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Method for Modeling Dynamic Modes of Nonlinear Control Systems for Thermoelectric Modules

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Abstract

Thermoelectric modules (TEM) based on Peltier elements have a number of advantages compared to classic compressor cooling systems based on refrigerants. Increasing the efficiency of TEM is associated with the search for new thermoelectric materials with improved properties and the use of temperature control systems for these modules. The complexity of modeling, analysis and synthesis of thermoelectric systems (TES) is determined by the significantly nonlinear characteristics of Peltier elements in a wide range of temperatures and control currents, as well as the influence of switching effects in pulse control. It is proposed to analyze the dynamic modes of TES using a spectral technique based on the approximation of the system characteristics by piecewise linear functions. This allowed us to obtain generalized expressions of transients for any order of the studied pulse system and arbitrary character of non-linearity, taking into account switching effects. The functional diagram and model of the current control system flowing through the Peltier element are presented. Dynamic modes of pulse TEM control systems with transfer functions of continuous parts of the 1st, 2nd and 3rd orders are simulated. The developed mathematical apparatus is the basis for further optimization and synthesis of TEM control systems.

Keywords: Peltier effect, thermoelectric module, control system, dynamic characteristic

INTRODUCTION

The use of thermoelectric modules (TEM) based on Peltier elements opens up wide prospects for the use of cooling systems in various fields of application, such as the radioelectronic industry [1,2], biology and medicine [3], the automotive industry [4-6], etc. The main advantages of thermoelectric systems (TES) in comparison with classic compressor-type cooling systems are: the absence of moving parts and, as a result, high manufacturability and reliability; the ability to control the temperature of the cooled object with high accuracy and programming time profiles of temperature changes; heating the object by simply switching the polarity of the voltage applied to the module. The key disadvantage of TEM compared to classic systems based on refrigerants, especially significant for devices with high power of more than hundreds of watts, is a relatively low energy efficiency. The main efforts of researchers aimed at eliminating this disadvantage are focused on the search for new thermoelectric materials with improved properties [7-10].

Another direction that increases the efficiency of TES is the use of temperature control systems based on various types of regulators: proportional integro-differential (PID), PI [11-13]. This direction is especially relevant for the temperature control of objects in an unstable environment, when the heat flow changes rapidly, and to compensate for this change, a rapid response of the control system is required. The complex nature of the influence of various factors on TPP determines the relevance of the use of adaptive neuro-fuzzy temperature control in TES [14]. Analysis and synthesis of thermoelectric systems is based on the use of dynamic TEM models, mainly the most simplified linear models are used. At the same time, in a wide range of temperatures and control currents, the Peltier element has a significantly nonlinear characteristic [15-19]. In addition, modeling Peltier elements and control systems based on them is further complicated when using pulse control with switching effects [20,21].

It is proposed to analyze the dynamic modes of TES using the spectral method previously proposed by the authors based on the approximation of the system characteristics by piecewise linear functions [22-25]. This will allow us to obtain generalized expressions of transients for any order of the studied pulse system, arbitrary character of nonlinearity and switching effects.

FUNCTIONAL DIAGRAM AND MODEL OF THE CURRENT CONTROL SYSTEM FLOWING THROUGH THE PELTIER ELEMENT

The control object–a TEM based on the Peltier effect–is shown in Fig. 1. Modern TEMs based on the Peltier effect are devices consisting of two ceramic plates-insulators, with series-connected thermocouples located between them (Fig. 1), and the design is made in such a way that each side of the module, depending on the polarity, contacts either p-n or n-p transitions.

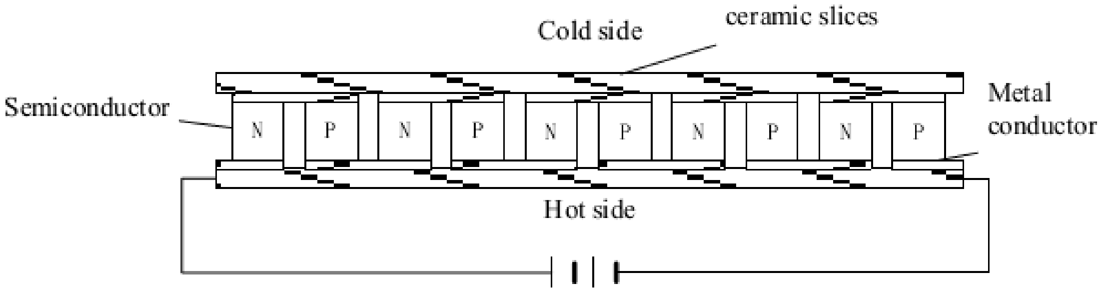

Fig. 1. The design of thermoelectric module based on the Peltier effect

The functional diagram of the system for regulating the current flowing through the Peltier element is shown in Fig. 2. [20]. The functional diagram shows: I_d–the desired current flowing through the Peltier element; u–the output signal of the controller (fill factor of pulse width modulation (PWM)); v–the PWM's output signal; $S_1…S_4$–the signals controlling the switching elements; U–the voltage applied to the smoothing filter; I–the current flowing through the Peltier element. MOSFET transistors can be used as switching elements (SE) [26], which are connected by a bridge circuit.

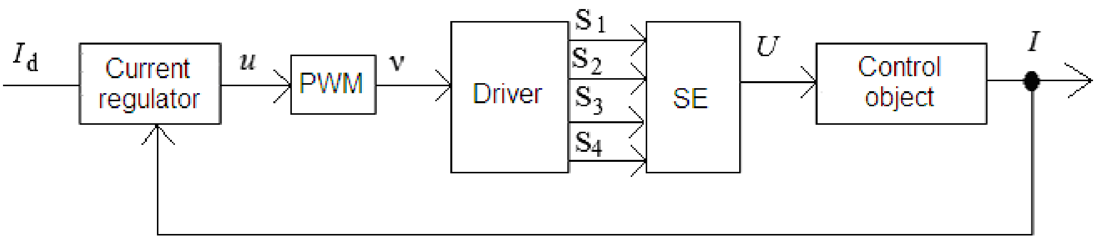

Fig. 2. Functional diagram of the current control system flowing through the Peltier element

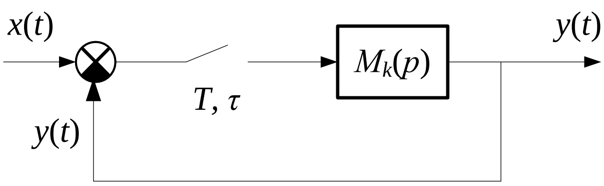

Fig. 3. Functional model of the system for analyzing transients

It is proposed to analyze the dynamic modes of TES using the spectral method previously proposed by the authors based on the approximation of the system characteristics by piecewise linear functions [22-25]. This will allow us to obtain generalized expressions of transients for any order of the studied pulse system, arbitrary character of nonlinearity and switching effects.

The functional model of the TEM control system for transient analysis is shown in Fig. 3. In the figure, denoted $x(t)$ and $y(t)$ is the forcing and the system response, T and τ are period of the switching key (switching element) and the duration of the pulse, $M_k(p)$ is the transmission coefficient of a continuous part for the section transition number k. The nonlinearity of the system is accounted for in the multiplier $M_k(p)$. Thus, the changed phase of the transition process k in this model indicates the actuation of the key (SE) and the transition of the nonlinear element on a new line plot mode.

EXPRESSIONS FOR THE ANALYSIS OF DYNAMIC MODES OF NONLINEAR TEM CONTROL SYSTEMS

As a rule, in practice, the influence of the Seebeck, Thomson, Joule and Fourier effects on the functioning of the Peltier element is quite small, and it can be ignored. A simplified equation of heat transfer to the Peltier element without taking into account these effects has the form [27]

$$E\frac{\partial \Delta T}{\partial t} + k^* A \Delta T = \Delta Q\,,$$

where E is the thermal coefficient of the thermoelement, ΔT is the temperature difference between the hot and cold side of TEM, k^* is coefficient of heat transfer, A is the area of the contact area between the TEM and the environment, $\Delta Q = U^2/R$ is the cooling capacity of the TEM, U is voltage on TEM, R is the internal resistance.

Based on the Lagrange transform, the heat transfer equation can be represented as

$$\frac{\Delta T}{\Delta U} = \frac{\Delta Q}{\Delta U k^* A}\frac{1}{1+\dfrac{Ep}{k^* A}}\,,$$

where $p = d/dt$ is the Laplace operator. Thus, the transfer function "applied voltage-temperature change" of TEM is an inertial link of the 1st order:

$$G(p) = \frac{K_1}{1+\tau_1 p}\,,$$

where $K_1 = \dfrac{\Delta Q}{\Delta U k^* A}$ is the link gain, $\tau_1 = \dfrac{Ep}{k^* A}$ is the link time constant.

To obtain analytical expressions of the dynamic characteristics of the TES y(t), we define partial solutions of the differential equation of the system $y_k(t)$, each of which corresponds to an unchanged state of the key (closed or open). Here k=0…K-1 is the number of the current partial solution and the current key switching interval, K is the number of partial solutions and twice the number of closing pulses. We assume that the initial state of the system at t<0 is open, then the closing moments correspond to even values of k, and the opening moments correspond to odd values of k:

$$\tilde{t}_{2k} = kT, \qquad \tilde{t}_{2k+1} = kT + \tau \quad (1)$$

For the current time interval $\tilde{t}_k \le t \le \tilde{t}_{k+1}$, the partial solution of the TES's differential equation has the form

$$y_k(t) = \hat{x}_k(t) H_k(p), \quad (2)$$

where $\hat{x}_k(t) = x(t - \tilde{t}_k) q_k$ is the influence on the system with switching effects, $\tilde{t}_k$ are the switching moments specified by the expression (1), q_k is the key state parameter (q_{2k} =1 means the key is closed and q_{2k+1} =0 means the key is open), $H_k(p)$ is transfer functions for the system with the current key state:

$$H_{2k}(p) = H_з(p) = \frac{M(p)}{1 + M(p)}, \qquad H_{2k+1}(p) = H_{paз}(p) = M(p), \qquad (3)$$

$M(p)$ is the transmission coefficient of the continuous part.

For the analytical calculation of $y(t)$ by the formula (2) together with (3), we represent the transfer coefficient of the continuous part of a fractional-rational function

$$M_k(p) = N_k \sum_{i=0}^{I} \alpha_i p^i \Big/ \sum_{i=0}^{I} \beta_i p^i, \quad (4)$$

where N_k is the transfer coefficient of the nonlinear component for the section k, I is the filter order, also α_i and β_i are the filter coefficients.

Substitute (4) in (3) and also represent the resulting expression as a relation of two polynomials, we write as

$$H_k(p) = \sum_{i=0}^{I} \mu_{ik} p^i \Big/ \sum_{i=0}^{I} \nu_{ik} p^i, \qquad (5)$$

where $\mu_{ik} = \alpha_i$, $\mu_{i,2k} = \alpha_i + \beta_i$, $\mu_{i,2k+1} = \alpha_i$. We substitute (5) into equation (2) and replace the Laplace operator p with the differential d/dt in the resulting expression:

$$\sum_{i=0}^{I} \nu_{ik} \frac{d^i y_k(t)}{dt^i} = \sum_{i=0}^{I} \mu_{ik} \frac{d^i \hat{x}_k(t)}{dt^i}. \qquad (6)$$

Let's make an operator equation for (6) taking into account the initial conditions [28,29]:

$$y_k(t) \leftarrow Y_k(p),\ \ y'_k(t) \leftarrow pY_k(p) - y_k(0),\ \ y''_k(t) \leftarrow p^2 Y_k(p) - pY_k(0) - y'_k(0),$$

$$y^{i'}{}_k(t) \leftarrow p^i Y_k(p) - p^{i-1} y_k(0) - p^{i-2} y'_k(0) - y_k{}^{i-1}(0) = p^i Y_k(p) - \sum_{j=1}^{i} y_k{}^{(j-1)}(0) p^{i-j}. \quad (7)$$

We will present the image of the influence $\hat{x}_k(t)$ and its derivatives $\hat{x}^{i'}{}_k(t)$ in the same way (7). The initial values of all the higher derivatives of influence and response are assumed to be zero: $y^{(i)}{}_k(0)=0$, $\hat{x}^{(i)}{}_k(0)=0$ for i>1. This assumption will simplify the obtained expressions and will not make a significant error in the calculation of dynamic modes.

Given (7), expression (6) will take the form

$$Y_k(p)=\left\{\hat{X}_k(p)\sum_{i=0}^{I}\mu_{ik}p^i - C^{(x)}{}_k(p)+C^{(y)}{}_k(p)\right\}/\sum_{i=0}^{I}\nu_{ik}p^i, \quad (8)$$

where $C^{(x)}{}_k(p)=x_k(0)\sum_{i=0}^{I}\mu_{ik}p^i + x_k'(0)\sum_{i=0}^{I}\mu_{ik}p^i$,

$C^{(y)}{}_k(p)=y_k(0)\sum_{i=0}^{I}\nu_{ik}p^i + y_k'(0)\sum_{i=0}^{I}\nu_{ik}p^i$ are

polynomials of the initial conditions of influence and response.

We assume the initial conditions at the beginning of the transition process (k=0) to be zero:

$$\hat{x}_k(0)=\hat{x}^{i'}{}_k(0)=y_k(0)=y_k'(0). \quad (9)$$

Also null are the initial values of the influence and its derivative when the key is closed: $\hat{x}_{2k}(0)=\hat{x}'_{2k}(0)=0$.

For each subsequent interval (k=1...K-1), the values of the corresponding parameters (influence, response, or their derivatives) coincide at the moments of key switching:

$$\hat{x}_{2k+1}(0)=\hat{x}_{2k}(t_{2k}),\quad \hat{x}'_{2k+1}(0)=\hat{x}'_{2k}(t_{2k}),\quad y_{k+1}(0)=y_k(t_k),\quad y'_{k+1}(0)=y'_k(t_k), \quad (10)$$

where t_k is the duration of the current interval: $t_{2k}=\tau$, $t_{2k+1}=T-\tau$. With constant exposure and t_k values much greater than the time constants of the continuous part of the system, key switching occurs at times when the system is in steady state. As follows from the properties of the Laplace transform, in this case, the initial conditions can be determined by the formulas

$$y_{\mathrm{k}+1}(0)=y_{\mathrm{k}}{}^{(end)}=\lim_{p\to 0}pY_k(p),\qquad y'_{\mathrm{k}+1}(0)=y'_{\mathrm{k}}{}^{(end)}=0, \quad (11)$$

The general solution of the dynamic characteristic of the TES $y(t)$ is obtained by summing all K partial solutions $y_k(t)$ taking into account their time shifts $\tilde{t}_k$

$$y(t)=\sum_{k=0}^{K-1}y_k(t-\tilde{t}_k)Q_k(t-\tilde{t}_k), \quad (12)$$

where $Q_k(t)=0{,}5\left[\mathrm{sign}(t)-\mathrm{sign}(t-t_k)\right]$ is the activation function of the current partial solution, equal to 1 in the interval of the argument [0; t_k] and equal to 0 for other values of the argument.

Each partial solution is the original of the operator function (8). Searching for the original by its image is associated with cumbersome transformations with a high system order and (or) a complex form of input influence.

Applying the inverse Fourier transform to the output spectrum of the partial solution [30], we obtain

$$y_k(t) = \frac{2}{\pi}\int_{0+}^{\infty} S_R^{(k)}(\omega)\cos(\omega t)d\omega + \frac{1}{\pi}\int_{0-}^{0+} S_R^{(k)}(\omega)\cos(\omega t)d\omega, \qquad (13)$$

where $S_R^{(k)}(\omega)$ is the real component of the system's response spectrum. The influence's spectrum and its real part for substitution in (13) are obtained from its operator expression (8) by replacing the Laplace operator p with a complex frequency $p \to \sigma + j\omega$, where σ is the convergence abscissa, such as σ =0 for stable partial solutions and σ>0 for unstable ones.

Using piecewise linear approximation of the real spectrum avoids cumbersome integral transformations and defines partial solutions by the relation

$$y_k(t) = \hat{x}_k(t)\cdot H_k(0) + \frac{2}{\pi}\sum_{i=0}^{N-1} a_{0\,i}^{(k)}\omega_i \frac{\sin\omega_i^* t}{\omega_i^* t}\frac{\sin\Delta_i^* t}{\Delta_i^* t}, \qquad (14)$$

where $\omega_i^* = \omega_i + \Delta_i/2$ is the central frequency of the current approximation segment, $H_k(0)$ is the transmission coefficient of the system at $\omega = 0$, $a_{0\,i}^{(k)} = S_R^{(k)}(\omega_i) - S_R^{(k)}(\omega_{i+1})$ is the approximation coefficient.

This approach allows us to generally study the transients of nonlinear control systems of thermoelectric modules for any order of the pulse system, arbitrary character of nonlinearity and switching effects with a large number of closing pulses $K/2$.

Modeling of dynamic modes of TEM control systems

Let's calculate the dynamic modes of TES with transfer functions of the continuous part of the 1st, 2nd and 3rd order:

$$M_1(p) = \frac{K_0}{1+T_1 p},\ M_2(p) = K_0\frac{1+T_1 p}{p(1+T_2 p)},\ M_3(p) = K_0\frac{1+T_1 p}{p(1+T_2 p+T_3 p^2)}, \qquad (15)$$

where K_0 is the gain, T_1, T_2 and T_3 are the time constants.

Let's take K_0=2, T_1=10 s, T_2=3 s, T_3=0,9 s, the input effect is a Heaviside function of the unit amplitude: $x(t) = \mathbf{1}(t) = \begin{cases} 1, t \ge 0 \\ 0, t < 0, \end{cases}$ its image $X(p)$=1/p. The approximation parameters of the output spectrum in (14) are chosen as N=100, $\omega_0 = 0{,}01c^{-1}$, $\omega_{N-1} = 1c^{-1}$, σ =0, the period of the key's closing pulses t = 200 s, the pulse duration τ=130 s.

Fig. 4 shows the calculated transient characteristics of a TES with a transfer function of a continuous part of the first order (TES1), second and third orders (TES2 and TES3). The figure shows 3 partial solutions of the equation of the system with numbers k=0, 1, 2. The transition process of the system of the 1st and 2nd order is aperiodic, one with the 3rd order is oscillatory. With increasing order, the duration of the process of establishing an open system (a particular solution $y_1(t)$ in Fig. 4) decreases, for a closed system (partial solutions $y_0(t)$ and $y_2(t)$), the duration of the process increases, while its nature significantly depends on the initial conditions.

The conditions $(T, \tau, T - \tau) >> \max(T_1, T_2, T_3)$ are met, so the switching moments follow after the end of the transients. The established (end) values of partial solutions of the transition process for any order of the system are equal $y_{2k}^{(end)} = 1$, $y_{2k+1}^{(end)} = -1$, and coincide with those calculated by (11) for the transfer function.

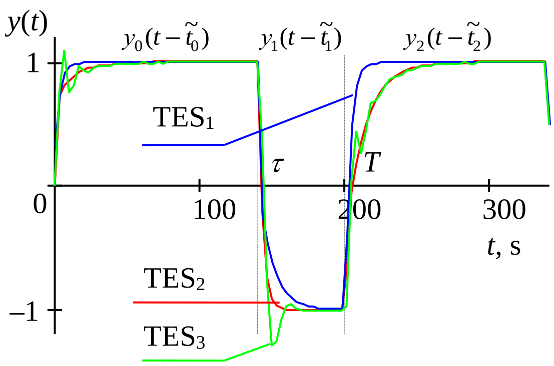

Fig. 4. Transient characteristics of the TEM control system with a continuous part of the 1st, 2nd and 3rd order

CONCLUSION

According to expressions (8-12), substitution of TES parameters can also be obtained as an analytical expression for the study of dynamic processes of a specific linear or nonlinear control system. Application of the proposed approach based on the spectral method and piecewise linear approximation (expressions (13-14)) allows to calculate transients of a nonlinear TEM control system of any order for any deterministic effects and a large number of switching pulses. The obtained expressions can also be used in the analysis of transient modes of specific pulse temperature control systems that use thermoelectric sensors in their composition. The model's error is mainly determined by the inaccuracy of the representation of the system's transfer characteristic by a fractional rational function. Changing the parameters of the system under study is taken into account by simply changing the coefficients in general expressions of dynamic characteristics. The developed mathematical apparatus is the basis for further optimization and synthesis of TEM control systems.

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