

POLARIZATION BREMSSTRAHLUNG

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This paper briefly reviews the present state of the art of the microscopic theory of polarization bremsstrahlung. Methods for the description of polarization bremsstrahlung at an isolated atom and in a condensed matter, in crystals included, are considered. In the last case, coherent polarization bremsstrahlung appears, which was given the name parametric X radiation. The relation between polarization bremsstrahlung and another type of radiation, such as resonance transition radiation, is followed. It is noted that polarization bremsstrahlung shows promise for structure analysis of materials and for new radiation sources.

In the last three decades a new field of research – polarization bremsstrahlung – has formed in radiation physics.

The essential contribution of the dynamic polarization of a medium to the radiation of a charged particle was first understood by Tamm and Frank [1], and this was a key point in the Vavilov–Cerenkov radiation theory. Actually, a refraction factor other than unity appears as a result of dynamic polarization of the medium, which, in turn, is a prerequisite for the Vavilov–Cerenkov effect. At a discontinuity, such as a boundary between two media, transition radiation (TR), predicted by Ginzburg and Frank [2], appears due to the difference in the refraction factors.

Later it has been shown that the discontinuity that induces a radiation may have the dimensions of the Debye “coat” of a structureless ion [3, 4] or an ion with an electronic structure in plasma and even atomic dimensions [5].

In the theory of bremsstrahlung at an atom, the term in the transition amplitude responsible for PBR appears in a consistent formulation of the problem of bremsstrahlung. This theory was developed in [6–9].

The original version of the quantum theory of bremsstrahlung, which is valid over a wide range of frequencies, was formulated in the classical works of Bethe and Heitler, and by Zauter and Sommerfeld [10] soon after the appearance of quantum mechanics. It is based on a certain physical pattern; namely, it is assumed that an incident electron or some other incident particle (proton, positron, etc.) is dissipated in the field of an atomic nucleus screened by the atomic electrons. Obviously, the incident and the atomic electron are described variously. For the interaction of the incident particle with the electromagnetic field, the interaction of the electromagnetic field with the atomic electrons is disregarded. The fact that each atomic electron has a kinetic energy is ignored as well.

Thus, there are two statements of the problem: The first one, exact, is associated with the Hamiltonian

$$\hat{H} = \hat{H}_e(\mathbf{r}) + \hat{H}_a(\mathbf{r}_a) + V_{fe}(\mathbf{r}) + V_{fa}(\mathbf{r}_a) + V_{ea}(\mathbf{r}_a, \mathbf{r}), \quad (1)$$

while the second one, approximate, leads to the Bethe–Heitler formula. This statement of the problem is associated with the Hamiltonian

$$\hat{H}_{\text{BH}} = \hat{H}_e(\mathbf{r}) + V_{fe}(\mathbf{r}) + V_{ea}(\mathbf{r}). \quad (2)$$

In formulas (1) and (2), \mathbf{r} and \mathbf{r}_a are, respectively, the coordinates of the incident particle and the coordinates of the atomic electrons in total; \hat{H}_e and \hat{H}_a are the respective Hamiltonians of the incident particle and the atom; V_{fe} , V_{fa} , and V_{ea} are the potential energies of the interaction of the incident particle and the atomic electrons with the electromagnetic field

and the potential energy of the interaction of the incident particle with the atomic electrons. This last interaction is described variously for the exact and the approximate statement of the problem. The screening approximation implies that the incident particle is subject to the action of the average effective field of the atomic electrons, which is determined by their “smeared” charge whose density, in turn, is specified by the atomic wave function. Therefore, the interaction $V_{ea}(\mathbf{r})$ in the Hamiltonian (2) depends only on the coordinates of the incident particle.

The simplest solution of the Schrödinger equation with the Hamiltonian (1) can be obtained in terms of the perturbation theory with the interactions V_{fe} , V_{fa} , and V_{ea} treated as small perturbations. The cross section for bremsstrahlung is defined by the squared modulus of the transition amplitude M (matrix element of the transition) and allows one to calculate the probability that the incident particle will be scattered within a solid angle element with spontaneous or forced absorption of a photon. The transition amplitude is the sum of two terms corresponding to two channels of the process. The first channel involves the scattering of the initial electron in the field of the screened charge of the nucleus. The second channel is associated with the scattering due to dynamic polarization of the atom that is induced by the charge of the incident particle or by a given external electromagnetic field, e.g., the field of a laser beam. Thus, we have

$$M = M_{\text{st}} + M_{\text{pol}}, \quad (3)$$

where M_{st} and M_{pol} are the respective amplitudes of the transitions via the static and the polarization channel. If we calculate the transition range using only the approximate Hamiltonian (2), we obtain only the first term in (3) that describes static bremsstrahlung, which takes its name from the fact that the scattering in this case occurs in the electrostatic field of an atom. The first and second terms describe the radiation of different dipole moments: the first one is associated with the first dipole moment of the incident particle and the second one with the variable dipole moment of the atom, and therefore they variously depend on the mass of the incident particle. Since for a given force acceleration is inversely proportional to mass, the static bremsstrahlung is much less intense for an incident proton than for an incident electron. Clearly this is not the case of polarization bremsstrahlung. In this sense, PB is similar to Vavilov–Cerenkov and transition radiations, which are also related to the dynamic polarizability of the medium. The PB of a proton is considered in [11]. Calculations performed for the aluminum atom [9] have shown that if the polarization amplitude is taken into account, good agreement between theory and experiment is achieved for 1-MeV protons in the range of photon energies roughly from 2 to 5 keV, the photons being emitted from an aluminum target which is bombarded by protons.

It should be stressed that invoking the quantum theory is necessary. Despite the difference in the mechanisms for the emission of radiation via the two channels, the transition amplitude M describes one and the same physical object, i.e., the bremsstrahlung of a charged particle. This shows up vividly in the interference of the partial transition amplitudes M_{st} and M_{pol} . The total probability of the transition is determined by the squared modulus of the transition amplitude

$$|M|^2 = |M_{\text{st}}|^2 + |M_{\text{pol}}|^2 + M_{\text{st}}M_{\text{pol}}^* + M_{\text{st}}^*M_{\text{pol}}. \quad (4)$$

Thus, the total probability is not equal to the sum of the probabilities associated with the two channels, but is given by this sum and additional interference terms.

Formula (4) implies another physical pattern of the bremsstrahlung where the incident particle is an electron. In Born’s approximation with the radiation frequencies ω being far in excess of the ionization energy of the atoms I ($\omega \gg I$), for long-distance collisions such that the distance of sighting is much greater than the atomic dimensions ($|q|^{-1} \gg a$, where q is the momentum transferred by the incident particle to the atom), the term in M_{st} that describes the screening is canceled out in the dipole approximation with M_{pol} . Thus a physical pattern arises which is just the reverse of the screening pattern: the electron radiates as if it were in the field of a “bare” unscreened nucleus. This result has a clear sense. Since the above assumptions allow us to consider an atomic electron as being free, the incident electron is treated as being scattered by the two other particles: the nucleus and the free electron. However, in terms of both quantum [12] and classical theory [13], in the dipole approximation, no radiation appears when particles with identical charge-to-mass ratios are scattered. Thus, the radiation that appears at the nucleus upon scattering of the incident electron persists [14].

In the region of low incident electron energies, the interference of the conventional radiation and the PB may result in a complete mutual suppression of the effect. It has been demonstrated [15] that this mutual suppression is possible for

atoms in which the Ramsauer effect is observed on elastic scattering. This leads, in particular, to the appearance of a transmission band for an electromagnetic radiation propagating in a weakly ionized gas. Quantum interference is pronounced in the cross section of a bremsstrahlung electron that is differential in the angle of electron scattering [16] (see also the review [17]).

The importance of invoking PB in the interpretation and prediction of experimental results was first demonstrated in the studies reported elsewhere [8, 9]. In the former of these papers, the so-called “giant” resonance in X-ray bremsstrahlung is explained. This phenomenon was studied experimentally for the scattering of electrons of energy 0.6 keV by gaseous xenon [18]. In [9] the role of the polarization effect in the absorption of radiation and the related mechanism for the laser breakdown of alkali metal vapors, experimentally studied in [19], are investigated.

The paper [9] is also of interest in the analysis of the conditions under which a rather simple formula for the cross section of nonrelativistic PB follows from quantum mechanical considerations. This formula largely coincides with the formula of Askarjan [20], who seems to be the first to consider the classical mechanism of the PB for an electron scattered by a perfectly conducting sphere. Unfortunately, the importance of the work of Askarjan for atomic physics was not realized in due time.

Subsequently considerable experimental work was performed which was specially devoted to the study of PB. In particular, we should like to mention the study of the PB of fast electrons in metallic lanthanum [21] and a detailed analysis of the PB in gaseous xenon [22–24].

It seems that when an incident particle is scattered by a multielectron atom, not only the dynamic polarization of the valence electron takes part in the radiation, but so does the dynamic polarization of the electrons of the inner atomic shells. Theoretical and experimental studies of the PB at multielectron atoms are described not only in [8, 9], but also in a later paper [22] and in the works cited in this paper.

The calculations were substantially simplified [22] due to the use of the Thomas–Fermi–Dirac model of an ion and the so-called local plasma frequency approximation. Comparison with multielectron calculations has appeared favorable for the assumptions of [23] and has made it possible to estimate their validity.

The use of laser radiation has naturally posed the problem of multiphoton transitions in bremsstrahlung and in the reverse braking effect. Initially a theory of the multiphoton bremsstrahlung in a given field of an atom was created [24] and later studies were performed where the polarization effect was taken into consideration [25–27]. A contemporary review of the relevant studies is given in [28]. An efficient application of a special quasiclassical approach, the so-called Kramers’ electrodynamics [29], to the calculation of bremsstrahlung in a strong magnetic field is discussed in [30].

In the foregoing we discussed triple collisions of an atom, an electron, and a photon, such that the states of the electron and the magnetic field change, while the atom retains its state. It is clear that in an actual experiment, an atom can be excited and simultaneously emit or absorb a photon. It was concluded [32] that this effect can be observed in a laser field. In the pioneering experiment [33], excitation of the 2^3S helium atom by this mechanism was discovered. This problem is discussed in detail in [34].

The range of use of the theory of polarization bremsstrahlung is far from being restricted to the emission of radiation by a system consisting of an atom and an incident fast particle. Currently PB is often considered as a macroscopic base for many coherent mechanisms of the emission of radiation by a fast particle in a medium. Conventionally, these mechanisms (transition radiation [2], resonance transition radiation [37], parametric radiation [38–41], diffraction radiation [39, 42]) are described in terms of macroscopic electrodynamics. The mentioned mechanism and some other mechanisms of the emission of radiation in condensed (crystalline and amorphous) media are currently considered as basic for the creation of efficient sources of quasimonochromatic X radiation. Therefore, they are extensively investigated experimentally at many accelerator centers (see, e.g., [43–49]).

The characteristics of the coherent radiation (including PB) of a fast particle in a dense medium may be substantially different from those of the radiation of a particle at an isolated atom. By now this difference has been demonstrated most vividly for relativistic particles in crystals where, due to the ordered disposition of atoms in the lattice, correlations appear between successive collisions of the incident particles with the crystal atoms. These correlations result in coherent summation of the amplitudes of the radiations of the fast particle at the atoms located along the particle trajectory within the boundaries of the zone where the radiation is formed. As this takes place, sharp maxima appear in the bremsstrahlung spectrum where the intensity of the radiation is over that of the radiation at an isolated atom [39]. The

theory and the results of experimental investigations of the coherent bremsstrahlung and the radiation emitted in channeling are described in a number of monographs [39, 50–52].

It can easily be understood that collective phenomena affect the properties of the polarization radiation of a fast particle more substantially than those of bremsstrahlung. This is due to the high values of the effective sighting parameters for the collision of a fast particle with an atom that are responsible for the appearance of polarization radiation. Obviously, the dynamic polarization of an atom, induced by the Coulomb field of an incident particle, will be a maximum if the sighting parameter is of the order of the atomic size a . In this case, all atomic electrons are on the same side of the trajectory of the fast particle and therefore they shift unidirectionally, generating a maximum current which is coherent with respect to the atom. Thus, the contributions of neighboring atoms to the polarization radiation of a fast particle moving in a condensed medium with average interatomic distance of the order of the atomic size are strongly correlated and cannot be considered independently. Analysis has shown that in terms of the simplest model of the PB of a fast particle in a condensed amorphous medium, these correlations are responsible for the abrupt suppression of the radiation yield in the range of low photon energies: $\omega < a^{-1}$ [53].

A more interesting physical pattern arises for the PB of a fast particle in a crystal. Summing of the radiation amplitudes over all crystal atoms and averaging the cross section over the thermal oscillations of atoms in the lattice result in a standard formula for the cross section

$$d\sigma = d\sigma^{\text{coh}} + d\sigma^{\text{inc}} \equiv d\sigma_0[S(\mathbf{q})e^{-q^2u^2} + N(1 - e^{-q^2u^2})]. \quad (5)$$

Here $d\sigma_0$ is the cross section for the PB at an isolated atom; $S(\mathbf{q})$ is the structure factor of the crystal ($S(\mathbf{q}) = N^2$ for the transferred momenta \mathbf{q} that coincide with the vectors of the reciprocal lattice of the crystal); N is the number of atoms in the crystal, and u is the rms amplitude of the thermal vibrations of the atoms.

Expression (6) is completely similar to the corresponding expression in the theory of coherent bremsstrahlung [39]. However, while for the conventional bremsstrahlung the effective transferred momentum $q \sim m \gg u^{-1}$ (with m being the mass of the radiating particle), so that $e^{-q^2u^2} \ll 1$ and the noncoherent component of the radiation in the crystal is only slightly different from that which follows from the Bethe–Heitler theory for an amorphous medium, for the PB we have $q \sim a^{-1} \ll u^{-1}$, so that $1 - e^{-q^2u^2} \approx q^2u^2 \sim u^2/a^2 \ll 1$ [54]. Thus, the noncoherent component of the polarization radiation of a fast particle in a crystal is almost completely suppressed, and this is where the given mechanism differs significantly from the conventional bremsstrahlung mechanism [54].

The first term in (5) that describes the coherent component of the PB in a crystal is of considerable interest as well. It appears [54, 55] that this component completely coincides with the parametric X radiation (PXR) of a fast particle in a crystal, which was predicted earlier based on a macroscopic description of the radiation of a fast particle in a medium with a periodically varying dielectric susceptibility [38–41]. By now the PXR, detected experimentally [44], has been studied rather adequately both theoretically [56–58] and experimentally [44–48, 59–61].

The macroscopic approach in the theory of parametric radiation, based on the summation of the amplitudes of the scattering of the Coulomb field of a fast particle by the electrons of atoms which are periodically distributed in space, is, in fact, the theory of coherent polarization radiation proposed a long ago by Dialetis [62] who, unfortunately, did not continue his investigations.

For a fast particle moving in a crystal, obviously both the polarization and the conventional breakdown mechanisms for the radiation are realized which interfere on the amplitude level, as this takes place during the interaction of a particle with an isolated atom [see relation (4)]. In certain conditions, the interference of the coherent components of the amplitudes may lead to a pronounced effect. This effect was predicted theoretically [54, 63] and detected experimentally [64, 65].

The polarization bremsstrahlung mechanism also appears to be very useful in analyzing the dynamics of fast particles moving in a medium with macroscopic nonuniformities in density (both regular and random). This process, which is called transition scattering, is generally considered in terms of the model of a medium with a macroscopically nonuniform permittivity [66]. The equivalence of the results of the macroscopic theory of transition radiation and the microscopic theory of coherent polarization radiation was demonstrated in [67, 68].

The characteristics of the radiation of a relativistic electron in a thin layer of condensed matter are well described by the macroscopic theory of bremsstrahlung in the range of angles $\Theta \sim \gamma^{-1}$ (with γ being the Lorentz factor). However, for

the photon emission angles that are considerably greater than the characteristic angle γ^{-1} , a correct description of the process should use a macroscopic description of the PB. In a recent experiment [69], the emission of X radiation at an angle $\Theta = 19^\circ$ resulting from the interaction of electrons with thin metal foils was investigated, and it has been shown that the major contribution to the X radiation in this range is from the PB.

The pronounced dependence of the characteristics of the coherent component of a polarization radiation on the mutual disposition of atoms in a condensed medium makes it possible to expect that methods for diagnosing the structure of solids based on the variations in the spectral-angular distributions of this radiation will be developed. First experimental investigations of the polarization radiation of relativistic electrons in thin films of various materials [70–72] have shown that the measured radiation spectra differ radically from those predicted by the theory of the PB of a fast particle at an isolated atom. On the other hand, the data of measurements obtained in experiments with metal films agree with the theory of the PB in polycrystals [72], and the unexpected result [71] that the coherent component of the polarization radiation in a diamondlike carbon film is almost completely suppressed can be explained in terms of the model of the radiation of a relativistic electron in a finely dispersed material consisting of randomly oriented crystallites of very small diameter.

The approach discussed in [62] may appear fruitful in considering the PB spectrum in the region of photon energies approaching the photoabsorption edge [59]. In recent years the so-called EXAFS spectroscopy has been extensively developed which is used to investigate the structure of solids based on the examination of the fine structure of the absorption spectra near the absorption edge for the material under investigation. EXAFS spectroscopy makes use of an intense monochromatic X-radiation beam with a controllable wavelength, which is produced by an electron energy store with an energy of about 1 GeV.

It seems that the fine structure of the photoabsorption edge for crystalline materials could be investigated with the use of PB. Experiment [73] has demonstrated that the “natural” width of the line of PB in a perfect crystal is not over several electron-volts.

In [74, 75] a new method for the generation of soft X rays is proposed which is based on passing an electron beam through a one-dimensional periodic nanostructure formed of layers of materials with different Z , each layer having a thickness of about 10 nm. The authors of [74, 75] made use of a classical description of the radiation in such a medium with the permittivity averaged over each layer. However, correct calculations call for a more rigorous approach based on the microscopic theory of PB in periodic structures.

The microscopic theory of PB can also be extremely useful in solving the problem of the radiation emitted by polarized particles (e.g., positrons) in their passage through a condensed medium. The scattering of polarized positrons by the polarized electrons of a magnetized ferromagnet is known to be harnessed for measuring the polarization of an incident nonrelativistic positron. It should be expected that the intensity of the PB of a polarized relativistic positron will also depend on the orientation of the positron spin. It is quite possible that this dependence could be used for the creation of a new type of polarimeter. The dependence of the PB characteristics on the polarization of the initial electrons was investigated based on the kinematic theory [76], and it has been shown that the asymmetry of the process under investigation is nonzero even for ordinary (nonmagnetic) crystals.

It was pointed out [5] that a broad spectrum of phenomena can be described based on an approach similar to the microscopic theory of PB. Let us give one more example, namely, the emission of radiation on collisions of relativistic ion and proton bunches in modern colliders [77]. The parameters of bunches are such that the field of a relativistic ion (proton) polarizes an incoming ion bunch, which is the reason for the emission of electromagnetic radiation. Despite the giant mass of the particles, the photon yield, which is proportional to $(ZN)^2$ (with Z being the charge of the ion and N the number of particles in the bunch), will be high enough to be detected. The authors of [77] suggest harnessing this effect for nonperturbing diagnostics of beams in colliders.

An interesting subject for investigations of coherent bremsstrahlung in a condensed matter is the generation of radiation through the excitation of collective oscillations of the electron density in metal clusters and fullerenes (see [35, 36] and the references involved).

Thus, three decades ago a new area in the physics of radiation of charged particles, namely PBR, arose that essentially altered the traditional notions about the character of the bremsstrahlung of fast particles at atoms. At present this approach is used more and more widely, uniting descriptions of all processes of emission of radiation based on the excitation of a dynamic polarization by the electromagnetic field of an incident particle in a medium with inhomogeneities of any type: both microscopic and macroscopic.

The subject of contemporary investigations is the emission of radiation by a fast particle interacting with a group of atoms where the properties of the emitted photons are determined simultaneously by both the character of the interaction of the incident particle with an individual atom and the interatomic correlations. Under certain conditions, the radiation yield coherent over a group of atoms may be much in excess of the coherent background. This offers strong possibilities of diagnosing the structure of materials and creating efficient sources of X radiation with unique intensity, directivity, and monochromatism.

At present intense experimental studies are under way in the field of the above applications, and the joint efforts of experimenters and theoretists engaged in PB research will accelerate the rate of development of the area of physics under discussion.

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