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SUBBAND ANALYSIS AND SYNTHESIS OF SIGNALS

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Abstract: Currently, a wide class of modern wireless telecommunications systems is based on the use of broadband signals. The problems of synthesis for their formation and processing algorithm are extremely important for such systems, especially under the conditions of various types of disturbances, which is impossible without scientifically based analysis of signal properties.

The article presents the results of signal property and synthesis sub-band analysis obtained by the division of the Fourier cosine transformation (transformants) definition domain for signal record into sub-bands (frequency representations). It is shown that the basis of the mathematical apparatus proposed for sub-band analysis and signal synthesis is a new class of matrices, called sub-band ones. The use of this mathematical apparatus allows to solve the tasks of sub-band analysis and signal synthesis without the transfer to the cosine transformation definition domain (without its calculation, but in the signal definition domain directly).
Some properties of the eigenvalues for these matrices are established; the problem of additive signal component optimal selection is formulated and solved.

Keywords: sub-band property analysis and signal synthesis, cosine transform, sub-band matrices, optimal sub-band properties of signals, energy share of an eigenvector in a given frequency interval.

I. INTRODUCTION

The problems of property analysis and synthesis of signal segments arise during many fundamental and practically important radio-physical problems, such as the analysis of non-equilibrium fluctuation processes in nonlinear media with powerful electromagnetic effects, the study of non-equilibrium modes of modern small electronic system element operation, the improvement and the optimization of precision measuring systems during the development of new classes of signal-code designs for modern telecommunication systems, particularly during their functioning in terms of various kinds of disturbances [1].

In this paper, based on the analysis of properties and the synthesis of signal segments $x(t), t \in [0,T]$ using discrete samples $x_k = x(k\Delta t), k = 1,..,N$, the following transformation is considered at the interval Δt:

$$X(z) = \sum_{k=1}^{N} x_k \cos(zk) , \quad (1)$$

which is periodic obviously and therefore the next interval of z axis is considered as the definition domain:

$$z \in [0,2\pi] , \quad (2)$$

Then the left side of (1) is called the transform of the cosine transformation.

Let's note that in the literature [2], the discrete set of interval points (2) is usually considered as the domain of the transformant definition (2), and therefore it is called the discrete cosine transform. It seems natural that the relation (1) with a continuous domain of transformant definition (2) is called the cosine transformation.

II. METHODOLOGY

The continuity of definition domain (2) makes it possible to formulate and solve the problems of sub-band analysis and synthesis for signal segments based on natural optimality criteria.

Moreover, taking into account the trigonometric identity

$$\cos((2\pi - z)k) = \cos(zk) \quad (3)$$

the following symmetry takes place

$$X(z) = X(2\pi - z), \; 0 \le z < \pi , \quad (4)$$

which allows us to consider only half of the definition domain (2).

In turn, the property of orthogonality is the following one:

$$2\int_0^{\pi} \cos(zk)\cos(zm)dz / \pi = \delta_{im} , \quad (5)$$

where δ_{im} - the Kronecker symbol [3], the following representation (inverse transformation) is true:

$$x_m = 2\int_0^{\pi} X(x)\cos(zm)dz / \pi , \quad (6)$$

as well as Parseval's equality with respect to Euclidean norms (energy)

$$\sum_{k=1}^{N} x_k^2 = 2\int_0^{\pi} X^2(z)dz / \pi \quad (7)$$

The last relation can be easily presented in a subband form.

$$\| \vec{x} \|^2 = \sum_{k=1}^{N} x_k^2 = \sum_{r=1}^{R} E_r(\vec{x}) , \quad (8)$$

where $\vec{x} = (x_1,...,x_N)'$; prime means transposition;

$$E_r(\vec{x}) = 2 \int_{Z_{1,r-1}}^{Z_{2,r}} X^2(z)dz / \pi . r = 1,...,R ; \quad (9)$$

$$Z_{1,0} = 0; Z_{2,R} = \pi . \quad (10)$$

It seems natural to describe the characteristic of the form (9) as the part of the signal segment energy that falls into the corresponding frequency interval. From the standpoint of signal analysis, it is advisable to consider the proportions of energy:

$$P_r(\vec{x}) = E_r(\vec{x}) / \| \vec{x} \|^2 , \quad (11)$$

Which characterize its relative distributions in the frequency domain.

Interest in such an analysis is associated with the possible presence of a quasi-periodicity in the segment. In the ideal case, when the signal samples are formed according to the ratio

$$x_k = \cos(vk), v \in (0,\pi) , \quad (12)$$

the following presentation is fair:

$$4X(z) = (\sin((2N+1)(z-v)/2) / \sin((z-v)/2) + \sin((2N+1)(z+v)/2) / \sin((z+v)/2) - 2)$$

It is easy to understand that the first term in brackets will reach the maximum value at the point coinciding with the frequency of the signal being analyzed, and about 90 percent of the energy will be concentrated in its vicinity of the following width:

$$\Delta z = 4\pi / (2N+1) \quad (13)$$

This example illustrates the usefulness of signal property consideration from the standpoint of some partitioning into sub-bands of the cosine transform transformant domain (sub-band analysis).

Another important trend of sub-band analysis is the selection of additive components from the signal (filtering):

$$\vec{x} = \vec{y}_r + \vec{u} , \quad (14)$$

based on idealized condition

$$Y_r(z) = X(z), \; z \in Z_r ; \; Y_r(z) \equiv 0, z \notin Z_r , \quad (15)$$

where subband is meant

$$Z_r = [Z_{1r}, Z_{2r}) ; \quad (16)$$

$$Y_r(z) = \sum_{k=1}^{N} y_{kr} \cos(zk) . \quad (17)$$

The generalization of this problem is to obtain the following on the basis of (15) subband components that satisfy the additivity relation:

$$\vec{x} = \sum_{r=1}^{R} \vec{y}_r , \quad (18)$$

At that, an important condition is to ensure that the components of the vectors in (18) depend only on the transformant (1) segments in the corresponding subbands.

Let's note that, in particular, such a formulation of the filtering problem corresponds to the conditions for information transfer with frequency multiplexing.

In turn, during the synthesis of signals, a great importance is attached to optimal subband signal property provision in the sense of a variational problem solution:

$$E_r(\vec{x}) = \max E_r(\vec{y}), \forall \vec{y} \in R^N \quad (19)$$

with the following limitation:

$$\| \vec{y} \|^2 = 1 . \quad (20)$$

The further content of this work is connected with the mathematical apparatus development that allows one to solve the problems of sub-band analysis and synthesis without going over to the domain of the cosine transform definition (without its calculation, but in signal definition domain directly).

Subband matrices and some of their properties

An adequate mathematical apparatus for subband analysis and signal synthesis in the domain of original definition can be developed if we put representation (1) in definition (9) and perform the corresponding integration after the permutation with summations. Thus, we obtain the following:

$$E_r(\vec{x}) = \vec{x}' B_r \vec{x}, \qquad (21)$$

where $B_r = \{b_{ik}^r\}; i,k = 1,...,N$ is the subband matrix with the following elements

$$b_{ik}^r = 2\int_{Z_{1r}}^{Z_{2r}} \cos(zi)\cos(zk)dz/\pi = a_{ik}^r + c_{ik}^r; \qquad (22)$$

$$a_{ik}^r = 2\sin((Z_{2r} - Z_{1r})(i-k)/2)\cos((Z_{2r} + Z_{1r})(i-k))/\pi(i-k) \qquad (23)$$

$$c_{ik}^r = 2\sin((Z_{2r} - Z_{1r})(i+k)/2)\cos((Z_{2r} + Z_{1r})(i+k))/\pi(i+k) \qquad (24)$$

It is clear that subband matrices are symmetric, and the relation (21) together with the definition (9) speak of their non-negative definiteness. Therefore, [4] the following representation is true:

$$B_r = Q_r L_r Q_r' \qquad (25)$$

where $Q_r = \{\vec{q}_1^r .. \vec{q}_N^r\}$ is the orthonormal matrix of eigenvectors, so

$$Q_r' Q_r = Q_r Q_r' = I = diag(1,...,1); \qquad (26)$$

$L_r = diag(\lambda_1^r,...,\lambda_N^r)$ - the diagonal matrix of eigenvalues, so that the following equalities take place:

$$\lambda_k^r q_{ik}^r = \sum_{m=1}^{N} b_{im}^r q_{mk}^r. \qquad (27)$$

Then, for simplicity, we assume that the non-negative eigenvalues and the corresponding eigenvectors are ordered by decrease.

$$\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_N \geq 0. \qquad (28)$$

Let's note that the set of eigenvectors of any of the subband matrices is an orthonormal basis, complete in the space of N - dimensional real vectors R^N. In other words, any vector $\vec{u}$ of such a dimension can be represented as a linear combination:

$$\vec{u} = \sum_{k=1}^{N} \alpha_{kr} \vec{q}_k^r, \qquad (29)$$

and the following relations are fair:

$$\alpha_{kr} = (\vec{u}, \vec{q}_k) = \sum_{m=1}^{N} u_m q_{mk}^r \qquad (30)$$

A more detailed study of eigenvector properties and subband matrix numbers is of interest.

Taking into account the definition (22), the relation (27) provides the following:

$$\lambda_k^r q_{ik}^r = 2\int_{Z_{1r}}^{Z_{2r}} G_k^r(z)\cos(zi)dz/\pi, \qquad (31)$$

where

$$G_k^r(z) = \sum_{m=1}^{N} q_{mk}^r \cos(zm). \qquad (32)$$

Thus, the components of the eigenvectors are completely determined by the cosine transformation segments in the corresponding subband.

In accordance with the definition of the scalar product (30) it is easy to get the following equality from (31):

$$\lambda_k^r(\vec{q}_k^r, \vec{q}_i^r) = 2\int_{Z_{1r}}^{Z_{2r}} G_k^r(z)G_i^r(z)dz/\pi. \qquad (33)$$

Thus, the property (26) implies the dual orthogonality property of eigenvector cosine transformations for subband matrices (orthogonality in the subband and in the whole definition domain):

$$2\int_{Z_{1r}}^{Z_{2r}} G_k^r(z)G_i^r(z)dz/\pi = 2\int_{0}^{\pi} G_k^r(z)G_i^r(z)dz/\pi = 0, i \neq k. \qquad (34)$$

Assuming that the subscripts of eigenvectors coincide, taking into account their normalization from (33), we obtain the following equality:

$$\lambda_k^r = 2\int_{Z_{1r}}^{Z_{2r}} (G_k^r(z))^2 dz/\pi. \qquad (35)$$

Thus, the eigenvalue is equal to the energy fraction of the corresponding eigenvector falling into the subband. Here, the normalization of eigenvectors (26) is taken into account, and from the validity of Parseval's equality (7) we obtain the important inequality:

$$\lambda_k^r \leq 2\int_{0}^{\pi} (G_k^r(z))^2 dz/\pi = 1, \qquad (36)$$

which clarifies the subband properties of the component combinations (29). In particular, the ratio for their coefficients (30) in the domain of the cosine transformation definition has the following form:

$$\alpha_{kr} = 2\int_{0}^{\pi} U(z)G_k^r(z)dz/\pi. \qquad (37)$$

Therefore, if an eigenvector corresponds to an eigenvalue close to unity, then the following approximate equality will be true:

$$\alpha_{kr} \approx 2\int_{Z_{1r}}^{Z_{2r}} U(z)G_k^r(z)dz/\pi, \qquad (38)$$

that is, only the dependence on the signal transformant segment shows up in the subband.

In turn, if the eigenvalue is close to zero, then the following approximation takes place:

$$\alpha_{kr} \approx 2\int_{z \notin Z_r} U(z)G_k^r(z)dz/\pi, \qquad (39)$$

that is, in this case, the coefficient of the linear combination (30) reflects the signal properties that appear outside the considered subband.

On the basis of computational experiments in the framework of this work, they established that only:

$$J_r = [N(Z_{2r} - Z_{1r})/\pi] + 2 \qquad (40)$$

of eigenvalues are significantly different from zero in the sense that the following inequality is performed:

$$1 - \sum_{m=1}^{J_r} \lambda_m^r / trecB_r << 1 \qquad (41)$$

where the denominator symbol means the trace of the matrix. They take into account the fact that the trace of a symmetric non-negative definite matrix is equal to the sum of its eigenvalues.

Analysis and synthesis of signals based on subband matrices.

Within this section, the relationships are given that allow for the computation of the above defined subband characteristics.

First of all, let's note that in accordance with the representations (21), (25) and the property (40), the relation is valid, which can significantly reduce the complexity of energy part calculations as compared with the quadratic form (21)

$$E_r(\vec{x}) = \sum_{m=1}^{J_r} \lambda_m^r \alpha_{mr}^2 . \qquad (42)$$

During the selection of signal components, it is proposed to use a subband measure of two signals proximity, which has the following form:

$$\phi(\vec{x}, \vec{y}) = E_r(\vec{x} - \vec{y}) == 2 \int_{Z_{1r}}^{Z_{2r}} (X(z) - Y(z))^2 dz / \pi = (\vec{x} - \vec{y})' B_r (\vec{x} - \vec{y}) \qquad (43)$$

Then, as the measure of the ideal requirement fulfilling accuracy (15), it is natural to use the following functional:

$$\Phi_r(\vec{x}, \vec{y}_k) = \beta E_r(\vec{x} - \vec{y}_k) + (1 - \beta)(\| \vec{y}_k \|^2 - E_r(\vec{y}_k)) , \qquad (44)$$

where β determines the weight component of the performance inaccuracy of either the first condition or the second one, and

$$0 \le \beta \le 1 . \qquad (45)$$

Let's note that the second term on the right-hand side of (44) is obtained on the basis of the Parseval's equality of the form (7) in respect of desired vector. To derive the relation for the desired vector, it is natural to use the functional minimization (44). The result of this variation problem solution is the following representation:

$$\vec{y}_r^{\beta} = \beta(\beta I + (2\beta - 1)B_r)^{-1} B_r \vec{x} . \qquad (46)$$

An important special case is equal weight when

$$\beta = 0,5 , \qquad (47)$$

which gives (the superscript is not used here for record simplicity)

$$\vec{y}_r = B_r \vec{x} . \qquad (48)$$

Bearing in mind the representations (23) and (24), it is easy to show the validity of the following formula

$$\sum_{r=1}^{R} B_r = \sum_{r=1}^{R} A_r = I = diag(1,...,1) \qquad (49)$$

Therefore, the formula (48) provides the following:

$$\sum_{r=1}^{R} \vec{y}_r = \vec{x} , \qquad (50)$$

which meets the additivity requirement (18) of the separated components.

In turn, the component record of the representation (48) has the following form:

$$y_{mr} = \sum_{k=1}^{N} b_{mk}^r x_k .$$

The substitution of element definition code for the subband matrix (22), taking into account the definition of the cosine transform (1), provides the following formula:

$$y_{mr} = 2 \int_{Z_{1r}}^{Z_{2r}} X(z) \cos(zm) dz / \pi \qquad (51)$$

Thus, the components of the vectors allocated on the basis of (48) are completely determined by the transformant segment of the original signal in the subband. This property was formulated as the second main requirement for the properties of the selected additive components in the first section of the work.

It can be shown that the vectors of the form (46) also satisfy the component dependence requirement on the segments of the cosine transformation transformant in the band under consideration. To do this, the right-hand side of (46) has to be substituted by the representation (25) of the subband matrix and the orthonormal property (26) of eigenvector matrix has to be taken into account. Thus, it is not difficult to get the following formula:

$$\vec{y}_r^{\beta} = Q_r L_r^{\beta} Q_r' \vec{x} , \qquad (52)$$

Where,

$$L_r^{\beta} = diag(\mu_1^{\beta r}, ..., \mu_N^{\beta r}) ;$$

$$\mu_k^{\beta r} = \beta \lambda_k^r / (\beta + (2\beta - 1)\lambda_k^r), k = 1,...,N . \qquad (53)$$

Thus, the formula (53) can be rewritten in the following form:

$$\vec{y}_r^{\beta} = Q_r L_r^{\beta} \vec{\alpha}_r , \qquad (54)$$

where the vector components are determined by the relations (30). Comparing (3) and (31) we get the following:

$$\lambda_k^r \alpha_{kr} = 2 \int_{Z_{r-1}}^{Z_r} G_k^r(z) \sum_{m=1}^{N} x_m \cos(zm) dz / \pi = 2 \int_{Z_{r-1}}^{Z_r} G_k^r(z) X(z) dz / \pi . \qquad (55)$$

Thus, in accordance with (54) and (56)

$$\gamma_{kr}^{\beta} = \mu_{kr}^{\beta}\alpha_{kr} = 2\int_{Z_{r-1}}^{Z_r} G_k^r(z)X(z)dz/(\pi(\beta + (2\beta - 1)\lambda_k^r) . \tag{56}$$

the coefficients depend only on the transformant segment of the cosine transformation from the considered subband. Therefore, the representation of the sought component obtained from (55):

$$\vec{y}_r^{\beta} = \sum_{k=1}^{N} \gamma_{kr}^{\beta}\vec{q}_k^{\,r} \tag{57}$$

has the same property.

It is clear that the thing is about the finiteness property (40) of nonzero eigenvalues of a subband matrix, and in this case one can assume that

$$\mu_k^{\beta r} \cong 0, k > J_r . \tag{58}$$

Therefore, the formula (57) can be transformed to the next computationally efficient form:

$$\vec{y}_r^{\beta} = \sum_{k=1}^{J_r} \gamma_{kr}^{\beta}\vec{q}_k^{\,r} . \tag{59}$$

Now let's consider the problem of subband synthesis of signals formulated in the first section of the article on the basis of variational problem (19), (20). To solve this problem, we use the representation (42), which, taking into account the ordering conditions (28), provides the following inequality:

$$E_r(\vec{y}) \le \lambda_1^r \sum_{k=1} \alpha_{kr}^2 = \lambda_1^r \| \vec{y} \|^2 . \tag{60}$$

Here they take into account that the terms of the first sum of the right-hand side are obtained after the orthonormal transformation and, therefore, the sum of their squares is equal to the square of the original vector Euclidean norm.

In the formula (60), the sign of equality is achieved on the eigenvector of the subband matrix corresponding to the maximum eigenvalue, so that the solution of the variational problem (19), (20) has the following form:

$$\vec{y} = \vec{q}_1^{\,r} . \tag{61}$$

III. SUMMARY

The article deals with the problem of subband analysis of properties and signal synthesis obtained by the Fourier transform domain (transformants) splitting into sub-bands of the signal samples (frequency representations). It is shown that the basis of the mathematical apparatus proposed for use in subband analysis and signal synthesis is a new class of matrices, called subband ones. Mathematical expressions are obtained, which allow estimating the fraction of the eigenvector energy infiltration beyond the selected frequency interval (subbands). As an important concretization of the proposed approach they considered the application of this mathematical apparatus for the solution of subband analysis and signal synthesis problem without going into the cosine transform definition domain (without its calculation, but in the signal definition domain directly).

Some properties of the eigenvalues of these matrices are determined; the problem of optimal selection of signal additive components is formulated and solved.

The relevance of the results is conditioned by the fact that at present one of the main approaches used for property analysis and the synthesis of various signals is the subband methodology, which is carried out from the position of the Fourier transform analysis (transformants) of the signal samples (frequency representations) into subbands. At that, the main tool of subband analysis, including wavelet analysis, which is widely used for these purposes, is bandwidth frequency filters, and first of all, the filters with finite impulse response (FIR filters) [2.5-12]. The approach of subband analysis and signal synthesis developed in the article is the contribution to the solution of this problem, which allows the formation of signals with a minimum level of their energy leakage beyond the allocated frequency interval.

IV. CONCLUSIONS

The results obtained can be used in the design, according to the principles of intellectual technology, called "cognitive radio" [13-15], a number of wireless telecommunication systems and their components.

V. REFERENCES

[1]. W. Zhou et al., "Power Frequency Interference and Suppression in Measurement of Power Transmission Tower Grounding Resistance," in IEEE Transactions on Power Delivery, vol. 30, no. 2, pp. 1016-1023, April 2015. doi: 10.1109/TPWRD.2014.2370037.

[2]. Malle S. Wavelets in signal processing. M.: Mir, 2005. Accessed from https://scholar.google.com/scholar?q=%22S.%20Malla,%20Wavelets%20in%20signal%20processing,%20Mir,%20Moskow,%202005.%22

[3]. Lanczosh K. Practical methods of applied analysis. M.: Nauka, 1961.

[4]. Gantmakher F.R. Theory of matrices. Moscow, Nauka, 1967. Accessed from https://books.google.co.ke/books?hl=en&lr=&id=ePFtMw9v92sC&oi=fnd&pg=PR3&ots=slbgpF9RG8&sig=auOpoh0XrqVZqdAmAnspbg5SsIY&redir_esc=y#v=onepage&q&f=false

[5]. Balashov V.A., Vorobienko P.P., Lyakhovetsky L.M. Transmission systems with orthogonal harmonic signals. M.: Ecotrends, 2012. – 228 p.

[6]. Chobanu M. Multidimensional Multi-Speed Systems of Processing of a Signal(Mnogomernye mnogoskorostnye sistemy obrabotki signalov), Moscow, 2009, 480 p. (in Russian.)

[7].Evgeny G. Zhilyakov, Sergey P. Belov, Denis V. Ursol About signals allowing to provide sustainability to impacts of short-term and or focused on spectrum interference. International Journal of Engineering & Technology, 7 (2.2) (2018). pp.16-20.

[8]. Victor Golikov, Marco Rodriguez-Blanco, Olga Lebedeva Robust multipixel matched subspace detection with signal-dependent background power// Journal of Applied Remote Sensing, Jan–Mar 2016 • Vol. 10(1). pp. 1975-1983.

[9]. Abramovits M., Stigan I.A (edited.). Hand book of Mathematical Functions with formulas, graphs and mathematical tables, National Bureau of Standards, Applied Mathematics, Washington, D.C., 1979. – 832 p.

[10]. Zhilyakov, E.G. Optimal sub-band methods for analysis and synthesis of finite-duration signals ,2015, Automation and Remote Control 76(4). pp. 589-602.

[11]. Zhilyakov, E.G. Constructing trends of time series segments, 2017, Automation and Remote Control, 78(3). pp. 450-462.

[12]. Mallat, S., A. Wavelet Tour of Signal Processing, New York: Academic Press, 1999. Translated under the title, 368 p.

[13]. Guryanov I.O. Cognitive radio: new approaches to RF resource provisioning for advanced radio technologies (Kognitivnoye radio: novyye podkhody k obespecheniyu radiochastotnymi resursami peredovykh radiotekhnologiy) / Elektrosvyaz, No. 8, 2012, pp.5-8. [in Russian]

[14]. Stevenson CR, Cordeiro C, Sofer E, Chouinard G. Functional Requirements for the 802.22 WRAN Standard. IEEE 802.22-05/0007r46. New York, NY, USA: IEEE, 2005

[15]. 15. Theory and methods of statistical solutions in radio engineering and communications: Textbook / Zhilyakov EG, Firsova AA. Belgorod: Publishing house of the National Research University "BelGU", 2014. 51-67 p.