{arccot}\,\lambda_0}{(1+\lambda_0^2)\,\Delta}$$

$$\times\frac{\Pr}{6a^2(\lambda_0 + (1-\lambda_0^2)\,\mathrm{arccot}\,\lambda_0)}\left(\lambda_0\,\mathrm{arccot}\,\lambda_0 + 3 - \frac{\mathrm{arccot}^2\,\lambda_0}{1-\lambda_0\,\mathrm{arccot}\,\lambda_0}\right)\int_V q_p dV\,.$$

Equating the total force to zero, we obtain the equation for the value of the velocity of ordered motion of the spheroidal particle

$$U_{\mathrm{p}} = -U^{(1)} , \quad U^{(1)} = U_{\mathrm{ph}} + U_{\mathrm{dh}} . \tag{18}$$

Here

$$U_{\mathrm{ph}} = \frac{f_{\mathrm{ph}}}{f_{\mu}} J n_z ; \quad U_{\mathrm{dh}} = \frac{f_{\mathrm{dh}}}{f_{\mu}} n_z .$$

Analysis of the Results Obtained. Formulas (17), (18) permit estimating the influence of the medium motion, i.e., taking into account the convective terms in the heat conduction equation, on the values of the photophoretic force and rate at small relative temperature drops in their vicinity.

To estimate the contribution of the medium motion to the rate of photophoresis of a large spheroidal solid aerosol particle, it is necessary to render concrete the nature of the heat sources nonuniformly distributed in its volume. As an example, consider the simple case where the particle absorbs radiation as a blackbody, i.e., particle heating occurs in a thin layer of thickness $\delta\varepsilon \ll \varepsilon_0$. In this case, the density of heat sources inside a layer of thickness $\delta\varepsilon$ is determined by the formula [23]

$$q_{\mathrm{p}} (\varepsilon, \eta) = \begin{cases} -\dfrac{\cosh \varepsilon \cos \eta}{c (\cosh^2 \varepsilon - \sin^2 \eta) \, \delta\varepsilon} I_0 , & \dfrac{\pi}{2} \le \eta \le \pi , \quad \varepsilon_0 - \delta\varepsilon \le \varepsilon \le \varepsilon_0 ; \\ 0 , & 0 \le \eta \le \dfrac{\pi}{2} , \end{cases}$$

where I_0 is the incident radiation intensity related to the average temperature T_{s} by the relation

$$T_{\mathrm{s}} = T_{\infty} + \frac{a \sqrt{1 + \lambda_0^2}}{4 \lambda_{\mathrm{g}}} I_0 \operatorname{arccot} \lambda_0 .$$

As a result, we obtain the following expressions for the photophoretic force and rate of absolutely black large solid aerosol particles of spheroidal form with and without account for the medium motion, respectively:

$$U_{\mathrm{p}}^{*} = f_{\mathrm{p}}^{*} n_z , \tag{19}$$

$$U_{\mathrm{p}}^{**} = f_{\mathrm{p}}^{**} n_z , \tag{20}$$

$$f_{\mathrm{p}}^{*} = \frac{b}{a} K_{\mathrm{t.s}} \frac{\nu_{\mathrm{g}} I_0}{2 t_{\mathrm{s}} \lambda_{\mathrm{p}} T_{\infty}} \left(\lambda_0 - (1 + \lambda_0^2) \operatorname{arccot} \lambda_0 \right) \frac{\sqrt{1 + \lambda_0^2}}{\Delta} \left(\frac{1}{\lambda_0} - \operatorname{arccot} \lambda_0 \right)$$

$$\times \left[1 - \frac{\mathrm{Pr}}{4} \frac{1}{\sqrt{1 + \lambda_0^2} \, (\lambda_0 - (1 + \lambda_0^2) \operatorname{arccot} \lambda_0)} \left(\lambda_0 \operatorname{arccot} \lambda_0 + 3 - \frac{\operatorname{arccot}^2 \lambda_0}{1 - \lambda_0 \operatorname{arccot} \lambda_0} \right) \right] ,$$

$$f_{\mathrm{p}}^{**} = \frac{b}{a} K_{\mathrm{t.s}} \frac{\nu_{\mathrm{g}} I_0}{2 t_{\mathrm{s}} \lambda_{\mathrm{p}} T_{\infty}} \left(\lambda_0 - (1 + \lambda_0^2) \operatorname{arccot} \lambda_0 \right) \frac{\sqrt{1 + \lambda_0^2}}{\Delta} \left(\frac{1}{\lambda_0} - \operatorname{arccot} \lambda_0 \right) .$$

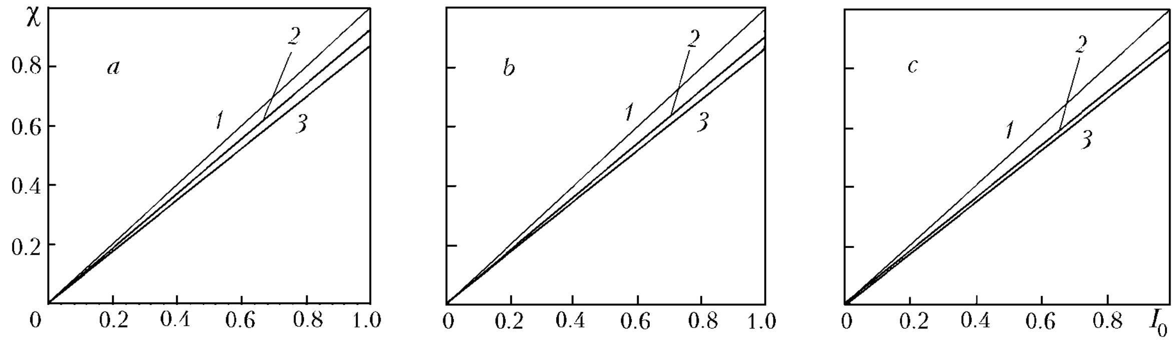

Fig. 1. Ratio $\chi = f_p^*/f_p^{**}$ versus the incident radiation intensity I_0 for borated graphite particles with spheroidal (1, 2) and spherical (3) forms of the surface [1) without account for motion of the medium; 2) with account for motion of the medium): a) $b/a = 0.2$; b) 0.5; c) 0.7. $T_\infty = 300$ K; $P_e = 1$ atm, $\lambda_p = 55$ W/(m·K). I_0, 10^5 W/cm^2.

The influence of the medium motion and of the ratio of the spheroid semiaxes on the photophoresis rate [19] for borated graphite particles suspended in the air is shown in Fig. 1.

Conclusions. Numerical analysis has shown that at a fixed semiaxes ratio with increasing incident radiation intensity I_0, the total contribution of the medium motion leads to a monotonic decrease in the photophoresis rate, and this decrease depends on the equatorial radius of the spheroid.

NOTATION

a, b, spheroid semiaxes; c_{pg}, heat capacity coefficient of the gas; e_η, e_ε, unit vectors of the spheroidal coordinate system; I_0, incident radiation intensity; $K_{t.s}$, coefficient of thermal slip which is determined by the methods of the kinetic theory (at tangential momentum and energy accommodation coefficients equal to unity $K_{t.s} \approx 1.152$ (in the case of the spherical particle) [17, 18]; Kn, Knudsen number; L, characteristic size of the particle; P_g, gas pressure; Pr, Re, Prandtl and Reynolds numbers respectively; q_p, distribution of the density of heat sources inside the particle; r, radius vector marking the position of the particle points whose origin coincides with the particle center; R, radius of the sphere; T_g, T_p, gas and particle temperature, respectively; T_∞, P_∞, gas temperature and pressure at a large distance from the particle; U_ε, U_η, components of the mass velocity of the gas U_g in the spheroidal system of coordinates; V, particle volume; λ, mean free path of the gaseous medium molecules; λ_g, λ_p, heat conductivity coefficients of the gas and particle; ν_g, μ_g, ρ_g, kinematic and dynamic viscosities and density of the gaseous medium. Subscripts: dh, medium motion; g, gas; p, particle; ph, photophoresis; s, values of the quantity at the average temperature of the spheroid surface; ∞, values of the physical quantities at a distance from the particle (on infinity); t.s, thermal slip.

REFERENCES

1. N. A. Fuks, *Mechanics of Aerosols* [in Russian], Izd. AN SSSR, Moscow (1955).
2. G. M. Hidy and J. R. Brock, Photophoresis and the descent of particles into the lower stratosphere, *J. Geophys. Res.*, **12**, 455–460 (1967).
3. V. B. Kutukov, E. R. Shchukin, and Yu. I. Yalamov, On the photophoretic motion of an aerosol particle in the optical radiation field, *Zh. Tekh. Fiz.*, **46**, No. 3, 626–627 (1976).
4. V. B. Kutukov and Yu. I. Yalamov, Transverse photophoretic motion of particles in a laser beam, in: *Proc. 4th All-Union Symp. on Laser Radiation Propagation in the Atmosphere* [in Russian], Tomsk (1972), pp. 145–146.
5. S. P. Lin, On photophoresis, *Coll. Inter. Sci.*, **51**, No. 1, 66–74 (1975).
6. E. R. Shchukin, On the motion of aerosol particles with an inhomogeneous distribution of heat sources in the field of external temperature and concentration gradients, *Zh. Tekh. Fiz.*, **50**, 1332–1335 (1980).

7. S. A. Beresnev, V. G. Chernyak, and G. A. Fomyagin, The kinetic theory of the photophoretic motion of an aerosol particle, *Abstracts of papers of the 14th All-Union Conf. "Urgent Problems of the Physics of Aerodisperse Systems"* [in Russian], Vol. 1, Odessa (1986), P. 64.
8. A. I. Bogoletov and G. G. Bystryi, Experimental study of the photophoretic motion in gases by the method of Knudsen number-based modeling, *Abstracts of papers of the 14th All-Union Conf. "Urgent Problems of the Physics of Aerodisperse Systems"* [in Russian], Vol. 1, Odessa (1986), pp. 153–154.
9. E. R. Shchukin and N. V. Malai, Photophoretic, thermophoretic, and diffusophoretic motion of heated nonvolatile aerosol particles, *Inzh.-Fiz. Zh.*, **54**, No. 4, 628–635 (1988).
10. E. R. Shchukin, N. V. Malai, and Yu. I. Yalamov, Motion of drops heated by internal heat sources in binary gas mixtures, *Teplofiz. Vys. Temp.*, **28**, No. 5, 1020–1024 (1988).
11. A. I. Bogoletov, P. E. Suetin, S. A. Beresnev, G. P. Bystryi, and V. G. Chernyak, Experimental and theoretical investigation of the photophoresis in a rarefied gas, *Teplofiz. Vys. Temp.*, **29**, No. 4, 750–758 (1991).
12. A. I. Bogoletov, G. P. Bystryi, S. A. Beresnev, V. G. Chernyak, and G. D. Fomyagin, Photophoresis of model aerosol particles, *Teplofiz. Vys. Temp.*, **34**, No. 5, 751–756 (1996).
13. Z. L. Shulimanova, E. R. Shchukin, and T. M. Eremchuk, On the photophoresis of a solid moderate spherical particle with the thermal conductivity depending on the radial coordinate, *Pis'ma Zh. Tekh. Fiz.*, **22**, Issue 18, 33–36 (1996).
14. S. Tehranian, F. Giovane, J. Blum, Y. L. Xu, and B. A. S. Gustafson, Photophoresis of micrometer-sized particles in the free-molecular regime, *Int. J. Heat Mass Transfer*, **44**, 1649–1657 (2001).
15. S. A. Beresnev, F. D. Kovalev, L. B. Kochneva, V. A. Runkov, P. E. Suetin, and A. A. Cheremisin, On the possibility of photophoretic levitation of particles in the stratosphere, *Opt. Atmos. Okeana*, **16**, No. 1, 52–57 (2003).
16. S. A. Beresnev and L. B. Kochneva, The factor of radiation absorption asymmetry and the photophoresis of aerosols, *Fiz. Atmos. Okeana*, **16**, No. 2, 134–141 (2003).
17. N. V. Malai, E. R. Shchukin, A. A. Pleskanev, and A. A. Stukalov, Characteristic features of the photophoretic motion of moderate aerosol spherical particles, *Opt. Atmos. Okeana*, **19**, No. 5, 413–418 (2006).
18. T. M. Eremchuk, *Thermo-, Photo-, and Diffusiophoretic Motion of Large and Moderate Inhomogeneous (as to the Thermophysical Properties) Spherical and Cylindrical Particles*, Candidate Dissertation (in Physics and Mathematics), Moscow (2001).
19. C. W. Oseen, *Hydrodinamik*, Akademische Verlag, Leipzig (1927).
20. I. Praudman and J. R. A. Pearson, Expansion at small Reynolds number for the flow past a sphere and a circular cylinder, *J. Fluid. Mech.*, **2**, 237–262 (1957).
21. A. Acrivos and T. D. Taylor, Heat and mass transfer from single spheres in Stokes flow, *J. Phys.*, **5**, No. 4, 387–394 (1962).
22. A. B. Poddoskin, A. A. Yushkanov, and Yu. I. Yalamov, The theory of thermophoresis of moderate aerosol particles, *Zh. Tekh. Fiz.*, **52**, Issue 11, 2253–2261 (1982).
23. J. Happel and H. Brenner, *Low Reynolds Number with Special Applications to Particular Media* [Russian translation], Mir, Moscow (1976).