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MEASUREMENT OF THE STATUS OF COMPLEX SYSTEMS IN MULTIDIMENSIONAL PHASE SPACES

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Abstract: It is proposed to use the principle of corresponding states and the principle of distance invariance in multidimensional phase spaces to create models of systems of various nature based on experimental or statistical information presented in the form of temporal data. This scientific idea is closely connected with the logical construction of measuring scales in natural science, and, in particular, in thermodynamics. A method for complex measurement of the state of objects in relation to the reference state and the reference process has been developed, based on obtaining state equations in the form of phenomenological relationships and using multidimensional scaling methods. As examples, the equations of states are obtained and measuring scales for two complex systems are constructed. The first example is related to the study of the similarity of biological species in the three-dimensional phase space of state variables. The second example uses statistical data on the state and development of Russian cities. The measuring scale is also constructed for the three-dimensional phase space of socio-economic variables. The proposed approach is universal and can be used for a comprehensive assessment of the development processes of countries, regions and cities, the study of human development and ranking of socio-economic objects by a set of indicators, while analyzing the environmental pollution by indicators of environmental safety, studying the similarity of objects of one class for several variables and etc.

Keywords: complex systems, temporal data arrays, multidimensional phase spaces, state equations, measuring scales

I. INTRODUCTION

Today, one of urgent tasks of modern science is connected with the search for an answer to the question: is a general approach possible in modeling objects, processes and phenomena in various subject areas? The epistemological aspects of modeling are associated with the construction of models of systems in the phase space of state variables characterizing the properties of these systems. To do this, it is necessary to link the process of building theories closely with the presentation, processing and analysis of experimental data. In current study, an attempt is made to propose methods for creating phenomenological models of systems based on the use of a single structure representing the experimental data in multidimensional phase spaces of variables. There is reason to believe that on this basis, patterns that are isomorphic for objects of different nature can be identified. The works of the authors indicate that a certain isomorphism for some classes of objects should be sought precisely in the universal form of representation of experimental data characterizing the processes of state change of the studied systems [1–8]. It is also necessary to work out methods for describing the states of objects, based on the general approaches of mathematical modeling. In this regard, the theory of thermodynamics gives a positive example of constructing phenomenological models, based on the general logic of data presentation, the construction of measurement scales, the identification of patterns and the formation of computed dependencies.

II. OBJECTIVES

Since all measurements and observations of processes and phenomena are based on changes in various quantities, the problem arises from experimental data to establish patterns that are characteristic of a particular system and which can mathematically be described by phenomenological relationships.

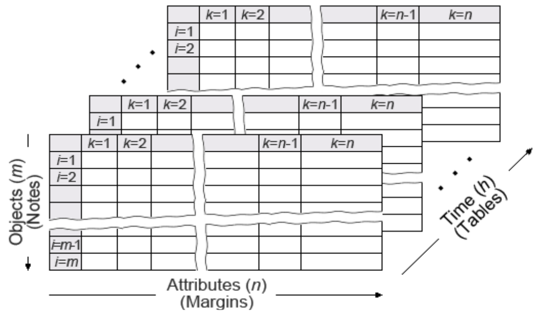
On this basis, the purpose of the article is to search for general principles for constructing scales for measuring the state of objects and establishing, on the basis of this, phenomenological regularities for systems of different nature that can be used as the basis for applied theories. An important task is also to illustrate the relevant methods with concrete examples.

This task is relevant in the study of complex systems for which significant amounts of experimental data have been accumulated and which can be presented in the form of temporal arrays of quantitative information. The proposed method in its essence is a logical development of the method of thermo-dynamics in relation to systems of various natures.

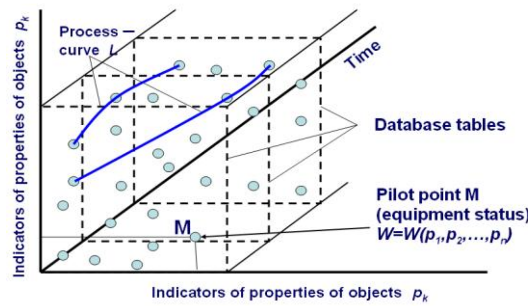
III. PRINCIPLES AND METHODS OF MODELING

To solve this problem, it is proposed to use empirical measures in the geometric form or probabilistic values of the multidimensional phase space with the aim of complex characterization of the states of objects, as well as to develop systems for measuring these measures based on the application of natural science principles. Among such principles one should particularly emphasize the principle of the corresponding states, according to which the states of objects can obey a single equation, if this equation is expressed in terms of some given variables. In turn, when modeling it is necessary to take into account also the principle reflecting a certain similarity with respect to the processes that objects can perform. For this, the principle of invariance of geometric distances in multidimensional phase spaces can be used. According to this principle, geometric objects are similar if, when comparing them, the relationship between some observable quantities that can be measured on a single scale is preserved. We will use these principles in modeling.

Data. The results of observations are often presented as data arrays characterizing the totality of objects of the same type when their states change with time [9]. Such data is called temporal and they look like multidimensional time series. In this case, we are talking about arrays of discrete data that have the structure of tables in the form of “object-parameters”, and the corresponding number of tables is ordered in time. The structure of temporal data and the corresponding state space of objects are shown in Figure 1. In temporal data arrays, objects of the same type are classes (entities) characteristic of real objects. As parameters (attributes), there can be different physical, chemical, biological and socio-economic quantities that have a quantitative measurement.



(a)



(b)

Figure 1. Temporal data sets, characterizing state changes of the objects:
a) structure of temporal data sets; b) states space of the objects

Phase state space. For temporal data, a modeling environment is constructed in the form of a phase state space. Suppose that for m objects of the same class, temporal data arrays contain quantitative information about n parameters z_k $(k = 1, 2, ..., n)$ characterizing the properties of the objects being studied. We take these parameters as state variables.

Thus, we define the n-dimensional phase space E^n, where $z = (z_1, z_2, ..., z_n)$, $z \in E^n$. Any state in n-dimensional space at each moment in time will be displayed by a multidimensional point $M = M(z_1, z_2, ..., z_n)$, and the process of changing the state of an object in time – by a multidimensional curve.

Suppose that in a n-dimensional phase space of states E^n there is a certain number of discrete points M_i

$i = 1, 2, ..., q$, which are experimental data. Let us represent these points as a certain limited sample from a continuous hypothetical medium of an infinite number of states for objects of the same class. We will use the continual principle of representing discrete information in space E^n [3], according to which the field of states is considered continuous, with each element of space associated with all neighboring elements, taking into account the patterns inherent in the complex system under study. Thus, the concept of a continual field of a certain quantity (field variable) is used, which characterizes the state of objects in general.

Also we will consider that with respect to observations the principle of invariance is valid, when in the state space the discrete data form a certain "image", reflecting to a certain extent the essence of the continual regularities of the field quantity. In this case, the invariance of the image will be associated with the isometry of space E^n when the distances between points are maintained M_i.

Further, let each point M_i be assigned a value that has the property of immutability with respect to the transformations of the variables $z_1, z_2, ..., z_n$. We define this field value as an empirical measure of the states of objects in space E^n in the form $W = W(M)$.

The main hypothesis of the study is connected with the possibility of describing the field regularities by establishing a connection between the measure W and a priori given geometrical metric of space E^n. The empirical measure will represent the features of the observed states and processes as geometric images (points and curves), based on field representations and can be described by a scalar field $W = W(M)$ that is invariant under coordinate transformations. An empirical measure should comprehensively characterize the states of objects, as well as their changes, correspond to the notion of scalar values, have a definition range from zero to + ∞ or from –∞ to + ∞, and make it poss

ible to evaluate the states and processes of state changes of objects on basis of the universal measurement system applicable to the entire space E^n.

In turn, the metric of the state space will be associated with variables $z_1, z_2, ..., z_n$ and can be described by a scalar function depending on these quantities. This value will reflect the geometric structure of the space E^n, based on one or another accepted mathematical model. Depending on the specifics of the problem being solved, the space metric E^n will be presented as dependencies with respect to all n indicators: additive, multiplicative, power, homogeneous or other dependencies, or as various measures of similarity: Euclidean, Manhattan or power distances, Chebyshev, Minkowski and others.

Quantitative knowledge about the properties and patterns of behavior of various systems is usually represented in the form of equations of state, where some parameters of the systems are expressed in terms of others. Usually, to build such equations, a reference object or a reference state is selected, and all other states correspond to the selected point in space E^n. In the general case, the principle of the corresponding states can be formulated in the form: for complex systems and objects, regularity can be observed when different states are associated with specially selected states in the same way. The fairness of the principle in each case is verified according to the available experimental data.

This principle allows building a scale for the relative comparison of the states of objects with each other. The procedure for constructing such scales is well-developed in thermodynamics [10, 11]. For example, we will use the appropriate logic for constructing scales for a comprehensive assessment of the similarity of objects, based on the representation of state spaces in the form of geometric Euclidean spaces. In general, the essence of the method lies in the choice in space E^n of both the reference state M_0 and a certain reference process.

Based on this, we construct a scale for a relative comparison of the states of objects as follows. Let us choose a certain linear reference process l0, on the lines of which we note the supporting state M_0 (Fig. 2). On the reference process, we note the second reference state M_0' and the two indicated states are connected by a straight line. The resulting segment is divided by a given number of odin-nakovnyh intervals, for example 100, and set the length of the obtained segments σ. Further from the origin of coordinates we draw a ray OM_0 and find the length of the segment OM_0. The scale of measurements of the states of objects is formed in the form θ of a certain index in relation to a beam OM_0 with a unit of measurement σ, and the length of the segment OM_0 in this scale of measurements will be $\theta_0 = l_{OM_0} / \sigma$. For definiteness and formation of differences from thermodynamics, let's call this index θ a mensoura (from lat. Mensura – measure) and set the corresponding unit of measure in the form of a degree mensura $°M$, which will be geometrically equal to length σ. Now, by drawing the radius vector $\vec{r}$ to each experimental point M_i and determining its module $|\vec{r}|$, it is possible to measure each state in degrees scale in the resulting scale.

The lengths of the segments in the Euclidean space will be determined on the basis of the known metric by the formula:

$$l_{ab} = \sqrt{(z_{1b} - z_{1a})^2 + (z_{2b} - z_{2a})^2 + .. + (z_{nb} - z_{na})^2}, \quad (1)$$

Where a and b – beginning and end of a segment ab.

Now for space E^n you can search for a phenomenological model in the form of an equation of state:

$$\theta = f\left(\frac{z_1}{z_{1_0}}, \frac{z_2}{z_{2_0}}, \ldots, \frac{z_n}{z_{n_0}}\right), \qquad (2)$$

where $z_{1_0}, z_{2_0}, \ldots z_{n_0}$ – variable values for the reference object (state).

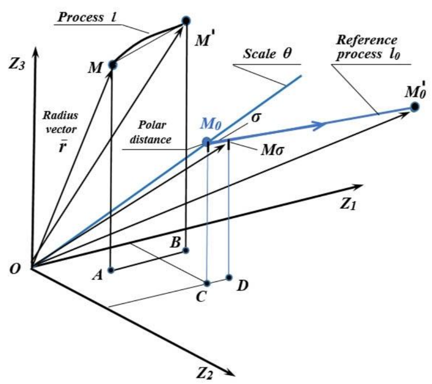


Figure 2: System of construction of the index scale θ in relation to the reference state and the reference process

The fact of existence for objects of one class of an equation of the form (2) must be confirmed by the available experimental data. If, on the basis of these data, a qualitative equation is obtained, then in this case one can speak about the validity of the principle of the corresponding states. This makes it possible to empirically substantiate the concept of mensura, as a special function characterizing the state of objects in a multidimensional space E^n. Based on equation (2), mensuru can be defined as a geometric measure of the deviation of the state of the object being studied from the reference state, which is standardized for a particular class of objects.

From the above, it follows that in justifying the concept of mensura, methods of constructing measuring scales, like temperature scales in thermodynamics, are used. It is known that in thermodynamics the role of measure for the relative comparison of the states of thermodynamic systems with each other is performed by a special function, called temperature. Similarly, the mensura scale can be represented as a system of comparable values of geometric quantities for assessing the states of objects in space E^n.

IV. EXAMPLES OF BUILDING SCALES AND OBTAINING EQUATIONS OF STATE

We illustrate the use of the proposed methods and dependencies using the example of obtaining state equations and dependencies for assessing the similarity of biological and socio-economic objects.

Biological objects. We will use the AnAge database [12] to obtain state equations and relations for assessing the similarity of biological species and, in particular, vertebrate animals. As variables, we use the following values from the AnAge database: the maximum lifetime in captivity z_1, years; weight of adult specimen z_2, kg; metabolic rate z_3, watts.

As the first reference object (point M_0) in the construction of the linear scale of Mensura, let us choose the biological state of the house mouse species (Mus musculus), which is one of the most numerous mammalian representatives and the most studied model animal. The main biological indicators of a house mouse are: $z_1 = 4$ years; $z_2 = 0.0205$ kg; $z_3 = 0.271$ W. We take the biological state of the species gray rat (Rattus norvegicus) as the second reference point M_0'. This species is in its heyday, distributed in a man-made environment, and is bred in large numbers as domestic and laboratory animals. The biological indicators of the gray rat are: $z_1 = 3.8$ years; $z_2 = 0.300$ kg; $z_3 = 1.404$ W. When analyzing the data, we will use combinations of the given indicators of the species from a sequence of values z_1, z_2, z_3, as well as their dimensionless values, related to the values of the reference state.

Construct a straight line between the states M_0 and M_0', determine the length of the resulting segment according to (1) and divide this segment into 100 equal parts. As a result, we have the standard of one degree $1°M$, equal to the length σ of the elementary segment. After the operations associated with the construction of the scale, the value of the mensura for the reference point M_0 was found, which, for example, to the state defined by the indicators z_1, z_2, z_3, is equal $338.62°M$. Some of the obtained state equations are given in Table 1. The multiple correlation coefficients of the regression dependencies have a high value, which allows concluding that the principle of the corresponding states for the studied class of objects is valid.

Table 1 Equations of state for biological species.

The indicators of types	The amount of types	Value θ at the point M_0, °M	Equation of state of objects	Correlation coefficient
z_1, z_2, z_3	546	338.62	$\theta = 323.45\left(z_1/z_{1_0}\right)^{0.951}\left(z_2/z_{2_0}\right)^{0.064}\left(z_3/z_{3_0}\right)^{0.0}$	0.96
z_1, z_2	2456	1163.82	$\theta = 1096.6\left(z_1/z_{1_0}\right)^{0.896}\left(z_2/z_{2_0}\right)^{0.101}$	0.95
z_2, z_3	545	23.23	$\theta = 20.23\left(z_2/z_{2_0}\right)^{0.346}\left(z_3/z_{3_0}\right)^{0.564}$	0.99
z_1, z_3	531	351.96	$\theta = 344.85\left(z_1/z_{1_0}\right)^{0.983}\left(z_3/z_{3_0}\right)^{0.085}$	0.97

Socio-economic objects. We will use a database of information on the state and development of Russian cities of the Federal State Statistics Service [13]. Based on this source, a temporal array of data describing the state of the economies of cities with a population of more than 100 thousand people was formed (only 154 cities, without Moscow and St. Petersburg). For each city, information is available on the main indicators in the period from 2003 to 2015 (step one year). For example, we use four indicators for analysis: population size Z_1, thous. people; the volume of goods produced, work performed and services using own resources by type of economic activity “Manufacturing Production” Z_2, million rubles; retail trade turnover, million rubles; the amount of work performed by the type of activity “Construction”, mln. rub.

Due to the fact that the pace of urban development depends on the number of population and regional characteristics, in the analysis we will use the specific values of the species $z_k = Z_k / Z_1$ (million rubles / thousand people), which we take for state variables. As a reference point M_0, we take a conditional state with minimum values $z_{k_0} = z_{k\ \min}$ in the group of cities that were observed in 2003. The corresponding conditional object for which these values were observed will be considered a city with a population of 100 thousand people. As a reference process, we choose the process of development of this city, while we believe that the values characterizing its development in each year from 2003 to 2015 changed linearly. As a second reference point M_0', we take the state of a given city in 2015 with indicators also equal to the minimum values of these quantities.

As in the previous case, we find the standard of one degree $(1\,^\circ M)$, equal to the length σ of the elementary segment. After the operations were performed, the value of the mensura for the reference point was determined M_0, which for the condition of the conditional city, determined by the values z_2, z_3, is 61, 18 $^\circ M$ .. Some of the resulting equations of state are shown in Table 2.

Table 2. The equation of state for cities of Russia.

Indicators cities'	Year	Equation of state of objects	Correlation coefficient
z_2, z_3, z_4	2003	$\theta = 78.93\left(z_2/z_{2_0}\right)^{0.670}\left(z_3/z_{3_0}\right)^{0.246}\left(z_4/z_{4_0}\right)$	0.97
z_2, z_3, z_4	2015	$\theta = 78.65\left(z_2/z_{2_0}\right)^{0.778}\left(z_3/z_{3_0}\right)^{0.151}\left(z_4/z_{4_0}\right)$	0.98
z_2, z_3	2003	$\theta = 61.77\left(z_2/z_{2_0}\right)^{0.716}\left(z_3/z_{3_0}\right)^{0.307}$	0.98
z_2, z_3	2015	$\theta = 61.06\left(z_2/z_{2_0}\right)^{0.849}\left(z_3/z_{3_0}\right)^{0.168}$	0.99
z_2, z_4	2003	$\theta = 588.6\left(z_2/z_{2_0}\right)^{0.899}\left(z_4/z_{4_0}\right)^{0.080}$	0.98
z_2, z_4	2015	$\theta = 726.4\left(z_2/z_{2_0}\right)^{0.858}\left(z_4/z_{4_0}\right)^{0.098}$	0.98
z_3, z_4	2003	$\theta = 40.86\left(z_3/z_{3_0}\right)^{0.836}\left(z_4/z_{4_0}\right)^{0.099}$	0.98
z_3, z_4	2015	$\theta = 31.96\left(z_3/z_{3_0}\right)^{0.936}\left(z_4/z_{4_0}\right)^{0.064}$	0.99

Thus, the use of the proposed principles in modeling the states of various classes of objects allows us to obtain state equations and formulate general methodological principles for processing temporal data to obtain phenomenological patterns.

V. CONCLUSION

The proposed approach allows the use of mathematical tools and data processing techniques that are inherently close to the basic relations and dependencies of thermodynamics when modeling systems. This makes it possible to use proven methods for modeling various systems. The proposed multidimensional scaling method for complex measurement of the states of systems uses the principle of the corresponding states and the principle of distance invariance in multidimensional phase spaces with respect to the reference state and the reference process. Based on this, measurement scales can be developed for a relative comparison of the states of different systems among themselves. At the same time, it should be noted that on the basis of the use of measuring scales, only objects of the same class can be compared with each other.

VI. REFERENCES

[1]. Averin, G.V., 2014. System dynamics. Donetsk, Donbass, Pages: 405. Available at: http://www.chronos.msu.ru/ru/rnews/item/sistemodinamika (accessed 24 April, 2019). (in Russian).

[2]. Zviagintseva, A.V., 2016. Probabilistic Methods of a Complex Assessment of Natural and Anthropogenic Systems. Cpektr Publishing House, Russia, Moscow, Pages: 257. Available at: http://dspace.bsu.edu.ru/handle/123456789/17837 (accessed 24 April, 2019). (in Russian).

[3]. Averin, G.V., Konstantinov, I.S., Zviagintseva, A.V., 2016. On the Continual Approach to the Model Data Presentation. Bulletin of Computer and Information Technologies, 10: 47–52. (in Russian).

[4]. Zviagintseva, A.V., Averin ,G.V., 2016. Integration of the several multivariate Pfaff equations which are important in application. Scientific Bulletin of Belgorod State University. Series. Mathematics. Physics, 27(248). Issue 45: 102–114. (in Russian).

[5]. Averin, G.V., Konstantinov, I.S., Zviagintseva, A.V., Tarasova, O.A., 2015. The Development of Multi-Dimensional Data Models Based on the Presentation of an Information Space as a

Continuum. International Journal of Soft Computing, 10(6): 458–461.

[6]. Averin, G.V., Zviagintseva, A.V., Shevtsova, M.V., Kurtova, L.N., 2017. On Representation of Discrete Information of Temporal Databases in the Continuous Form. Journal of Engineering and Applied Sciences, 12 (15): 3884–3889, DOI: 10.3923/jeasci.2017.3884.3889.

[7]. Averin, G.V., Zviagintseva, A.V., Konstantinov, I.S., Shvetsova, A.A., 2018. Forecasting indicators of territorial entity based on phenomenological models of collective behavior. Amazonia investiga, 7(13): 42–49.

[8]. Averin, G.V., Zviagitrseva, A.V., 2017. On justice of the principle of corresponding conditions for various systems. Scientific Bulletin of Belgorod State University. Series. Economy. Computer science, 16(265). Issue 43: 104–112. Available at: http://dspace.bsu.edu.ru/handle/123456789/21056 (accessed 15.09.2018). (in Russian).

[9]. Kostenko, B.B., Kuznetsov, S.D., 2019. History and actual problems of temporal databases. Citforum. URL: http://citforum.ru/database/articles/temporal/ (accessed 24 April, 2019).

[10]. Gukhman, A.A., 1986. About the thermodynamic bases. Moscow, Energoatomizdat, 383 (In Russian).

[11]. Roberts, J.K., Miller, A.R., 1951.Heat and Thermodynamics. 4th ed. Blackie, London, 274 p.

[12]. AnAge: The Animal Ageing and Longevity Database. Available at: http://genomics.senescence.info/species/ (accessed 24 April, 2019).

[13]. Database of the Federal State Statistics Service [Electronic source] office.site. URL: http://www.gks.ru/ (accessed 7 April, 2019).