POLARIZATION OF TRANSITION RADIATION ON SOME SORTS OF TARGETS

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The problem of transition radiation under impact of relativistic particles under small angle to the atomic string of a crystal is considered. The conditions under which the non-uniformity of the electron density along the string is not substantial are obtained. In this case the problem of transition radiation is reduced to the problem of the particle radiation on the thin fiber-like dielectric target. The formulae for the spectral-angular distribution of transition radiation under both regular and random collisions of a particle with the set of fiber-like targets are obtained. The radiation of the particle on a single atomic plane and on a set of planes in crystal is also considered.

1. INTRODUCTION

The transition radiation arises under crossing by charged particle the boundary between two media with different dielectric properties (see [1-4] and references in them). For relativistic particle this radiation is concentrated in the region of small angles along the direction of the particle motion. The process of radiation develops in large spatial region along the particle velocity, that is called as the coherence length [2,5,6]. If the particle in the limits of this region crosses some boundaries of different media, the interference of radiation emitted under crossing of every boundary is substantial. It was shown in [7] that for long waves not only longitudinal, but also transverse dimensions of the region of radiation formation could have macroscopic sizes. If the transverse size of the target satisfies the condition $L_\perp \le \gamma\lambda$, where λ is the length of the radiated wave and $\gamma = (1-v^2)^{-1/2}$ is the particle's Lorentz-factor (we use the system of units, in which the velocity of light is equal to unit), than the transverse sizes of the target and its geometrical shape make substantial influence on the transition radiation.

In the presented paper the problems on transition radiation by relativistic particle on dielectric targets of some particular geometries are considered. We consider the region of high frequencies of radiated waves, for which the dielectric function of the medium can be presented in the form:

$$\varepsilon_\omega \approx 1 - \frac{\omega_p^2}{\omega^2}, \quad \omega > \omega_p, \qquad (1)$$

where $\omega_p = \sqrt{4\pi e^2 n_e(\vec{r})/m}$ is the plasma frequency, m is the electron mass, $n_e(r)$ is the electron density in the target.

2. TRANSITION RADIATION IN THE FRAMES OF PERTURBATION THEORY

For the case of high frequencies of radiated waves, $\omega >> \omega_p$, the second term in the dielectric function of the medium (1) can be considered as a small perturbation. In the frameworks of perturbation theory which was built, the spectral-angular density of radiation with given polarization is determined by relation

$$\frac{dE}{d\omega\, do} = \frac{\omega^2}{4\pi^2}\left|\vec{e}\vec{I}\right|^2, \qquad (2)$$

where e is the polarization vector, $e \perp k$, $|e| = 1$, k is the wave vector of the radiated wave, and the value I in our case can be written in the form

$$\vec{I} \approx \frac{1}{4\pi\omega}\int d^3r e^{i\vec{k}\vec{r}}\omega_p^2(\vec{r})\vec{E}_\omega^{(0)}(\vec{r}), \qquad (3)$$

where $E_\omega^{(0)}(r)$ is the Fourier component of the non-disturbed Coulomb field of the uniformly moving relativistic particle,

$$\vec{E}_\omega^{(0)}(\vec{r}) = \int \frac{d^3\kappa}{\pi} ie \frac{\vec{\kappa} - \omega\vec{v}}{\omega^2 - \kappa^2}\delta(\omega - \vec{\kappa}\vec{v})e^{i\vec{\kappa}\vec{r}}. \qquad (4)$$

The spectral-angular distribution of radiation summed over polarizations is determined by equation

$$\frac{dE}{d\omega\, do} = \frac{1}{4\pi^2}\left|\vec{k}\times\vec{I}\right|^2. \qquad (5)$$

Using the Fourier transformation of the electron density distribution $n_e(r)$,

$$n_q = \int d^3r n_e(r) e^{-iqr},$$

we can rewrite (5) in the form

$$\frac{dE}{d\omega\, do} = \frac{e^6}{m^2}\left|\frac{k}{\omega}\times\vec{J}_k\right|^2, \qquad (6)$$

where

$$\vec{J}_k = \int \frac{d^3q}{2\pi^2} n_q \frac{k - q - \omega\vec{v}}{\omega^2 - (k-q)^2}\delta(\omega - (\vec{k} - \vec{q})\vec{v}). \qquad (7)$$

3. TRANSITION RADIATION ON A DIELECTRIC FIBER AND ATOMIC STRING

Let us consider the transition radiation by relativistic particle incident on a thin dielectric fiber under small angle $\psi << 1$ to its axis . The atomic string in crystal [6] or nanotube [8] can be treated as such fiber in the case when the length of radiation formation (the coherence length) exceeds in much the atomic string thickness along the particle motion direction:

$$l_{coh} \sim \frac{2\gamma^2}{\omega} \frac{1}{1+\gamma^2\theta^2} >> \frac{2R}{\psi}, \qquad (8)$$

where R is the screening radius of the atomic potential (Thomas-Fermi radius), θ is the radiation angle (the angle between the wave vector k of the radiated wave and the particle velocity).

Let us take the electron density distribution in the fiber in Gaussian form,

$$n_e(\vec{r}) = \frac{n_e}{2\pi R^2} \exp\left[-\frac{(x-\psi z)^2 + (y-y_0)^2}{2R^2} \right], \qquad (9)$$

where z axis is directed along the particle velocity v, the axis of the fiber is parallel to (x, z) plane, y_0 is the distance between the particle trajectory and the axis of the fiber, n_e is the electron density per unit length of the fiber. The Fourier transformation of this distribution has the form:

$$n_q = 2\pi\, n_e e^{iq_y y_0} \delta\,(q_x tg\psi + q_z) \times$$
$$\times \exp\left[-\frac{(q_x^2 + q_y^2)R^2}{2} \right]. \qquad (10)$$

The spectral-angular distribution of radiation (6) must be averaged over all possible values of the impact parameter y_0,

$$\left\langle \frac{dE}{d\omega\, do} \right\rangle = \frac{1}{a_y} \int_{-\infty}^{\infty} dy_0 \frac{dE(y_0)}{d\omega\, do}, \qquad (11)$$

where a_y is the distance between the atomic strings in the crystal along the y axis. After substituting (7) with n_q determined by (10) into (6) and averaging according to (11), one can demonstrate that for small radiation angles ($\theta << 1$) and for small angles of incidence ($\psi << 1$) the condition (8) leads to the possibility to neglect the second exponential factor in (10), that corresponds to the case of zero thickness of the fiber. In other words, the details of the electron density distribution in the fiber are not substantial under such conditions.

For the case of infinitely thin dielectric fiber we obtain:

$$\left\langle \frac{dE}{d\omega\, do} \right\rangle = \frac{e^6 n_e^2 \gamma}{a_y m^2 \omega \psi^2} F(\theta, \varphi), \qquad (12)$$

where

$$F(\theta,\varphi) = \frac{1 + 2\left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\gamma\psi} \right)^2}{\left[1 + \left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\gamma\psi} \right)^2 \right]^{3/2}}, \qquad (13)$$

φ is the azimuth angle (the angle between the x axis and the projection of the wave vector k onto the plane (x, y)). This function is plotted on the Fig. 1. One can see that the angular distribution possesses the axial symmetry relatively to the axis of the fiber ($\theta = \psi$, $\varphi = 0$) (it can be easy shown analytically from (13)). Near the axis of symmetry of the angular distribution the intensity has rather high level, but with incraesing of the incidence angle ψ the minimum in the center developes itself. For $\psi \sim 10\gamma^{-1}$ the distribution has the shape of narrow double ring.

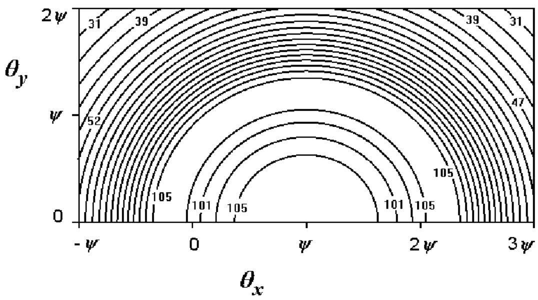

Fig. 1. *Surface plot (view from above) of the function* $100 \cdot F(\theta,\varphi)$ *for* $\psi = 10^{-3}$, $\gamma = 2000$ *(*$\theta_x = \theta\cos\varphi$, $\theta_y = \theta\sin\varphi$*). The lines of equal level of the surface are shown*

Choosing the polarization vectors in the form

$$\vec{e}^{(1)} = \frac{k \times e_x}{|k \times e_x|},\ \vec{e}^{(2)} = \frac{k \times e^{(1)}}{\omega}, \qquad (14)$$

we find that radiation is partially polarized in $e^{(1)}$ direction with polarization

$$P = \frac{1}{1 + 2\left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\gamma\psi} \right)^2}. \qquad (15)$$

This function is plotted on Fig. 2.

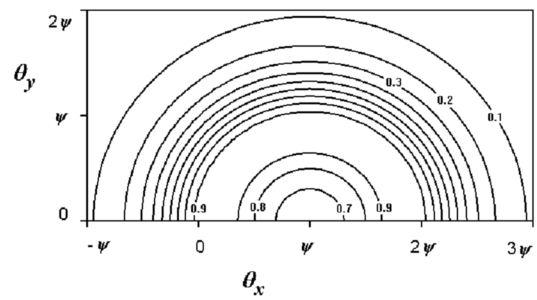

Fig. 2. *Polarization of radiation determined by Eq. (15) at the same conditions as in Fig. 1*

When our particle moves through the cristall it collides with the periodical set of parallel atomic strings. The motion of the particle in this case can be both regular and chaotic [9]. If the particle moves chaotically in the plane perpendicular to atomic strings (under the constant incidence angle ψ), we can neglect the interference of radiation produced by interaction of the particle with different strings. In this case the symmetry of angular distribution of radiation intensity, described above, leads to the same form of the angular distribution (13), as in the case of the single string. The total intensity of radiation will be proportional to the number of strings under collision.

If the particle motion in the crystal is regular, the account of interference effects is necessary.

4. TRANSITION RADIATION ON ATOMIC PLANE IN A CRYSTAL

Now let us consider the transition radiation of the particle on the atomic plane in crystal in the case when this atomic plane can be treated as a uniform thin dielectric plate. The problem of radiation on a plate is not new in the theory of transition radiation (see for example [1-3]), but our perturbation theory permits to obtain results in a rather simple way.

The Fourier transformation of the electron density distribution under condition (8) can be written in the form

$$n_q = (2\pi)^2 n_e \delta(q_y)\delta(q_x tg\psi + q_z)\,. \qquad (16)$$

Here n_e denotes the electron density per unit area of the plane. Substitution (16) into general formulae (6), (7) gives us the following result for the spectral-angular distribution of transition radiation:

$$\frac{d\mathrm{E}}{d\omega\, do} = \frac{4e^6 n_e^2 \gamma^2}{m^2 \omega^2 \psi^2} F_p(\theta,\varphi)\,, \qquad (17)$$

where

$$F_p(\theta,\varphi) = \frac{(\gamma\theta\sin\varphi)^2 + \left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}{\left[1 + (\gamma\theta\sin\varphi)^2 + \left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2\right]^2} \,. \qquad (18)$$

The angular distribution (18) is plotted on the Fig. 3 for the case $\psi = 4\gamma^{-1}$. One can see that the distribution is symmetrical relatively to the reflection by the atomic plane. The shape of distribution looks like two empty cones, one of which is directed almost along the particle velocity, and the second is "reflected" by the atomic plane.

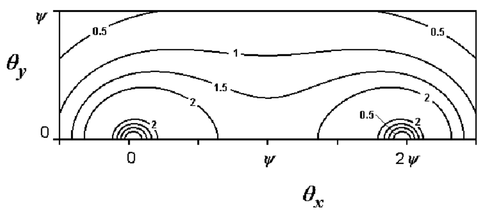

Fig. 3. *3-dimentional plot of the function* $F_p(\theta,\varphi)$ *(18)* *(*$\theta_x = \theta\cos\varphi$*,* $\theta_y = \theta\sin\varphi$ *) for* $\psi = 2\cdot 10^{-3}$, $\gamma = 2000$

The polarization of radiation is determined by Stocks parameters:

$$\xi_1 = \frac{2\gamma\theta\sin\varphi\left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\psi\gamma}\right)}{\gamma^2\theta^2 - \theta\cos\varphi\,\dfrac{1+\gamma^2\theta^2}{\psi} + \left(\dfrac{1+\gamma^2\theta^2}{2\psi\gamma}\right)^2}\,,$$

$$\xi_3 = \frac{\gamma^2\theta^2\sin^2\varphi - \left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\psi\gamma}\right)^2}{\gamma^2\theta^2 - \theta\cos\varphi\,\dfrac{1+\gamma^2\theta^2}{\psi} + \left(\dfrac{1+\gamma^2\theta^2}{2\psi\gamma}\right)^2}\,. \qquad (19)$$

The radiation is linearly polarized with polarization $P = \sqrt{\xi_1^2 + \xi_3^2}$. It is easy can be seen that for Stocks parameters (19) we have $P = 1$ for all values of θ and φ. The angle between vector $e^{(1)}$ and direction of maximum polarization α is determined in our case of 100% linear polarization by relation $\xi_1 = \sin 2\alpha$, $\xi_3 = \cos 2\alpha$ (see Fig. 4).

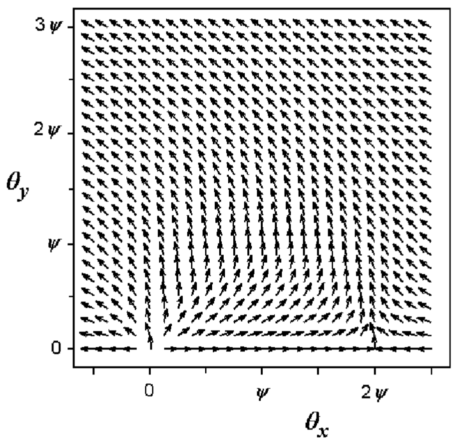

Fig. 4. *Vector plot of polarization of radiation at the same conditions as on Fig. 3. Directions of arrows indicate the direction of maximal polarization (respectively to* $e^{(1)}$ *direction)*

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