Incoherent bremsstrahlung in flat and bent crystals

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Abstract

The bremsstrahlung cross section for relativistic electrons in a crystal is split into the sum of coherent and incoherent parts (the last is due to a thermal motion of atoms in the crystal). Although the spectrum of incoherent radiation in crystal is similar to one in amorphous medium, the incoherent radiation intensity could demonstrate substantial dependence on the crystal orientation due to the electrons' flux redistribution in the crystal. In the present paper we apply our method of the incoherent bremsstrahlung simulation developed earlier to interpretation of some recent experimental results obtained at the Mainz Microtron MAMI.

Key words: bremsstrahlung, incoherent radiation, crystalline undulator, simulation

1. Introduction

It is well known (see, e.g. [1, 2, 3]) that high energy electron beam incident on an oriented single crystal produces the coherent radiation that is due to the spatial periodicity of the lattice atoms, and the incoherent one, that is due to the thermal spread of atoms from their positions of equilibrium in the lattice. For the first look, the incoherent part of radiation is similar to the last in amorphous medium (with Bethe-Heitler spectrum), and do not depend on the crystal orientation in relation to the particles beam.

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However, in [4, 5] it was paid attention to the fact that some features of the particle's dynamics in the crystal (channeling effect etc.) could lead to various substantial orientation effects in the hard range of the spectrum, where (for $\varepsilon \sim 1$ GeV electrons) the incoherent part is predominant. The semi-numerical approach developed in [4, 5, 6] was used for interpretation of early experimental data [7].

The ideas of [5] had been referred by the authors of recent experiments [8] to interpret some of their results. The results of simulation of the incoherent radiation under the conditions of the experiment [8] are presented in [9]. A good agreement with the experimental data confirms the interpretation given in [8].

Here we present the results of simulation using the improved procedure taking into account the crystal deformations. The simulation was carried out under the conditions of the recent experiment performed at the Mainz Microtron MAMI [10] to explore the radiation emission from periodically bent crystal. The possibility of application of such crystals as undulators is discussed during last years [10, 11, 12].

2. Bremsstrahlung in dipole approximation

Radiation of relativistic electron in matter develops in a large spatial region along the particle's momentum. This region is known as the coherence length (or formation length) [1, 2, 3] $l_{\text{coh}} \sim 2\varepsilon\varepsilon'/m^2c^3\omega$, where ε is the energy of the initial electron, ω is the radiated photon frequency, $\varepsilon' = \varepsilon - \hbar\omega$, m is the electron mass, c is the speed of light. In the large range of radiation frequencies the coherence length could exceed the interatomic distances in crystal:

$$l_{\text{coh}} \gg a. \tag{1}$$

In this case the effective constant of interaction of the electron with the lattice atoms may be large in comparison with the unit, so we could use the semiclassical description of the radiation process. If, in addition to that, the electron's scattering angle on the coherence length ϑ_l satisfies the condition

$$\vartheta_l \ll \gamma^{-1},$$

where $\gamma = \varepsilon/mc^2$ is the electron's Lorentz factor, the dipole approximation is valid [2]. In this approximation the spectral density of bremsstrahlung under subsequent collisions on atoms could be described by the formula

$$\frac{dE}{d\omega} = \frac{e^2\omega}{2\pi c^2} \int_\delta^\infty \frac{dq}{q^2} \left[1 + \frac{(\hbar\omega)^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right] \left| \sum_n \boldsymbol{\vartheta}_n e^{icqt_n} \right|^2 , \qquad (2)$$

where $\delta = m^2c^3\omega/2\varepsilon\varepsilon' \sim l_{\text{coh}}^{-1}$, $\boldsymbol{\vartheta}_n$ is the two-dimensional electron scattering angle under collision with the n-th atom, t_n is the time moment of the collision.

Consider now the radiation of the electron incident onto the crystal under small angle ψ to one of its crystallographic axes. It is known [1, 2] that averaging of the value $|\sum_n \boldsymbol{\vartheta}_n e^{icqt_n}|^2$ over the thermal vibrations of atoms in the lattice leads to the split of this value (and so the radiation intensity) into the sum of two terms describing coherent and incoherent effects in radiation:

$$\left\langle \left| \sum_n \boldsymbol{\vartheta}_n e^{icqt_n} \right|^2 \right\rangle = \sum_{n,m} e^{icq(t_n - t_m)} \left\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n + \mathbf{u}_n) \right\rangle \left\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_m + \mathbf{u}_m) \right\rangle \qquad (3)$$

$$+ \sum_n \left\{ \left\langle (\boldsymbol{\vartheta}(\boldsymbol{\rho}_n + \mathbf{u}_n))^2 \right\rangle - \left(\left\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n + \mathbf{u}_n) \right\rangle \right)^2 \right\} , \qquad (4)$$

where $\boldsymbol{\rho}_n = \boldsymbol{\rho}(t_n) - \boldsymbol{\rho}_n^0$ is the impact parameter of the collision with the n-th atom in its equilibrium position $\boldsymbol{\rho}_n^0$, $\boldsymbol{\rho}(t)$ is the trajectory of the electron in the plane orthogonal to the crystallographic axis (which could be obtained by numerical integration of the equation of motion), and $\mathbf{u}_n$ is the thermal shift of the n-th atom from the position of equilibrium. In the range of radiation frequencies for which

$$l_{\text{coh}} \ll a/\psi, \qquad (5)$$

where a is the distance between two parallel atomic strings the closest to each other, the incoherent term (4) makes the main contribution into the bremsstrahlung intensity (2).

The radiation by the uniform beam of particles is characterized by the radiation efficiency, that is the radiation intensity (2) integrated over impact parameters of the particles' incidence onto the crystal in the limits of one elementary cell. So, the efficiency is the classical analog of the quantum cross section. In the further consideration we shall compare the radiation efficiency in the crystal to the Bethe-Heitler efficiency of bremsstrahlung in amorphous medium.

For further computational details see [4, 5, 6].

3. Origin of the orientation dependence of the incoherent bremsstrahlung

When charged particles are incident onto the crystal under small angle θ to one of the atomic planes densely packed with atoms, the channeling phenomenon could take the place (see, e.g., [2, 3]). Under planar channeling the electron moves in the potential well formed by the attractive continuum potential of the atomic plane (see figure 1a, trajectory 1). The largest incidence angle, for which the capture into the channel is possible, is called as the critical channeling angle θ_c [2, 3].

Under $\theta \ll \theta_c$ the most part of the incident electrons would move in the planar channeling regime. These electrons will collide with atoms at small impact parameters more frequently then in amorphous medium, that leads to the increase of the incoherent bremsstrahlung efficiency (see figure 1b). For $\theta \sim \theta_c$ the above-barrier motion in the continuum potential takes the place for the most part of the particles (figure 1a, trajectory 2). Above-barrier electrons rapidly cross the atomic plane, with reduced number of close collisions with atoms comparing to the case of amorphous medium. This leads to the decrease of the incoherent bremsstrahlung efficiency (figure 1b).

Incoherent multiple scattering on the thermal vibrations on the lattice atoms (described by Eq.(4) and hence proportional to the incoherent radiation efficiency) leads to dechanneling of the particles and, as a consequence, to smoothing of the orientation dependence described above [5] (compare dashed and solid curves of figure 1b). Hence, relative height of the radiation efficiency maxima produced by different atomic planes in the crystal could be used as a measure of the dechanneling rate.

4. Results and discussion

The simulation was carried out under the conditions of the recent experiment performed at the Mainz Microtron MAMI [10] to explore the radiation emission from silicon crystal with 4-period bent (110)-planes (period of oscillations $\lambda_U = 7\ \mu m$, amplitude $A = 4.8$ Å, electron energy $\varepsilon = 855$ MeV). The radiation yield with the photon energy $\hbar\omega = \varepsilon/2$, for which the incoherent radiation mechanism is predominant, had been registered.

For the simulation we let the crystal is aligned on the goniometer in such a way that the zero angle of incidence to (110)-plane is achieved for the

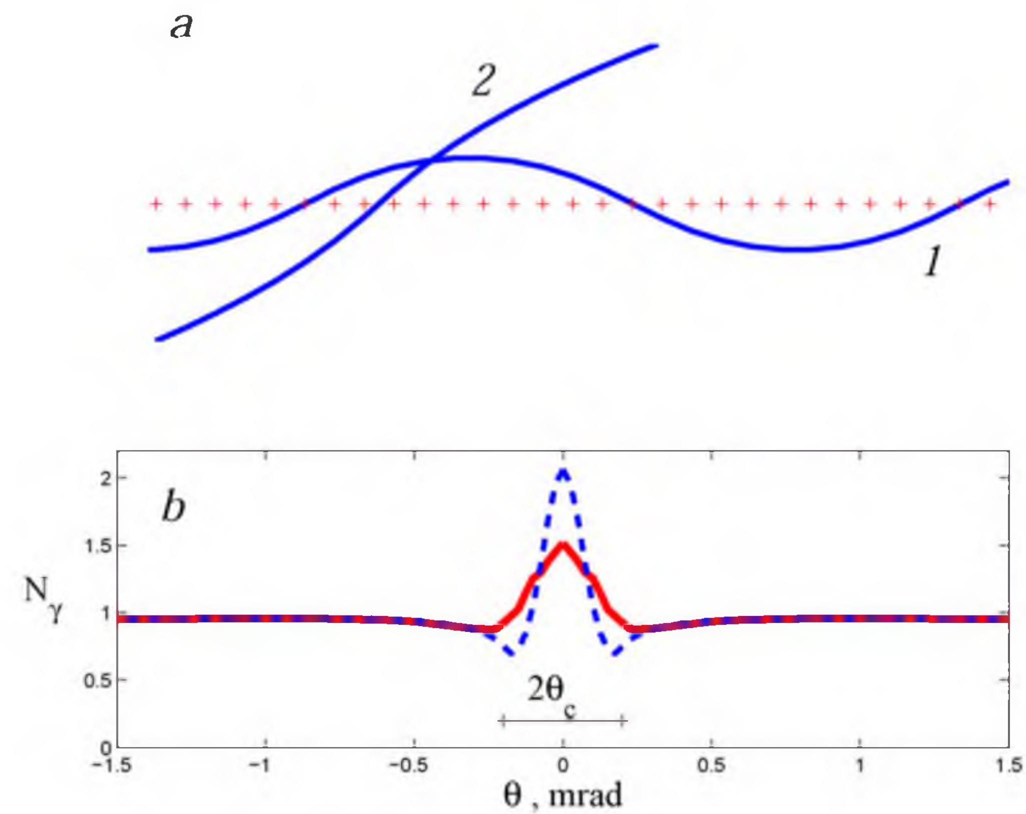

Figure 1: **(a)** Typical trajectories of the electrons under planar channeling (*1*) and above-barrier motion *(2)*. Pluses mark the positions of atomic strings (perpendicular to the figure plane) forming the atomic planes of the crystal. The horizontal scale of the figure is highly compressed. **(b)** Simulated incoherent bremsstrahlung efficiency (in ratio to the Bethe-Heitler efficiency in amorphous medium) from 1 GeV electrons (solid line) vs incidence angle θ to (110) plane of 30 μm thick Si crystal [4]. Dashed line corresponds to the trajectories simulated neglecting thermal vibrations of atoms.

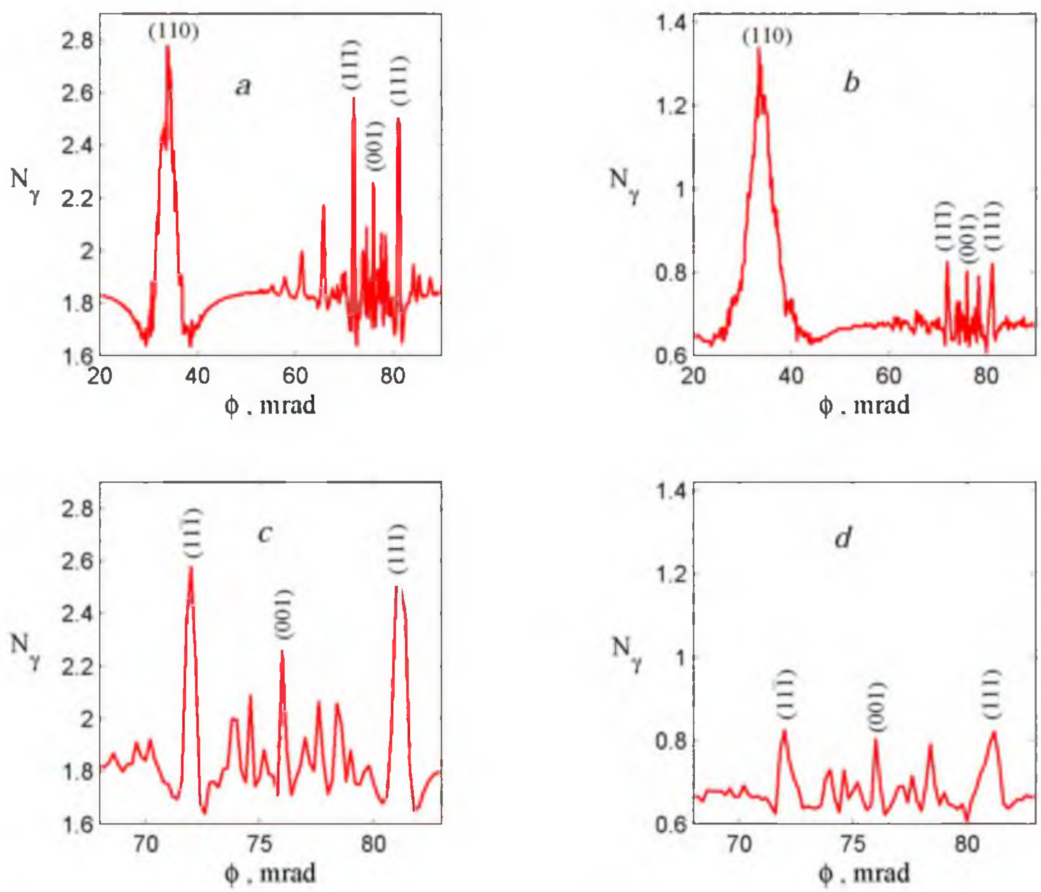

Figure 2: Results of simulation for the incoherent bremsstrahlung intensity (in ratio to Bethe-Heitler intensity in amorphous medium) from 855 MeV electrons in flat (left plot) and sinusoidally bent (right plot) silicon crystals under scanning of the goniometric angle like in the experiment [10]. We can see characteristic structures similar to that on figure 1b, generated by different crystallographic planes.

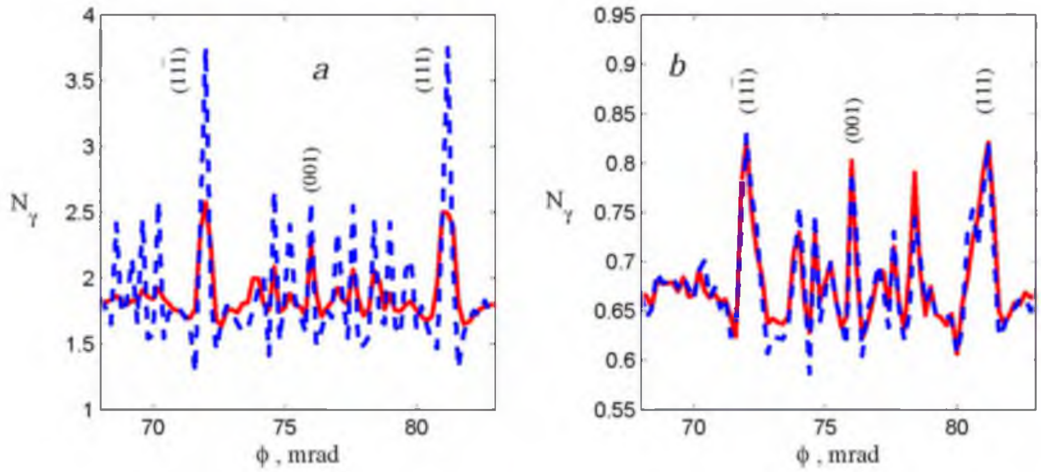

Figure 3: The same as on figure 2 c and d (solid lines) and the results of simulation neglecting the effect of scattering of the electrons on the thermal vibrations of atoms (dashed lines).

goniometer angle $\phi \approx 34$ mrad, and the zero angle of incidence to (001)-plane is achieved for $\phi \approx 76$ mrad, like in the experiment [10].

The results of simulation (figure 1) demonstrate (at least qualitative) agreement with the experimental data. Under scanning the angle ϕ different crystallographic planes generate characteristic maxima and minima similar to that on figure 1b.

Comparing left and right panels on figure 2, we can see the decrease of the incoherent bremsstrahlung efficiency in the bent crystal due to the increase of the dechanneling rate.

Some features of the dechanneling process could be illuminated comparing the results of simulation with and without account of the scattering of the electrons on thermal vibrations of atoms (figure 3). In flat crystal that scattering is the only cause of the dechanneling. It leads to softening of the orientation dependence of the value N_γ (compare dashed and solid curves on figure 1b and figure 3a). That softening would gradually increase with the increase of the crystal thickness [5].

Bending of the crystallographic planes increases the dechanneling rate so highly that the scattering on the thermal vibrations of atoms already have no substantial influence on the incoherent radiation efficiency (compare solid and dashed curves on figure 3b).

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