Orientation effects in the incoherent bremsstrahlung by high energy particles in a crystal

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Abstract

A simulation procedure based on a semiclassical description of the bremsstrahlung process in crystals is developed. Using this procedure it is demonstrated that the intensity of hard incoherent radiation possesses substantial orientation dependence, when the particles are incident to crystallographic axes or planes at angles close to the critical channeling angle. It is demonstrated that the orientation dependences for the electrons and positrons substantially differ from each other. The orientation dependence on the crystal thickness is also considered. The results of simulations are in a good agreement with the experimental data.

Keywords: Bremsstrahlung; Incoherent radiation; Semiclassical approximation; Simulation

1. Introduction

During the motion of fast charged particles in a crystal at a small angle to one of crystallographic axes or planes the channeling phenomenon is possible, when the particles move in channels formed by uniform potentials of atomic strings or planes in the crystal [1–3]. The influence of such uniform potentials leads to redistribution of the particle flux in the crystal. The character of this redistribution substantially varies with crystal orientation with respect to the incident beam by the angles of order of the critical channeling angle [3–5]. This leads to substantial orientation dependence of yields of the processes connected to small impact parameters. Such orientation dependences have been observed earlier for nuclear reactions yields, delta-electron yield, and a number of other processes [2,3,6–8].

The present paper is devoted to the analysis of the orientation dependence of the yield of incoherent radiation of relativistic electrons and positrons during their motion in the crystal near planes or axes. An electron passing through the oriented crystal produces the coherent bremsstrahlung, which is due to the periodicity of the spatial arrangement of atoms in the lattice, and the incoherent one, which is due to the thermal spread of the atoms around their equilibrium positions [3,9]. The spectrum of the incoherent part of radiation is similar to the Bethe–Heitler spectrum of radiation in an amorphous medium. For the incident electrons of energy about 1 GeV the incoherent part of radiation is predominant in the hard range of the bremsstrahlung spectrum.

At first consideration, the incoherent part of bremsstrahlung should not depend on the crystal orientation in relation to the electron beam. However, it was demonstrated in [10] (using a semi-numerical procedure) that details of the particles' dynamics in a crystal could lead to substantial orientation dependence of the incoherent

radiation efficiency. In the present article we shall develop the improved variant of the simulation procedure described in [10] and compare the results of computations with the experimental ones.

2. Simulation procedure for incoherent radiation

The radiation of relativistic electrons in matter develops in a large spatial region along the particle's momentum. This region is known as the coherence length (or formation length) [3,9]. If the electron collides with a large number of crystal atoms in the coherence length, the effective constant of the interaction of the electron with the lattice atoms may be large in comparison with the unit, so we could use the semiclassical description of the radiation process [3,11]. In the dipole approximation the spectral density of bremsstrahlung is described by the formula (see Eq. (68.1) in [3])

$$\frac{dE}{d\omega} = \frac{e^2\omega}{2\pi c^4}\int_\delta^\infty \frac{dq}{q^2}\left[1 + \frac{(\hbar\omega)^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right]|\mathbf{W}_q|^2, \qquad (1)$$

where $q = \frac{\varepsilon}{\varepsilon'}(\omega - \mathbf{kv})$, ω and $\boldsymbol{k}$ are the frequency and the wave vector of the radiated photon, ε is the energy of the initial electron, $\boldsymbol{v}$ is its velocity, $\varepsilon' = \varepsilon - \hbar\omega$, $\mathbf{W}_q = \int_{-\infty}^{\infty}\dot{\mathbf{v}}_\perp(t)e^{iqt}dt$ is the Fourier component of the electron acceleration in the direction orthogonal to $\boldsymbol{v}$, $\delta = \frac{m^2c^3\omega}{2\varepsilon\varepsilon'}$. Note that the coherence length l_{coh} is about δ^{-1}.

For the case of radiation of an electron in the field of single atom (using the screened Coulomb potential $U(r) = Ze\frac{e^{-r/R}}{r}$ as the potential of the atom, where Z is the atomic number, R is Thomas–Fermi radius) we have:

$$\mathbf{W}_q^{(1)}(\rho_0) = \frac{2Ze^2c}{\varepsilon}\sqrt{q^2 + R^{-2}}K_1\left(\rho_0\sqrt{q^2 + R^{-2}}\right)\frac{\boldsymbol{\rho}_0}{\rho_0}, \qquad (2)$$

where $K_1(x)$ is the modified Bessel function of the third kind, $\boldsymbol{\rho}_0$ is the impact parameter. Since the characteristic values of q making the main contribution to the integral in Eqn. (1) are $q \sim \delta \ll R^{-1}$, we can take $q = 0$ in Eqn. (2):

$$\mathbf{W}^{(1)}(\rho_0) = \frac{2Ze^2c}{\varepsilon R}K_1\left(\frac{\rho_0}{R}\right)\frac{\boldsymbol{\rho}_0}{\rho_0}. \qquad (3)$$

Substituting this value into the general formula, we obtain the Bethe–Heitler result for the radiation efficiency of a unit particle flux in the field of the atom:

$$\hbar\omega\frac{d\sigma_{BH}}{d\omega} = \int\frac{dE}{d\omega}d^2\rho_0 = \frac{16}{3}\frac{Z^2e^6}{m^2c^2}\left(1 + \frac{3}{4}\frac{(\hbar\omega)^2}{\varepsilon\varepsilon'}\right)\ln\left(\frac{mRc}{\hbar}\right). \qquad (4)$$

Note that the integral over the impact parameter diverges at small values of ρ_0. The divergence results from the use of the dipole approximation, which is valid at $\rho_0 \geqslant \hbar/mc$. We take this constraint into account by introducing the lower limit of integration ($\rho_{\min} = \hbar/mc$ that is the Compton wavelength of the electron), so this result is obtained with logarithmic accuracy.

Consider now the radiation of the electron interacting with a crystal which is a system of atoms periodically arranged in space. The case of our interest is the electron incidence onto the crystal under small angle ψ to one of its crystallographic axes (the z axis). It is known [3,9] that averaging of the equation for the $|\mathbf{W}_q|^2$ over the thermal vibrations of atoms in the lattice leads to the split of this value (and so the radiation intensity) into the sum of two terms describing coherent and incoherent effects in radiation:

$$\left\langle|\mathbf{W}_q|^2\right\rangle = \sum_{n,m}e^{iqc(t_n - t_m)}\left\langle\mathbf{W}_q^{(1)}(\boldsymbol{\rho}_n - \mathbf{u}_n)\right\rangle\left\langle\mathbf{W}_q^{(1)}(\boldsymbol{\rho}_m - \mathbf{u}_m)\right\rangle + \sum_n\left\{\left\langle\left(\mathbf{W}_q^{(1)}(\boldsymbol{\rho}_n - \mathbf{u}_n)\right)^2\right\rangle - \left(\left\langle\mathbf{W}_q^{(1)}(\boldsymbol{\rho}_n - \mathbf{u}_n)\right\rangle\right)^2\right\}, \qquad (5)$$

where the indexes n and m numerate the atoms under collisions, t_n is the time moment when the electron collides with the n-th atom, $\boldsymbol{\rho}_n = \boldsymbol{\rho}(t_n) - \boldsymbol{\rho}_n^0$ is the impact parameter of the collision with the n-th atom in its equilibrium position $\boldsymbol{\rho}_n^0$, $\boldsymbol{\rho}(t)$ is the trajectory of the electron in the plane orthogonal to z axis, and $\mathbf{u}_n$ is the thermal shift of the n-th atom from the position of equilibrium.

In the range of frequencies for which

$$l_{\mathrm{coh}} \sim \delta^{-1} \ll a/\psi, \qquad (6)$$

where a is the distance between two parallel atomic strings the closest to each other, the incoherent component of radiation is predominant [3]. As in the case of the radiation on a single atom, we put $q = 0$ and then, substituting the formula for $W^{(1)}$ from Eqn. (3), we find the following expression for the incoherent part of the quantity of interest:

$$|W_{\mathrm{incoh}}|^2 = \frac{4Z^2e^4c^2}{\varepsilon^2R^2}\sum_n F(\boldsymbol{\rho}_n), \qquad (7)$$

where

$$F(\boldsymbol{\rho}) = \left\langle\left(K_1\left(\frac{|\boldsymbol{\rho} + \mathbf{u}|}{R}\right)\right)^2\right\rangle - \left\langle\frac{\boldsymbol{\rho} + \mathbf{u}}{|\boldsymbol{\rho} + \mathbf{u}|}K_1\left(\frac{|\boldsymbol{\rho} + \mathbf{u}|}{R}\right)\right\rangle^2. \qquad (8)$$

For computing the value $F(\rho)$ we substitute $\rho = \rho_{\min}$ when $\rho < \rho_{\min}$ to take into account the constraint under small distances. Averaging over thermal vibrations is made by integration with a Gaussian distribution. This integration can be carried out only numerically, so the values of the function $F(\rho)$ in Eq. (7) are determined by interpolation of the results of numerical integration for the finite set of the values $\boldsymbol{\rho}$.

It is convenient to compare the efficiency of incoherent radiation in the crystal with the radiation efficiency in an amorphous medium (with equal numbers of collisions with

atoms N in both cases). The ratio of these two values is equal to

$$N_\gamma = \frac{\int d^2\rho_0 \left(\frac{dE}{d\omega}\right)_{\text{incoh}}}{N\hbar\omega \frac{d\sigma_{BH}}{d\omega}} = \frac{\int d^2\rho_0 \sum_n F(\boldsymbol{\rho}_n)}{2\pi N R^2 \ln\left(\frac{mRc}{\hbar}\right)}, \tag{9}$$

where integration over $d^2\rho_0$ means the integration over all possible points of incidence of the beam onto the crystal surface. This integration can be effectively reduced to the integration over one elementary cell in the plane (x, y). It can be carried out using Monte–Carlo techniques.The impact parameters of collisions with atoms $\boldsymbol{\rho}_n$ are determined using the trajectory of the electron in the crystal $\boldsymbol{\rho}(t)$, which is determined by the equation of motion [1–3]

$$\ddot{\boldsymbol{\rho}} = -\frac{c^2}{\varepsilon}\frac{\partial}{\partial\boldsymbol{\rho}}\sum_s U_R(\boldsymbol{\rho} - \boldsymbol{\rho}_s^0), \tag{10}$$

where $U_R(\boldsymbol{\rho} - \boldsymbol{\rho}_s^0)$ is the potential energy of the electron's interaction with the uniform potential of the atomic string parallel to the z axis, $\boldsymbol{\rho}_s^0$ is the position of the string in the transverse plane.

However, the thermal spread of atoms from their positions of equilibrium in the lattice leads to incoherent scattering of incident electrons on thermal vibrations of atoms along with their coherent scattering on the uniform potentials of atomic strings (see [3], section 57). The scattering angle of the electron on a single atom $\vartheta^{(1)}(\boldsymbol{\rho}_0)$ can be found from the equation of motion; it is described by the formula that differs from (3) for $W^{(1)}$ only by the coefficient c. After summation of the scattering angles on all atoms constituting the atomic string in the crystal and averaging its squared value over thermal vibrations, we obtain the equation analogous to Eqn. (5). So, the average value of the squared angle of the incoherent scattering is described by the formula that is distinct from Eqn. (7) only by the factor c^2:

$$\langle\vartheta^2\rangle_{\text{incoh}} = \frac{4Z^2e^4}{\varepsilon^2R^2}\sum_n F(\boldsymbol{\rho}_n) \tag{11}$$

Hence during simulation of the electron's trajectory in the crystal the incoherent scattering could be taken into account by addition to each component of the electron's velocity in the transverse plane in the time moment of collision with the n-th atom the random value distributed by the Gaussian law with the dispersion equal to $(2Z^2e^4c^2/\varepsilon^2R^2)F(\boldsymbol{\rho}_n)$.

3. Orientation dependence of the incoherent radiation of relativistic electrons and positrons in the crystal

Specific parameters of the simulation are chosen corresponding to the conditions of the experiment in [7]. Photons of energy $\hbar\omega = 1.1$ GeV emitted by electrons of energy $\varepsilon = 1.2$ GeV incident on a silicon ($Z = 14$, $R = 0.194$ Å) crystal under small angle θ to the (1 1 0) plane were registered. Under these conditions the angle of electron

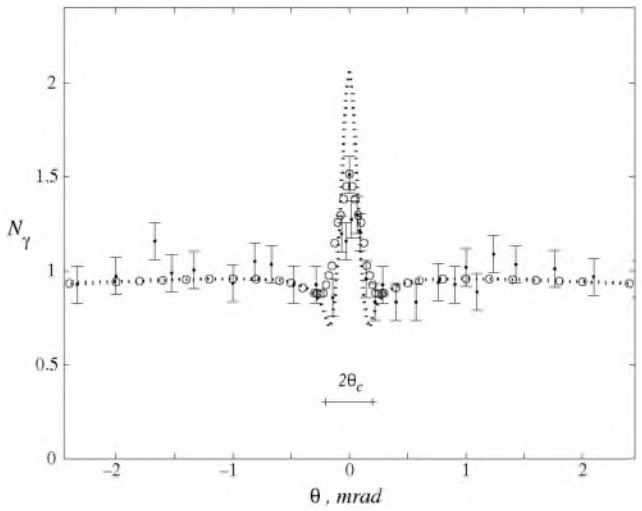

Fig. 1. Photon yields of energy of 1.1 GeV versus incidence angle of 1.2 GeV electrons to the (1 1 0) plane of a 30 μm thick silicon crystal [7] (error bars) and the efficiency of incoherent radiation of electrons as a result of simulation based on Eq. (9) with (circles) and without (dotted line) taking account of incoherent scattering of the electrons by thermal vibrations.

incidence to the ⟨0 0 1⟩ axis (the z axis) was large enough to ensure the absence of axial channeling, $\psi \sim 100\psi_c$, where the critical angle of axial channeling is

$$\psi_c = \sqrt{\frac{4Ze^2}{\varepsilon a_z}} \approx 3.5 \times 10^{-4}\text{rad} \tag{12}$$

(the lattice constant along the ⟨0 0 1⟩ axis in silicon crystal is $a_z = 5.431$ Å)[1]. Note that planar channeling takes place under $\theta \leqslant \theta_c$, where θ_c is the critical plane channeling angle (for the case under discussion $\theta_c \approx 2 \times 10^{-4}$ rad). The experimental data (see Fig. 2 in [7]) and the results of simulations are presented on Fig. 1. When the simulation procedure takes into account not only the electron's motion in uniform potentials of atomic strings (as in [10]), but also its incoherent scattering on thermal vibrations of separated lattice atoms, we see a good agreement between the experimental data and the results of simulation.

We can see that the incoherent radiation efficiency demonstrates substantial dependence on the crystal orientation in relation to the electron beam. The similar (but opposite) situation takes place for the radiation from positrons (see Fig. 2). In both cases the maxima and minima of the radiation efficiency are determined by the characteristics of the particle trajectories in the crystal. For θ values close to zero, planar channeling takes place for the most part of points of incidence of the particle onto the crystal. The electron under planar channeling spends most of the motion through the crystal in close vicinity to atomic planes, with small impact parameters of collisions with atoms that leads to the maximum in the efficiency of inco-

[1] In the case of large ψ we can approximate the real electron's trajectory by broken line, that leads to economy of computation time.[10].

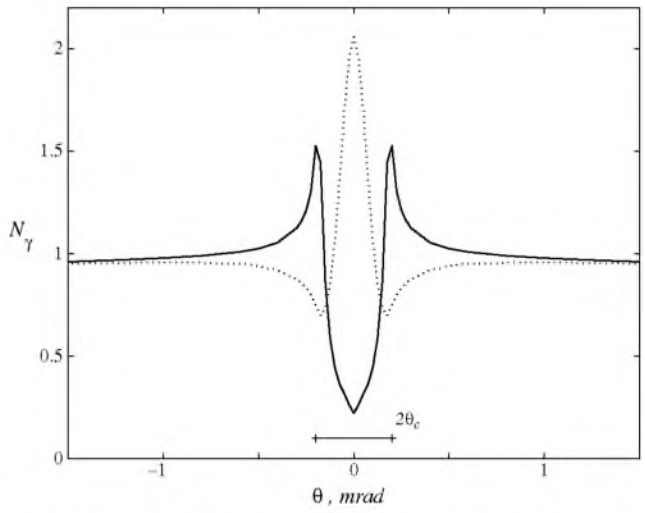

Fig. 2. Efficiency of incoherent radiation from positrons (solid line) and electrons (dotted line) simulated under the same conditions as Fig. 1, without taking into account particle scattering by the thermal vibrations.

herent radiation. On the other hand, the positron under planar channeling alignment spends most of the time far from atomic planes, leading to a minimum in the incoherent radiation intensity.

For angles close to the critical angle of planar channeling θ_c the above-barrier positrons spend most of the time in close vicinity to atomic planes, leading to a maxima in the incoherent radiation efficiency. In contrast, the above-barrier electrons rapidly move through atomic planes, leading to a minima in the incoherent radiation intensity.

For $\theta \gg \theta_c$ the energy of transverse motion of the particle $\varepsilon_\perp = \varepsilon\theta^2/2$ [3] exceeds by far the height of the potential barrier formed by the uniform potential of the atomic planes in the crystal. In this case the trajectory of the particle is almost rectilinear. For such a trajectory all possible impact parameters of collisions with atoms are almost equiprobable, like in an amorphous medium, and the incoherent radiation efficiency becomes equal to that in an amorphous medium and independent of the crystal orientation.

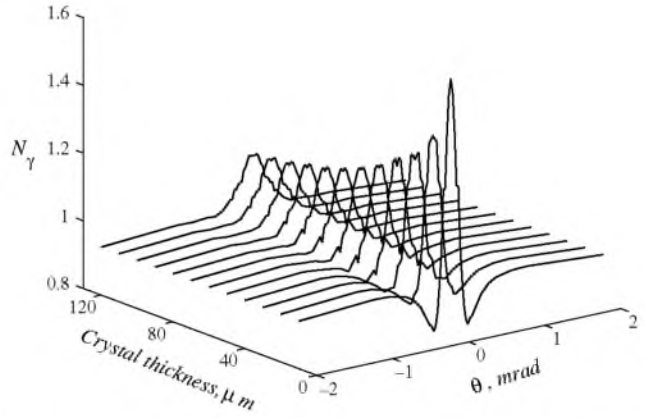

Fig. 3. Changing of orientation dependence of the incoherent radiation efficiency for the same conditions as Fig. 1 with increasing crystal thickness.

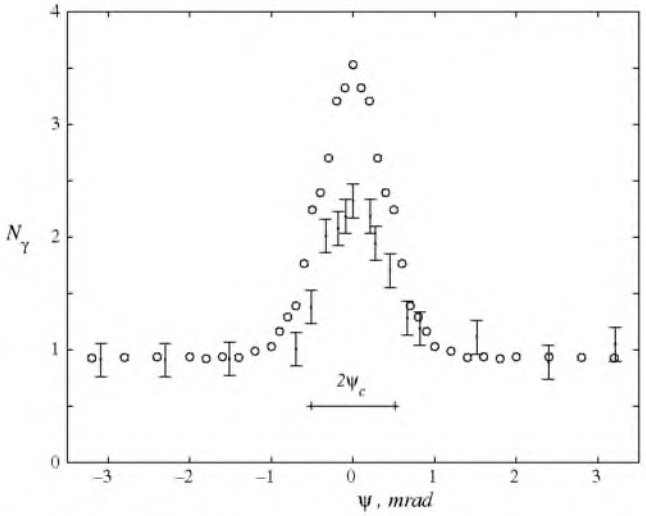

Fig. 4. Photon yields of energy 700 MeV versus incidence angle of 800 MeV electrons to the ⟨1 1 1⟩ axis of a 30 μm thick silicon crystal [7] (error bars) and the efficiency of incoherent radiation simulated taking into account the incoherent scattering of the electrons by thermal vibrations and beam divergence (circles).

The incoherent scattering of the particles by thermal vibrations of atoms leads to a dechanneling effect and, as a consequence, to a weakening of the orientation dependence described above (see Fig. 3). Note that under small values of the crystal thickness the maximal value of the radiation efficiency decreases exponentially rapidly, which is connected to the exponential character of the dechanneling process. On the other hand, for large crystal thicknesses that decrease becomes slower because of the concurrence of dechanneling and rechanneling effects.

We can also see the orientation dependence of the incoherent radiation efficiency in the case of the particles inci-

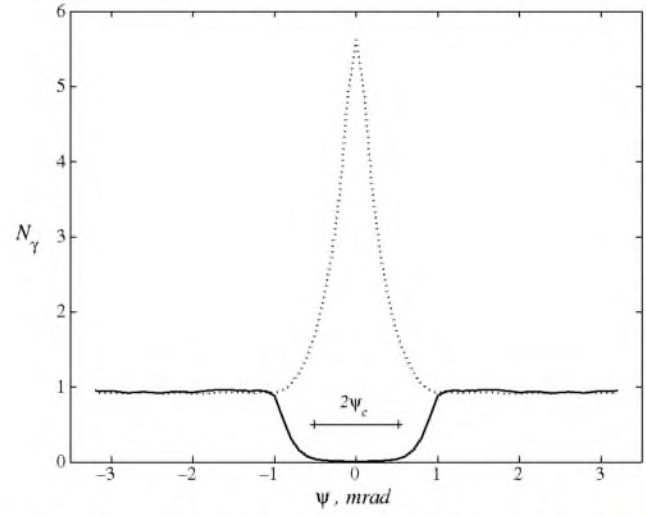

Fig. 5. Incoherent radiation efficiency from positrons (solid line) and electrons (dotted line) simulated under the same conditions as Fig. 4 without taking into account incoherent scattering of the particles and beam divergence.

dence at a small angle ψ to one of crystallographic axes which are densely packed with atoms. When axial channeling takes the place, the electrons spend most time near atoms and radiate stronger than in an amorphous medium (see Fig. 4). The situations for positrons is the opposite (Fig. 5).

4. Conclusion

The results of the simulations demonstrate substantial orientation dependence of the incoherent radiation yield for angles of incidence to the atomic planes or axes in the crystal close to the critical channeling angles. These orientation dependences substantially differ for relativistic electrons and positrons. In the case of electrons the simulations were compared to the experimental data obtained earlier at KIPT [7] on the orientation dependence of the incoherent radiation yield in the region of photon energies close to the energy of the radiating electron. The agreement between the results of the simulation and the experimental ones is quite good.

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