Multiscale Approach of Retinal Blood Vessels Segmentation Based on Vessels Segmentation with Different Scales

Chernomorets Daria, Mikhelev Vladimir, Chernomorets Andrey
Institute of engineering technologies and natural sciences
Belgorod State University (BelSU)
Belgorod, Russia
daria013ch@yandex.ru

Abstract—**In this work, we developed retinal blood vessels segmentation approach using contrast limited adaptive histogram equalization, morphological filtering, k-means clustering, matched filtering for thin and thick vessels selection. We also applied matched filtering for thin vessels selection using the kernels which were built in order to determine the existence of line segments with different length and orientation. Our approach has shown promising results.**

Keywords—eye fundus; segmentation; morphological filtration; k-means clustering; matched filtering

I. Introduction

The analysis of state of the circulatory system's vessels of the eye fundus is of great interest in the diagnosis and treatment of various diseases such as hypertension, diabetic retinopathy, stroke, and cardiovascular diseases.

The eye fundus is the only part of the human body, the circulatory system of which can be observed directly. The circulatory system of the eye fundus consists of arteries and veins, which can be observed in the images of the eye fundus (fig. 1).

With the correct selection (segmentation) of blood vessels in the image, it becomes possible to supply a more accurate diagnosis, and it is important in the treatment of the patient.

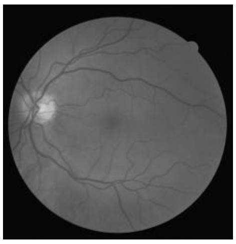

Fig. 1. Image of eye fundus.

The selection of the vascular system manually is a rather complex process, taking a considerable amount of time and effort, and sometimes impossible due to the too complicated structure of the vascular tree or low PSNR in the image [1].

Because of these problems, the computer analysis of eye fundus images has become widespread, it has become the main tool for medical diagnostic systems, which greatly improved the quality of diagnostics. Modern medicine is one of the most high-technology industries, the most important task of which is the development of new effective methods for early diagnosis of various pathologies. Now almost all methods of examination in ophthalmology, cardiology and other fields of medicine are computerized.

Segmentation (recognition) of vessels and determination of their morphological features are the key stages of automated methods of diagnostic analysis of the vascular system, because the results of diagnosis depend on the accuracy of the selection and measurement of vessels elements.

II. The Main Approaches to Select Retinal Vessels

Now the main approaches to select retinal vessels on the eye fundus images deal with the segmentation of blood vessels, allowing to select the vessel tree in one iteration, and deal with the tracking of vessels (trace).

Next approaches are used to develop algorithms of retinal blood vessels segmentation.

A. *Matched filtering*

Matched filtering is a convolution of the image with the two-dimensional filter. Kernel of filter models some details of the image, the filter response indicates the presence or absence of the required parts. Kernel of filter is based on the three main assumptions:

1. the vessels can be approximated by the broken line;

2. the diameter of the vessels decreases while the moving from the optical disk;

3. the cross sectional profile of the vessel is similar to the Gaussian curve.

The use of matched filtering is quite effective in combination with other methods of image processing [3].

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B. Morphological processing

Morphological processing assumes that the vessels are composed of interconnected linear segments. Morphological operations are used for the vessels segmentation and the selection of microaneurysms [4-6].

Two similar operations are used during the segmentation of medical images: TopHat transformation [7] and the watershed method [8]. Vessels emphasis is performed using TopHat transformation (its effect is associated with using of closing and opening morphological operations), it estimates the local background and then it subtracts it from the original image, therefore we select the vessels. A priori we assume that the basic structure of the vascular system can be represented by a set of connected linear segments.

Morphological processing for certain forms identification has the advantage of high speed and resistance to noise.

The main disadvantage of the exclusively morphological methods using is that they do not use information about the shape of the vessels. In addition, the searching of oblong structures may cause the loss of strongly twisting vessels [2].

C. Multiscale analysis

The cross profile of a vessel can be approximated by Gauss's function, in local scale it is similar to a rectilinear object with smoothly decreasing diameter. The basic idea of using the spatially-scale analysis for vessels extraction is to define the vessels information which is scale independent [9, 10]. It is possible to consider multiscale structures of the second order (Hesse's matrix) for vessels selection filter development. The algorithm of vessels retrieval is based on the analysis of eigenvalues and eigenvectors of the Hesse matrix. It defines the main directions in which the local structure can be decomposed into the components giving the directions of the smallest curvature along the vessel channel.

D. The approaches based on models

These approaches use obvious vessels models for extraction of vascular system [11]. These approaches are divided into 2 categories: (1) vessel profile models; (2) deformable models.

1) Vessel profile models. In this approach the cross profile of a vessel is approximated by means of Gauss's curve or by a combination of the curves in case of the central reflex presence. Also the second derivative of Gauss function, cubic splines or Hermite's polynoms can be used. More difficult models join bright or dark damages of a retina and the characteristics of a background to increase segmentation accuracy on difficult images.

2) Deformable models. Techniques of vessels segmentation that base on deformable models are divided into 2 categories [12]: (a) parametrical models; (b) geometrical deformable models.

a) Parametrical models. The active contours method also known as snakes [13] is based on object contour form changing under the influence of internal and external forces. External and internal forces are arranged in such a way that under their influence the contour will be compressed upto object borders. Ability to adapt to objects of various shapes and ability to find states with the energy minimum in which internal and external forces are counterbalanced belongs to the advantages of this method in comparison with other approaches. The method of active contours can be used for tracking of objects both in space and in time. The main restriction of this method is using of information about object borders only, without other parameters of the image. It is necessary to set the contour initial position rather close to an object for correct use of the method. The active contours method is often used in such areas as trace, recognition of object shape, segmentation and delimitation of an object.

b) Geometrical models. The geometrical models of active contours are based on the theory of geometrical shapes evolution. These models are usually implemented with use of numerical algorithms [12]. One of such models realization, the LSM method (level set method), is a numerical algorithm to trace of certain surfaces and shapes [14]. Advantage of a LSM method is that numerical processing of various curves and surfaces can be carried out without indication of these objects parameters.

III. METHOD

The analysis of the eye fundus images characteristics [2] showed that the brightness of the pixels corresponding to thick blood vessels have a greater contrast to the brightness of the background compared to the brightness difference of the background pixels with the brightness of the pixels corresponding to the thin vessels.

Considering this fact, we proposed to perform the selection on the image separately the thin and thick vessels using multiscale approach.

Preliminary computational experiments that were implemented by authors showed that when clustering pixels of the eye fundus image for k classes using the values of their brightness, the senior class S_k contains majority of the pixels corresponding to thick blood vessels, while a significant number of pixels corresponding to all the blood vessels of the eye fundus are contained in two senior classes S_{k-1} and S_k (which will be demonstrated later).

We think that one of the main objectives during development of multiscale approach to vessels segmentation is the selection of thin vessels from class S_{k-1} which also contains the non-vessels pixels. For this purpose we applied matched filtering [3] for thin vessels selection using the kernels which were built in order to determine the existence of line segments with different length and orientation.

Thus, the main steps of the procedure segmentation of blood vessels of eye fundus can be formulated as follows:

1. To execute preliminary processing of the image in order to remove noises.

2. To carry out a clustering of a set of the processed image I_0 pixels to k classes.

3. To execute segmentation of thick vessels on the eye fundus image I_{B0} (to create the binary image, using pixels of the senior class S_k).

4. To create the image I_1 that contains pixels of class S_{k-1} .

5. To execute the multiscale processing allowing to select thin vessels (binary image I_{B1}) on the image I_1 .

6. To add result I_{B1} of thin vessels segmentation to the image I_{B0} which is result of segmentation of thick vessels.

There is often a need of preprocessing of the image by means of various filters during the solution of problems of retinal blood vessels segmentation because segmentation of the initial image usually doesn't give the expected results due to irregular illuminating intensity, insufficient contrast, etc.

In this work we used the algorithm [15] of preprocessing of the eye fundus image which essence is as follows:

1. To transform the initial color image, which dimension is $N_1 \times N_2$ pixels, to the grayscale image (fig. 2a).

2. To increase the contrast of the grayscale image, using a method of contrast limited adaptive histogram equalization (CLAHE) (fig. 2b). Let I – result of performance of this step.

3. The morphological filtering is carried out for further selection of vessels (fig. 2c):

$$I_0 = I - (I \bullet Se) \circ Se \, , \tag{1}$$

where I_0 – the image received during a morphological filtering, I – the image after a contrast limited adaptive histogram equalization, Se – a structural element (in work we applied the disk with a radius of 8 pixels), operations $\circ$, $\bullet$ – morphological operations of open and close.

Fig. 2. Results of the image processing:
a – the initial grayscale image, b – result of application of contrast limited adaptive histogram equalization, c – result of application of a morphological filtering.

In this work we suggested to realize clustering of a pixels set of the processed image I_0 on k classes S_1, S_2,…, S_k on the basis of the k-means method, which is the one of the most widespread methods of a clustering [16, 17].

In case of application of the k-means method partitioning on classes is carried out in such way to minimize a summary square deviation σ of clusters points from centers of these clusters:

$$\sigma = \sum_{i=2}^{k} \sum_{x_j \in S_i} (x_j - m_i)^2 , \tag{2}$$

where k – number of clusters, S_i – the received classes, $i = 1,2,\ldots,k$, m_i – center of masses of points $x_j \in S_i$.

For formation binary image I_{B0} , that correspond to thick vessels of the eye fundus (fig. 3a), pixels of the senior class S_k are used:

$$I_{B0}(i,j) = \begin{cases} 1, & (i,j) \in S_k, \\ 0, & \text{otherwise}, \end{cases} \quad i = 1,2,\ldots,N_1, \quad j = 1,2,\ldots,N_2 \tag{3}$$

Further the image I_1 (fig. 3b) containing values of pixels which have been included in a class S_{k-1} is formed:

$$I_1(i,j) = \begin{cases} I_0(i,j), & (i,j) \in S_{k-1}, \\ 0, & \textit{otherwise}, \end{cases} \quad i = 1,2,\ldots,N_1, \quad j = 1,2,\ldots,N_2 \tag{4}$$

To increase contrast of thin vessels on the image I_1 (4) in work we used the matched filtering $F(I_1, h_L, \theta)$ which is based on convolution of the image I_1 with a kernel h_L , which has dimensional $L \times L$ pixels and correspond to a segment which is carried out at an angle θ to a horizontal axis. Parameter L accepts values {7, 9, 11, 13, 15}, parameter θ accepts values $\theta_l = 15(l-1)$ of degrees, $l = 1,2,\ldots,12$.

In this case the matched filtering allows to allocate linearly extended areas on the image I_1 which have various scale and various orientation (in the fig. 4 at $L = 15$ the kernel elements, that are equal 1, are displayed in the form of white squares, and that are equal 0 – in the form of black squares).

In the article we offered to use the result of the matched filtering (the image I_{S1} (fig. 5a)) for the pixels belonging to class S_{k-1} only:

$$I_{S1}(i,j) = \begin{cases} I_{F1}(i,j), & (i,j) \in S_{k-1}, \\ 0, & \textit{otherwise}, \end{cases} \quad i = 1,2,\ldots,N_1, \quad j = 1,2,\ldots,N_2 \, , \tag{5}$$

where

$$I_{F1} = \sum_{\alpha=3}^{7} \sum_{l=1}^{12} F(I_1, h_{2\alpha+1}, \theta_l) \tag{6}$$

For selection of thin vessels in work we applied the k-means method to split a set of pixels of the image I_{S1} (5) on n classes R_1, …, R_n, at the same time pixels of the senior class R_n are considered as pixels of thin vessels.

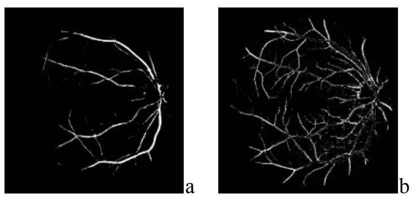
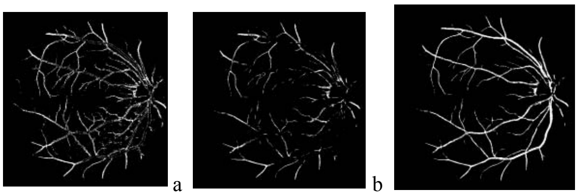


Fig. 3. Binary images that correspond to different classes:
a– binary image I_{B0} of thick vessels (class S_5 , $k = 5$), b– image I_1 correspond to class S_4 .

a b c d e

Fig. 4. Examples of kernels, that are used in matched filtering, $L = 15$: a – when $\theta = 0°$, b – when $\theta = 30°$, c – when $\theta = 90°$, d – when $\theta = 135°$, e – when $\theta = 150°$.



a b c

Fig.5. Vessel images: a– result I_{S1} of implementation of matched filtering, *b*– Binary image I_{B1} of thin vessels (class R_3 , $n = 3$), *c*– Result I_{vessel} of developed method implementation.

Then, the creation of the binary image containing thin vessels (fig. 5b) is carried out on the basis of the following expression:

$$I_{B1}(i,j) = \begin{cases} 1, & (i,j) \in R_n, \\ 0, & otherwise, \end{cases} \quad i = 1,2,\ldots,N_1, \quad j = 1,2,\ldots,N_2 \tag{7}$$

At the final stage of formation of the eye fundus vessels binary image I_{vessel} the addition of the images I_{B0} (3) and I_{B1} (7), corresponding to thick and thin vessels (fig. 5c), is carried out:

$$I_{vessel} = I_{B0} + I_{B1} \tag{8}$$

IV. Experimental Results

To estimate the operability of segmentation methods in [18] they offered to use next criteria: the accuracy δ_{Acc} , sensitivity δ_{Sn} and specificity δ_{Sp} . These criteria are defined as follows:

$$\delta_{Acc} = (N_{tp} + N_{tn})/(N_{tp} + N_{fp} + N_{tn} + N_{fn}), \tag{9}$$

$$\delta_{Sn} = N_{tp}/(N_{tp} + N_{fn}), \tag{10}$$

$$\delta_{Sp} = N_{tn}/(N_{tn} + N_{fp}), \tag{11}$$

where N_{tp} – amount of correctly identified vessel pixels (true positive), N_{tn} – amount of correctly identified background pixels (true negative), N_{fp} – amount of incorrectly identified vessel pixels (false positive), N_{fn} – amount of incorrectly identified background pixels (false negative).

Accuracy δ_{Acc} (9) indicates the overall segmentation performance, sensitivity δ_{Sn} (10) is a measure of effectiveness in identifying vessels pixels (it is defined as the relation N_{tp} of correctly allocated vessels pixels to total number N_{vessel} of the pixels corresponding to vessels),

$$N_{vessel} = N_{tp} + N_{fn}, \tag{12}$$

specificity δ_{Sp} (11) is a measure of effectiveness in identifying background pixels.

Images from database DRIVE [19] of eye fundus images are used in the further computing experiments.

The purpose of the next computing experiments is to define the correspondence degree of the eye fundus vessels to pixels which are included in the senior classes which are received at the image I_0 by clustering on the basis of the k-means method.

In table 1 there are values of sensitivity δ_{Sn} (10) and first kind error E_{FP} for several separate images from base DRIVE,

$$E_{FP} = N_{fp}/(N_{tp} + N_{fn}), \tag{13}$$

where E_{FP} – relation of N_{fp} incorrectly identified vessel pixels to total number N_{vessel}, for senior class S_k (2), which were received during clustering on the basis of the k-means method for different values of classes amount k.

The data provided in table 1 show that during the pixels clustering on $k = 5$ classes in the senior class S_k there is in most cases a significant amount of the pixels corresponding to eye fundus vessels while the first kind error is insignificant. There is also in most cases an overwhelming number of correctly identified vessel pixels in two senior classes S_k and S_{k-1} .

Reduction of quantity of classes at a clustering increases the number of correctly identified vessel pixels, however, at the same time value of the corresponding error of first kind increases significantly.

For estimation of efficiency of the developed method in work we offered to use comparison with one of the known methods of vessel segmentation which basic postulates are stated in work [15].

Comparison of the developed method with the method that are offered in work [15] was carried out on the basis of criteria (9), (10), (11), using images from base of images DRIVE.

Criteria values (9), (10), (11) of developed method are given in table 2.

TABLE 1. EXAMPLES OF CORRESPONDENCE OF THE EYE FUNDUS VESSELS TO PIXELS WHICH ARE INCLUDED IN THE SENIOR CLASSES WHICH ARE RECEIVED AT THE IMAGE I_0

k \ Image			04_test	08_test	15_test	16_test	18_test.	19_test
5	S_k	δ_{Sn}	0,1897	0,3685	0,3045	0,2246	0,1501	0,3527
		E_{FP}	0,0002	0,0228	0,0051	0,0017	0,0111	0,0042
	$S_k \cup S_{k-1}$	δ_{Sn}	0,6032	0,8335	0,7128	0,6597	0,6919	0,8367
4	S_k	δ_{Sn}	0,3447	0,4381	0,3666	0,4523	0,4132	0,5699
		E_{FP}	0,0031	0,0415	0,0096	0,0160	0,0396	0,0186
	$S_k \cup S_{k-1}$	δ_{Sn}	0,8438	0,8973	0,7951	0,8979	0,9039	0,9533
3	S_k	δ_{Sn}	0,3856	0,4497	0,4906	0,5155	0,5120	0,6116
		E_{FP}	0,0056	0,0461	0,0351	0,0253	0,0633	0,0267
	$S_k \cup S_{k-1}$	δ_{Sn}	0,8886	0,9056	0,9391	0,9471	0,9516	0,9665

The data that are given in the table 2 show that variants of clustering on 5 and 3, 5 and 2 classes, and also on 4 and 3 classes are preferable respectively for segmentation of thick and thin vessels.

At the choice of a clustering on 5 and 3 classes the result of segmentation has rather high accuracy δ_{Acc} (9) and a small error of the first kind E_{FP} (13), however in this case 50-60% (value δ_{Sn} (10)) of the pixels corresponding to vessels are allocated.

At the choice of a clustering on 5 and 2 classes, and also on 4 and 3 classes results of segmentation have comparable values of criteria: at high accuracy δ_{Acc} the first kind error E_{FP} in most cases is slightly less when you use variant of a clustering on 5 and 2 classes, however with clustering variant on 4 and 3 classes more pixels corresponding to vessels are allocated.

TABLE 2. EXAMPLES OF CRITERIA CALCULATION δ_{Acc}, δ_{Sn} AND δ_{Sp} WHEN WE USED THE DEVELOPED METHOD

Image	k	n	δ_{Acc}	δ_{Sn}	δ_{Sp}	E_{FP}
04_test.tif	5	3	0,9482	0,4468	0,9990	0,0101
		2	0,9577	0,5881	0,9952	0,0475
	4	3	0,9031	0,7717	0,9165	0,8247
		2	0,9420	0,7590	0,9605	0,3899
	3	3	0,8801	0,7856	0,8896	1,0895
		2	0,7933	0,8848	0,7840	2,1316
08_test.tif	5	3	0,9066	0,7181	0,9243	0,8039
		2	0,9040	0,7625	0,9173	0,8784
	4	3	0,8253	0,7614	0,8313	1,7922
		2	0,8933	0,7774	0,9042	1,0172
	3	3	0,8083	0,7648	0,8124	1,9926
		2	0,7162	0,8985	0,6991	3,1966
15_test.tif	5	3	0,9623	0,5484	0,9942	0,0751
		2	0,9608	0,7003	0,9809	0,2478
	4	3	0,9588	0,6674	0,9812	0,2438
		2	0,9426	0,7844	0,9548	0,5863
	3	3	0,8616	0,8193	0,8648	1,7537
		2	0,7450	0,9341	0,7305	3,4969
16_test.tif	5	3	0,9517	0,4819	0,9984	0,0166
		2	0,9594	0,7290	0,9822	0,1788
	4	3	0,9586	0,6932	0,9849	0,1520
		2	0,8561	0,8924	0,8525	1,4857
	3	3	0,8441	0,8412	0,8444	1,5676
		2	0,7345	0,9414	0,7139	2,8822
18_test.tif	5	3	0,9588	0,5311	0,9956	0,0509
		2	0,9616	0,6410	0,9892	0,1255
	4	3	0,9310	0,8179	0,9408	0,6883
		2	0,8562	0,8986	0,8526	1,7133
	3	3	0,8297	0,8549	0,8276	2,0038
		2	0,7188	0,9470	0,6992	3,4959
19_test.tif	5	3	0,9620	0,5689	0,9976	0,0271
		2	0,9690	0,8053	0,9839	0,1785
	4	3	0,9613	0,7889	0,9769	0,2556
		2	0,9280	0,8919	0,9312	0,7601
	3	3	0,8261	0,8637	0,8227	1,9601
		2	0,7165	0,9604	0,6944	3,3786

For comparison of efficiency of the developed method application the values of criteria (9), (10), (11) are given in table 3 when we used the method described in work [15].

TABLE 3. RESULTS OF CALCULATING OF CRITERIA δ_{Acc}, δ_{Sn} AND δ_{Sp} WHEN WE USED METHOD [15]

Image	δ_{Acc}	δ_{Sn}	δ_{Sp}
04_test.tif	0,959	0,704	0,984
08_test.tif	0,962	0,782	0,975
15_test.tif	0,959	0,821	0,970
16_test.tif	0,958	0,726	0,977
18_test.tif	0,953	0,808	0,964
19_test.tif	0,956	0,760	0,977

The data that are given in the tables 2 and 3 show that the developed method in some cases has advantage in comparison with method [15] both in values of accuracy δ_{Acc}, and in values of sensitivity δ_{Sn} (identification of vessel pixels) and specificity δ_{Sp} (identification of background pixels).

For the visual analysis of results of using of the developed method of blood vessels segmentation of an eye fundus the corresponding images are given on test images from DRIVE base on the figures 6-11.

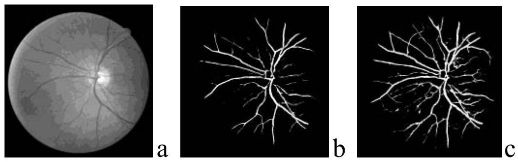


Fig. 6. Segmentation of image 04_test.tif:
a – initial image, b – segmentation at clustering on 5 and 3 classes,
c – segmentation at clustering on 4 and 3 classes.

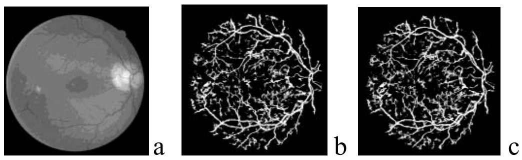


Fig. 7. Segmentation of image 08_test.tif:
a – initial image, b – segmentation at clustering on 5 and 3 classes,
c – segmentation at clustering on 4 and 3 classes.

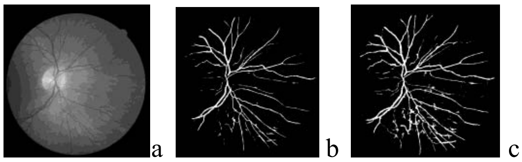


Fig. 8. Segmentation of image 15_test.tif:
a – initial image, b – segmentation at clustering on 5 and 3 classes,
c – segmentation at clustering on 4and 3 classes.

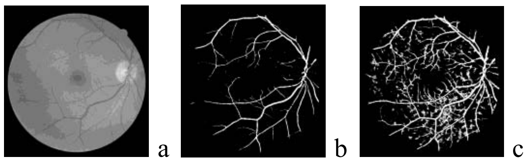


Fig. 9. Segmentation of image 16_test.tif:
a – initial image, b – segmentation at clustering on 5 and 3 classes,
c – segmentation at clustering on 4 and 3 classes.

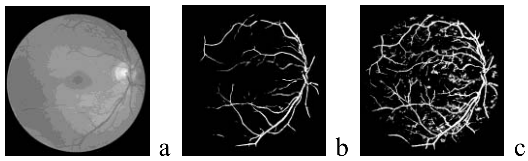


Fig. 10. Segmentation of image 18_test.tif:
a – initial image, b – segmentation at clustering on 5 and 3 classes,
c – segmentation at clustering on 4 and 3 classes

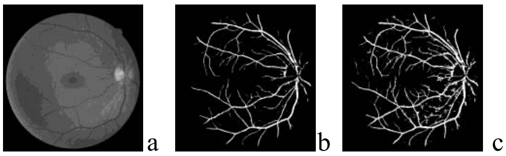


Fig. 11. Segmentation of image 19_test.tif:
a – initial image, b – segmentation at clustering on 5 and 3 classes,
c – segmentation at clustering on 4 and 3 classes.

Thus, results of computing experiments showed operability of the developed method of retinal blood vessels segmentation. Accuracy of calculations is comparable with results of using of the known method [15], and in some cases surpasses them (images 04_test.tif, 15_test.tif, 16_test.tif, 18_test.tif, 19_test.tif). However, in the image 08_test.tif method [15] works better, which is due to the presence of the branched network of thin vessels on this image. On the received binary images the result of the eye fundus vascular system segmentation is accurately visible.

V. CONCLUSION

Automatic segmentation of eye fundus blood vessels is an important step for detection of various pathologies in an eye of the person. In this work, we developed retinal blood vessels segmentation approach using contrast limited adaptive histogram equalization, morphological filtering, k-means clustering, matched filtering for thin and thick vessels selection. Preprocessing of the image allowed to select vessels on an eye fundus more contrastly. We also applied matched filtering for thin vessels selection using the kernels which were built in order to determine the existence of line segments of different length and orientation.

In the Matlab programming environment the program realization of the developed blood vessels segmentation method is developed. It showed its operability.

The method has been tested on images from public base DRIVE. Method has shown the efficiency in correct identification of vessel pixels and background pixels of the image.

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[19] Research Section, Digital Retinal Image for Vessel Extraction (DRIVE) Database, Utrecht, The Netherlands, University Medical Center Utrecht, Image Sciences Institute: http://www.isi.uu.nl/Research/Databases/DRIVE/.