ISSN 1027-4510, Journal of Surface Investigation: X-ray, Synchrotron and Neutron Techniques, 2020, Vol. 14, No. 5, pp. 922–928. © Pleiades Publishing, Ltd., 2020.
Russian Text © The Author(s), 2020, published in Poverkhnost', 2020, No. 9, pp. 66–72.

Effect of the Divergence of a Relativistic Electron Beam on the Diffracted Transition Radiation Excited by Them in a Single-Crystal Target

S. V. Blazhevich[a], M. V. Bronnikova[a], and A. V. Noskov[b, *]

[a] *Belgorod State University, Belgorod, 308015 Russia*

[b] *Belgorod State Technological University named after V.G. Shukhov, Belgorod, 308012 Russia*

**e-mail: noskovbupk@mail.ru*

Received December 14, 2019; revised January 23, 2020; accepted January 25, 2020

Abstract—The dependence of the angular density and the photon yield of collimated diffracted transition radiation (DTR) generated in a thin single-crystal plate by a beam of relativistic electrons on the beam divergence are studied. An expression describing the DTR angular density averaged over all the rectangular trajectories of electrons in the beam is derived. For the averaging, the two-dimensional Gaussian distribution is used. A significant dependence of the angular density of collimated DTR photons on the electron-beam divergence is shown. An expression describing the number of collimated DTR photons is obtained. The significant dependence of the photon number of the collimated DTR is shown.

Keywords: beam of relativistic electrons, beam divergence, diffracted transition radiation, single crystal, angular density

DOI: 10.1134/S1027451020040230

1. INTRODUCTION

When a charged particle crosses the input surface of a crystal plate, transition radiation (TR) occurs [1], which diffracts at a system of parallel atomic planes of the crystal, forming diffracted transition radiation (DTR) [2, 3]. When a charged particle crosses a crystalline plate, its Coulomb field is scattered at a system of parallel atomic planes of the crystal, generating parametric X-ray radiation (PXR) [4, 5]. In the general case of the asymmetric reflection of radiation from a plate, when the diffracting atomic planes make an arbitrary angle with the surface, dynamic effects for PXR and DTR were considered in [6–8], in which it was shown that by changing the asymmetry of reflection, one can significantly increase the radiation yields.

In this work, we study the possibilities of using coherent radiation excited by electrons in a single-crystal target to indicate the parameters of accelerated electron beams. The relevance of this problem is associated with the need to provide reliable information on the parameters of electron beams of various energies during fundamental and applied research conducted using modern accelerator installations.

When conducting fundamental and applied experimental studies using electron beams of different energies, scientists are faced with the problem of insufficient information on the parameters of beams. The important parameters of a beam are its transverse dimensions and angular divergence. The main problem for physicists working with beams of relativistic electrons in the energy range 100–1000 MeV is the provision of measurements of the transverse dimensions of the beam, since the angular divergence on modern electron accelerators is on the order of 0.001 mrad, which is not significant for electron beams with dimensions on the order of more than ten microns.

At present, two linear electron–positron colliders are being designed [9, 10]. In these installations, electrons and positrons will be accelerated to an energy of 250 GeV. The transverse dimensions of the beam are assumed to be very small (~5–100 nm) and the main problem will be measurement of the angular divergence. Obviously, the measurement process should not significantly affect the measured parameters of the beam, therefore, for an indication, it is necessary to use processes that would have a minimal effect on the measured parameters. The solution to this problem will allow a more accurate interpretation of the experimental data in fundamental and applied research.

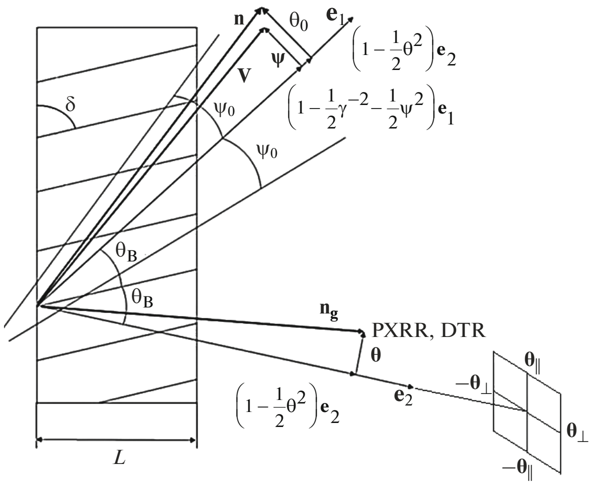

Fig. 1. Geometry of the emission process.

The expressions obtained in [8] show that the angular density of the DTR depends on the divergence of the electron beam to a greater extent than the angular density of the PXR. This is due to the fact that for electron beams with energies exceeding several hundred MeV, the emitted DTR photons have a smaller angular spread than PXR photons and, as a result, the angular density of the DTR becomes more sensitive to the beam divergence. It should be noted that with a further increase in the relativistic-electron energy, the angular distribution of DTR photons narrows, and the width of the angular distribution of PXR photons reaches saturation and ceases to change.

In this paper, we study the diffracted transition radiation of relativistic electrons in a single-crystal target in the Laue scattering geometry. Radiation is considered for the case of a very thin target, when the multiple scattering of electrons by target atoms is negligible, which is important when measuring the divergence of an electron beam, since it provides a sufficiently small change in the process of measurement. An expression is obtained that describes the number of DTR photons emitted at a given solid angle. It has been shown that the collimated DTR generated in a single crystal substantially depends on the electron-beam divergence.

2. GEOMETRY OF THE EMISSION PROCESS

Let a beam of relativistic electrons intersect a single crystal plate in the Laue scattering geometry (Fig. 1). We introduce the angular variables $\boldsymbol{\psi}$, $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_0$ in accordance with the definitions of the speed of a relativistic electron $\mathbf{V}$ (in units of the speed of light in free space) and unit vectors in the direction of a photon emitted close to the speed of an electron $\mathbf{n}$, and a photon emitted near the Bragg scattering direction $\mathbf{n_g}$

$$\mathbf{V} = \left(1-\frac{1}{2}\gamma^{-2}-\frac{1}{2}\psi^2\right)\mathbf{e}_1 + \boldsymbol{\psi}, \quad \mathbf{e}_1\boldsymbol{\psi} = 0,$$
$$\mathbf{n} = \left(1-\frac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \boldsymbol{\theta}_0, \quad \mathbf{e}_1\boldsymbol{\theta}_0 = 0, \quad \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B, \quad (1)$$
$$\mathbf{n_g} = \left(1-\frac{1}{2}\theta^2\right)\mathbf{e}_2 + \boldsymbol{\theta}, \quad \mathbf{e}_2\boldsymbol{\theta} = 0,$$

where $\boldsymbol{\psi}$ is the angle of deviation of an electron in the beam, measured from the axis electron beam $\mathbf{e}_1$, $\boldsymbol{\theta}_0$ is the angle between the axis $\mathbf{e}_1$ and the direction $\mathbf{n}$ of TR incident photon propagation, $\boldsymbol{\theta}$ is the angle between the direction $\mathbf{e}_2$ of the Bragg reflection of a photon incident along the axis of the electron beam and the propagation direction of the diffracted photon $\mathbf{n_g}$ (emission angle), $\gamma = 1\big/\sqrt{1-V^2}$ is the Lorentz factor of a particle, and $\mathbf{e}_1$ and $\mathbf{e}_2$ are the unit vectors.

In Fig. 1, ψ_0 is the angular divergence of the electron beam. Angle ψ_0 defines a cone bounding part of the electron beam, beyond which the electron density decreases by more than e times compared with the density on the beam axis. The angular variables are decomposed into components parallel and perpendicular to the plane of the figure $\boldsymbol{\theta} = \boldsymbol{\theta}_\parallel + \boldsymbol{\theta}_\perp$, $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_{0\parallel} + \boldsymbol{\theta}_{0\perp}$, $\boldsymbol{\psi} = \boldsymbol{\psi}_\parallel + \boldsymbol{\psi}_\perp$.

3. INFLUENCE OF DIVERSITY OF THE ELECTRON BEAM ON THE DTR

We use the formula obtained in [11], which describes the spectral-angular density of the DTR for a single crystal of arbitrary thickness:

$$\omega\frac{d^2N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\Omega^{(s)2} \times\left(\frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_\parallel+\psi_\parallel)^2} - \frac{1}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_\parallel+\psi_\parallel)^2-\chi_0'}\right)^2 R_{\mathrm{DTR}}^{(s)}, \quad (2a)$$

$$R_{\mathrm{DTR}}^{(s)} = \frac{\varepsilon^2}{\xi(\omega)^2+\varepsilon} \times\left[\exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(1)}\right)+\exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(2)}\right) - 2\exp\left(-b^{(s)}\rho^{(s)}\frac{1+\varepsilon}{\varepsilon}\right)\cos\left(\frac{2b^{(s)}\sqrt{\xi^{(s)2}+\varepsilon}}{\varepsilon}\right)\right], \quad (2b)$$

where

$$\Omega^{(1)} = \theta_\perp - \psi_\perp, \quad \Omega^{(2)} = \theta_\parallel + \psi_\parallel,$$
$$C^{(1)} = 1, \quad C^{(2)} = \cos 2\theta_B,$$
$$\Delta^{(2)} = \frac{\varepsilon+1}{2\varepsilon} + \frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}} + \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}},$$
$$\Delta^{(1)} = \frac{\varepsilon+1}{2\varepsilon} - \frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}} - \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}},$$
$$\varepsilon = \frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)}, \quad \kappa^{(s)} = \frac{\chi''_{\mathbf{g}} C^{(s)}}{\chi''_0}, \quad \rho^{(s)} = \frac{\chi''_0}{\left|\chi'_{\mathbf{g}}\right| C^{(s)}}, \tag{3}$$
$$b^{(s)} = \frac{1}{2\sin(\delta-\theta_B)}\frac{L}{L_{\text{ext}}^{(s)}}, \quad L_{\text{ext}}^{(s)} = \frac{1}{\omega\left|\chi'_{\mathbf{g}}\right| C^{(s)}},$$
$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1-\varepsilon}{2\nu^{(s)}},$$
$$\eta^{(s)}(\omega) = \frac{2\sin^2\theta_B}{V^2\left|\chi'_{\mathbf{g}}\right| C^{(s)}}\left(\frac{\omega(1-\theta_\parallel\cot\theta_B)}{\omega_B} - 1\right),$$
$$\chi'_{\mathbf{g}} = \chi'_0 \left(F(g)/Z\right)\left(S(\mathbf{g})/N_0\right)\exp\left(-g^2u_\tau^2/2\right),$$

where $\chi_0 = \chi'_0 + i\chi''_0$ is the average dielectric susceptibility, $F(g)$ is the form factor of an atom containing Z electrons, $S(\mathbf{g})$ is the structural factor of a unit cell containing N_0 atoms, and u_τ is the root-mean-square amplitude of the thermal vibrations of atoms of the crystal. The work deals with the range of X-ray frequencies ($\chi'_{\mathbf{g}} < 0, \chi'_0 < 0$).

At $s = 1$ expressions (2) describe σ polarized waves, and when $s = 2$ π polarized waves. Since the electromagnetic field emitted by a relativistic electron is transverse in the X-ray frequency range, the incident $\mathbf{E}(\mathbf{k},\omega)$ and diffracted $\mathbf{E}(\mathbf{k}+\mathbf{g},\omega)$ waves are determined by two amplitudes with different values of the transverse polarization:

$$\begin{aligned}\mathbf{E}(\mathbf{k},\omega) &= E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)},\\ \mathbf{E}(\mathbf{k}+\mathbf{g},\omega) &= E_{\mathbf{g}}^{(1)}(\mathbf{k},\omega)\mathbf{e}_{\mathbf{g}}^{(1)} + E_{\mathbf{g}}^{(2)}(\mathbf{k},\omega)\mathbf{e}_{\mathbf{g}}^{(2)},\end{aligned} \tag{4}$$

where the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to the vector $\mathbf{k} = k\mathbf{n}$, and vectors $\mathbf{e}_{\mathbf{g}}^{(1)}$ and $\mathbf{e}_{\mathbf{g}}^{(2)}$ are perpendicular to the vector $\mathbf{k}_{\mathbf{g}} = \mathbf{k} + \mathbf{g} = k_{\mathbf{g}}\mathbf{n}_{\mathbf{g}}$. The vectors $\mathbf{e}_0^{(2)}$ and $\mathbf{e}_{\mathbf{g}}^{(2)}$ lie in the plane of vectors $\mathbf{k}$ and $\mathbf{k}_{\mathbf{g}}$ (π polarization), and the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_{\mathbf{g}}^{(1)}$ perpendicular to it (σ polarization).

Parameter ε determines the degree of asymmetry of the reflection of the electron field relative to the target surface. We note that the angle of incidence of the electron on the target surface $(\delta - \theta_B)$ decreases with increasing parameter ε. Parameter $b^{(s)}$ is equal to half the electron path in the target $L_e = L/\sin(\delta-\theta_B)$, expressed in X-ray extinction lengths in a single crystal $L_{\text{ext}}^{(s)}$.

Expressions (2a) and (2b) describe the spectral-angular density of the DTR of a relativistic electron crossing the single-crystal plate, taking into account the deviation of the electron velocity direction (angle $\boldsymbol{\psi}(\psi_\perp, \psi_\parallel)$) relative to the axis of the electron beam $\mathbf{e}_1$. Expressions were obtained in the framework of the two-wave approximation of the dynamic theory of diffraction for the general case of the asymmetric reflection of radiation waves, when the angle between the reflecting system of parallel atomic planes of the crystal and the target surface (angle δ) can take on arbitrary values within the framework of Laue geometry.

We consider the emission of a beam of relativistic electrons in a thin crystal, that is, under the condition that the largest path length of the diffracted photon in the plate $L_f = L/\sin(\delta+\theta_B)$ will be much less than the absorption length of X-ray waves in the crystal $L_{\text{abs}} = 1/\omega\chi''_0$:

$$2\frac{b^{(s)}\rho^{(s)}}{\varepsilon} = \frac{L_f}{L_{\text{abs}}} \ll 1. \tag{5}$$

To fulfill this condition, as well as the conditions for the small influence of the indication process on the parameters of the electron beam (the angle of multiple scattering in the target is many times smaller than the initial divergence of the beam) in the X-ray range of radiation frequencies, the target thickness should be less than or on the order of tens of micrometers. In this case, the choice of the target thickness will depend on the density of the target substance, the energy of relativistic electrons, and the energy of the emitted DTR photons. In this case, from the expression (2b) it follows:

$$R_{\text{DTR}}^{(s)} = \frac{4\varepsilon^2}{\xi^{(s)2}+\varepsilon}\sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)2}+\varepsilon}}{\varepsilon}\right). \tag{6}$$

We find the angular density of the DTR by integrating expressions (2a) in frequency ω, using the ratio $\frac{d\omega}{\omega} = \frac{\left|\chi'_{\mathbf{g}}\right| C^{(s)}}{2\sin^2\theta_B} d\xi^{(s)}$, which follows from the expression for $\xi^{(s)}(\omega)$ in (3). The angular density of the DTR takes the form:

$$\frac{dN_{\rm DTR}^{(s)}}{d\Omega} = \frac{e^2}{8\pi^2}\frac{\left|\chi'_{\mathbf{g}}\right|C^{(s)}}{\sin^2\theta_{\rm B}}\Omega^{(s)2}\left(\frac{1}{\Omega} - \frac{1}{\Omega - \chi'_0}\right)^2 \times \int_{-\infty}^{\infty} R_{\rm DTR}^{(s)} d\xi^{(s)}(\omega), \quad (7)$$

where $\Omega = \gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2$.

We integrate the spectral function in (7) using in the case of $b^{(s)} \ll \sqrt{\varepsilon}$ the approximation: $\int_{-\infty}^{\infty} R_{\rm DTR}^{(s)} d\xi^{(s)}(\omega) \approx 4\pi\varepsilon b^{(s)}$.

Condition $b^{(s)} \ll \sqrt{\varepsilon}$ corresponds to the case when the path length of an electron in a single crystal is substantially less than the extinction length of X-ray waves. As a result, we obtain the expression

$$\frac{dN_{\rm DTR}^{(s)}}{d\Omega} = \frac{e^2\omega_{\rm B}\chi_0'^2\chi_{\mathbf{g}}'^2 C^{(s)2}}{4\pi\sin^2\theta_{\rm B}}\frac{\Omega^{(s)2}}{(\Omega - \chi'_0)^2\Omega^2}\frac{\varepsilon L}{\sin(\delta - \theta_{\rm B})}. \quad (8)$$

We study the dependence of the yield of DTR photons in the collimator on the initial divergence of the electron beam in order to use it to indicate the divergence of high and ultrahigh-energy electron beams. To do this, we average the expression for the angular density of the DTR (8) over the possible rectilinear paths of the electron in the beam. We average over the symmetric Gaussian distribution function: $f(\psi) = \frac{1}{\pi\psi_0^2}\exp\left[-\frac{\psi_\perp^2 + \psi_\parallel^2}{\psi_0^2}\right]$. Adding the angular densities (8) for two polarizations and averaging over $f(\psi)$, we obtain an expression describing the angular density of the DTR excited by a single electron, averaged over all possible rectilinear paths of the electron in the beam:

$$\left\langle\frac{dN_{\rm DTR}}{d\Omega}\right\rangle = \frac{e^2\omega_{\rm B}\chi_0'^2\chi_{\mathbf{g}}'^2}{4\pi\sin^2\theta_{\rm B}}\frac{\varepsilon L}{\sin(\delta - \theta_{\rm B})}Y_{\rm DTR}, \quad (9a)$$

where

$$Y_{\rm DTR} = \frac{1}{\pi\psi_0^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\left[(\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2\cos^2 2\theta_{\rm B}\right]\exp\left[-\frac{\psi_\perp^2 + \psi_\parallel^2}{\psi_0^2}\right]d\psi_\perp d\psi_\parallel}{(\Omega - \chi'_0)^2\Omega^2}. \quad (9b)$$

Function $Y_{\rm DTR}$ characterizes the effect of the electron beam divergence ψ_0 on the angular density of the DTR.

By integrating over the emission angles ($-\theta'_\perp \le \theta_\perp \le \theta'_\perp$, $-\theta'_\parallel \le \theta_\parallel \le \theta'_\parallel$) the angular density of the DTR (9a) incident on a rectangular collimator with dimensions of $2\theta'_\perp \times 2\theta'_\parallel$, we obtain an expression describing the number of photons of the collimated DTR, excited by a beam of relativistic electrons, normalized to the number of electrons in the beam:

$$N_{\rm DTR} = \frac{e^2\omega_{\rm B}\chi_0'^2\chi_{\mathbf{g}}'^2}{4\pi\sin^2\theta_{\rm B}}\frac{\varepsilon L}{\sin(\delta - \theta_{\rm B})}F_{\rm DTR}(\psi_0), \quad (10a)$$

$$F_{\rm DTR}(\psi_0) = \frac{1}{\pi\psi_0^2}\int_{-\theta'_\parallel}^{\theta'_\parallel}\int_{-\theta'_\perp}^{\theta'_\perp}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\left[(\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2\cos^2 2\theta_{\rm B}\right]\exp\left[-\frac{\psi_\perp^2 + \psi_\parallel^2}{\psi_0^2}\right]d\psi_\perp d\psi_\parallel d\theta_\perp d\theta_\parallel}{(\Omega - \chi'_0)^2\Omega^2}. \quad (10b)$$

Function $F_{\rm DTR}(\psi_0)$ describes the dependence of the number of DTR photons emitted to the collimator on the divergence of the electron beam ψ_0 and the size of the rectangular collimator $\pm\theta'_\perp$ and $\pm\theta'_\parallel$. Formula (10) for the given collimator defines the unique relationship between the number $N_{\rm DTR}$ of photons emitted into the collimator, and the divergence of the electron beam.

Let us consider the dependences of the angular density of the DTR and the number of photons of the collimated DTR on the divergence of the electron beam. For definiteness, we set $\chi'_0 = -10^{-5}$, whose value is determined by the radiation frequency and the target material. Figure 2 shows the curves of the angular distribution of the density of diffraction pulses constructed according to formula (9b) for beams of relativistic electrons with different divergence parameters

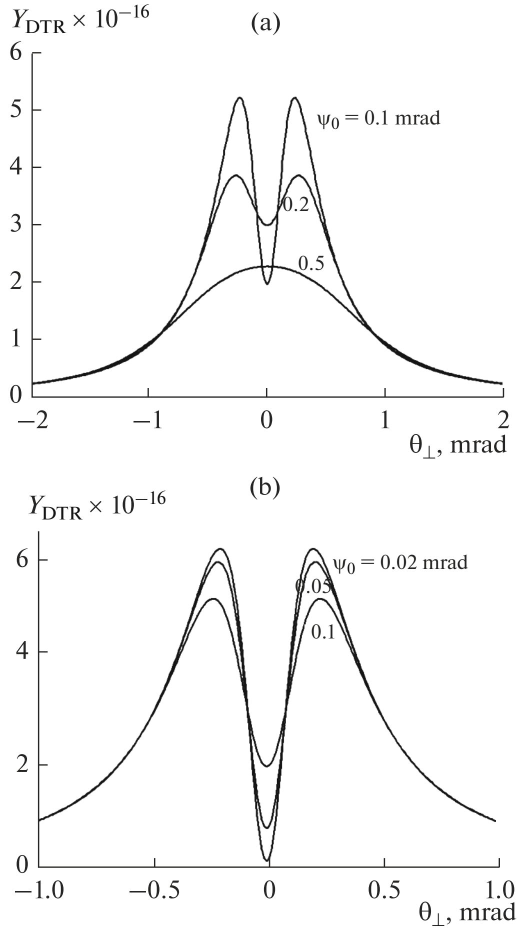

Fig. 2. (a) Dependence of the angular density of DTR on the divergence ψ_0 of the electron beam. $\theta_{||} = 0$, $\gamma = 5000$. $\psi_0 = 0.1$ mrad $< 1/\gamma = 0.2$ mrad, $\psi_0 = 1/\gamma = 0.2$ mrad and $\psi_0 = 0.5$ mrad $> 1/\gamma = 0.2$ mrad. (b) Same as in (a) but with less divergence ψ_0. $\psi_0 < 1/\gamma = 0.2$ mrad.

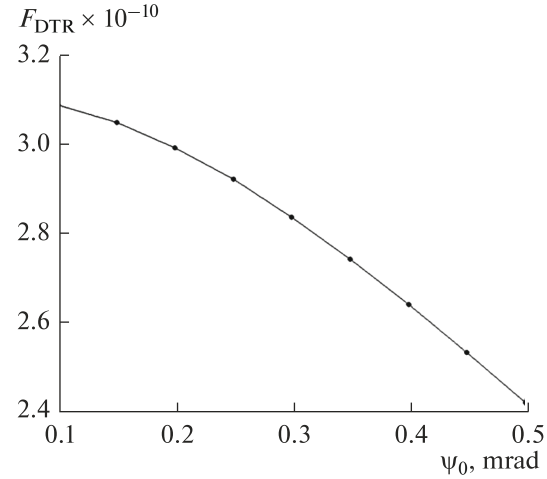

Fig. 3. Dependence of the number of DTR photons emitted to the collimator on the divergence of the electron beam ψ_0. Collimator size $\theta'_\perp = \theta'_{||} = 3\gamma^{-1}$, $\gamma = 5000$.

ψ_0. The curves constructed for the relativistic electron energy $E \approx 2.55$ GeV ($\gamma = 5000$) at a fixed value of the component of the angle of emission (observation) $\theta_{||} = 0$.

Figure 2a shows the curves plotted for values of the divergence parameter ψ_0 on the order of magnitude and more of the characteristic angle of emission $\frac{1}{\gamma}$, and in Fig. 2b, for smaller values of the divergence parameter ($\psi_0 < 0.2$ mrad $= 1/\gamma$).The figures show the presence of a significant dependence of the angular density of the DTR on the divergence of the electron beam, up to values of the divergence on the order of $\psi_0 = 0.02$ mrad for a given electron energy. It should be noted that when the divergence changes with respect to the value $\psi_0 = 1/\gamma$, there is a qualitative change in the shape of the angular distribution of the DTR (Figs. 2a and 2b).

Figures 3–5 show curves constructed by formula (10b) that describe the function $F_{DTR}(\psi_0)$ at various values of the collimation parameters of the photon beam $\theta'_\perp$ and $\theta'_{||}$. The dependence on the target thickness is present only in the coefficient before the function $F_{DTR}(\psi_0)$.

The curves in Figs. 3 and 4, constructed for a collimator with dimensions of, respectively, $\theta'_\perp = \theta'_{||} = 3\gamma^{-1} = 0.6$ mrad and $\theta'_\perp = \theta'_{||} = \gamma^{-1} = 0.2$ mrad, show a noticeable decrease in the number of DTR photons emitted to the collimator with increasing divergence of the electron beam. However, the curve in Fig. 5, constructed for a collimator with the parameters $\theta'_\perp = \theta'_{||} = 0.5\gamma^{-1} = 0.1$ mrad, shows a noticeable increase in the number of photons of the collimated DTR with an increase in the divergence of the electron beam. This reflects the fact that the angular density in the minimum characteristic of the DTR increases with increasing angular divergence (Fig. 2b), namely, in the region of this minimum, a collimator with the parameters $\theta'_\perp = \theta'_{||} = 0.5\gamma^{-1} = 0.1$ mrad. Figure 6 shows the function dependence curve for large collimator sizes $\theta'_\perp = \theta'_{||} = 10\gamma^{-1} = 2$ mrad, when almost all DTR photons are incident on it. In this case, there is a very weak dependence of the yield of DTR photons on the divergence of the electron beam.

Figure 7 shows the curves constructed by formula (9b), which demonstrate the dependence of the angu-

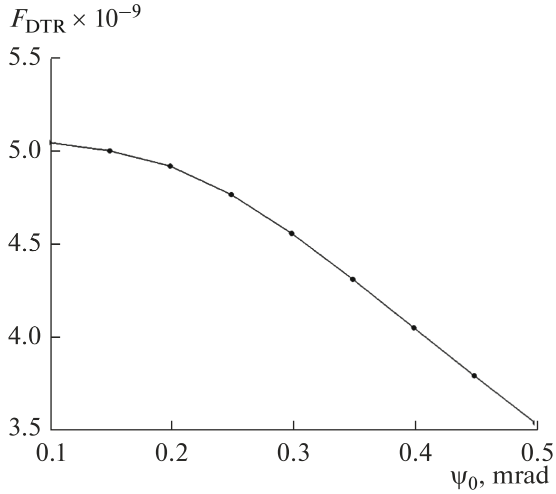

Fig. 4. The same as in Fig. 4 but for $\theta'_\perp = \theta'_\| = \gamma^{-1}$.

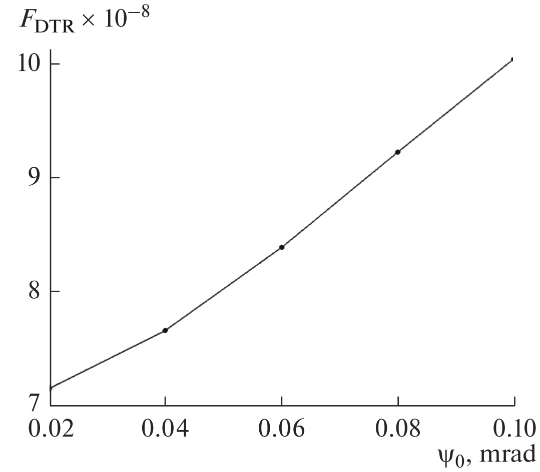

Fig. 5. The same as in Fig. 4 but for $\theta'_\perp = \theta'_\| = 0.5\gamma^{-1}$.

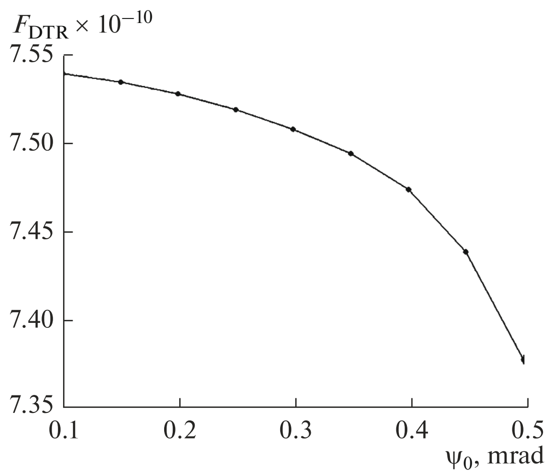

Fig. 6. The same as in Fig. 5 but for $\theta'_\perp = \theta'_\| = 10\gamma^{-1}$.

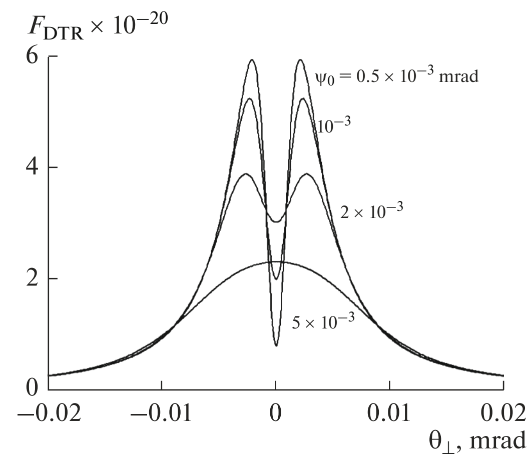

Fig. 7. Dependence of the angular density of DTR on the divergence ψ_0 of the electron beam. $\theta_\| = 0$, $\gamma = 500000$.

lar density of the DTR on the divergence of the electron beam at an energy of the relativistic electron of $\gamma = 500000$ ($E \approx 255$ GeV) and parallel components of the angle of observation $\theta_\| = 0$. From Fig. 7 it is seen that a noticeable dependence of the angular density of the DTR on the beam divergence for a given electron energy is observed in the range of the divergence parameter $5\times10^{-3} \le \psi_0 \le 5\times10^{-4}$ mrad. This means that the DTR can be used to indicate electron beams with such an energy. Figure 8 shows the dependence of the number of DTR photons emitted to the collimator. $\theta'_\perp = \theta'_\| = 3\gamma^{-1}$ on the divergence ψ_0 of an electron beam with $\gamma = 500000$.

CONCLUSIONS

In this paper, we theoretically study the diffracted transition radiation of a beam of relativistic electrons crossing a thin single-crystal plate in the Laue scattering geometry. An expression is obtained that describes the angular density of the DTR averaged over all possible rectilinear paths of an electron in the beam. For averaging, a two-dimensional Gaussian distribution is used. The performed calculations showed the existence of a significant dependence of the angular density of the DTR on the electron-beam divergence. For the case of a thin single-crystal target under consideration, an expression is obtained that describes the number of collimated DTR photons excited by a beam of relativistic electrons. It was shown that the number

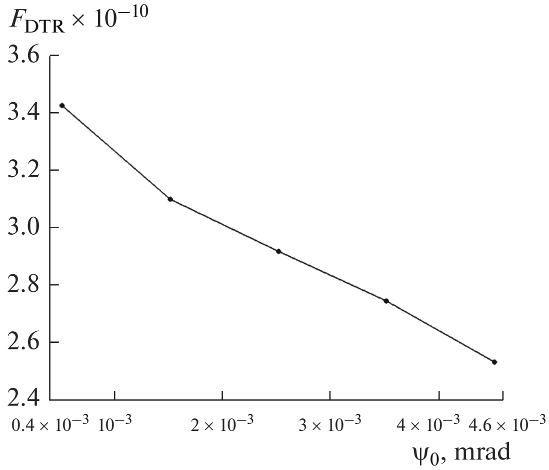

Fig. 8. Dependence of the number of emitted DTR photons in the collimator on the divergence of the electron beam ψ_0. Collimator size $\theta'_\perp = \theta'_\parallel = 3\gamma^{-1}$, $\gamma = 500000$.

of photons of the collimated DTR is also substantially dependent on the divergence of the electron beam, while the total yield of the study without collimation is practically independent of the divergence of the electron beam.

The expression obtained in this paper for the number of photons of a collimated DTR can be used to determine the divergence of beams of relativistic electrons, which greatly simplifies indication compared to using the angular distribution of the DTR.

FUNDING

This work was supported by the Ministry of Education and Science of the Russian Federation (state task no. 3.4877.2017/VU).

REFERENCES

1. G. M. Garibyan and Y. Shi, *X-ray Transition Radiation* (Akad. Nauk ArmSSR, Erevan, 1983) [in Russian].
2. A. Caticha, Phys. Rev. A **40,** 4322 (1989).
3. N. N. Nasonov, Phys. Lett. A **246,** 148 (1998).
4. G. M. Garibyan and Y. Shi, Sov. Phys. JETP **34,** 495 (1971).
5. V. G. Baryshevskii and I. D. Feranchuk, Sov. Phys. JETP **34,** 502 (1971).
6. S. V. Blazhevich and A. V. Noskov, Nucl. Instrum. Methods Phys. Res., Sect. B **252,** 69 (2006).
7. S. V. Blazhevich and A. V. Noskov, Nucl. Instrum. Methods Phys. Res., Sect. B **266,** 3770 (2008).
8. S. V. Blazhevich and A. V. Noskov, J. Exp. Theor. Phys. **120,** 753 (2015).
9. *International Linear Collider (ILC) Technical Design Report* (KEK, Tokyo, 2013).
10. *A Multi-TeV linear collider based on CLIC technology: CLIC Conceptual Design Report,* Ed. by M. Aicheler (CERN, Geneva, 2012). https://doi.org/10.5170/CERN-2012-007
11. S. V. Blazhevich, N. I. Moskalenko, T. V. Kos'kova, A. V. Noskov, and E. A. Tkachenko, J. Surf. Invest.: X-ray, Synchrotron Neutron Tech. **11,** 49 (2017).