Ionization Energy Losses of Fast Charged Particles Produced in Matter

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Abstract—Ionization energy losses of an ultrarelativistic electron produced in matter are considered. The interference of the proper Coulomb field of the product particle and the electromagnetic wave that this particle emits is shown to be significant at impact-parameter values that make a dominant contribution to the ionization energy losses. The effect is shown to exert virtually no influence on the ionization energy losses of the particle.

1. Ionization energy losses of clusters formed by fast charged particles occurring at small distances from one another may differ significantly from the total ionization energy losses of the individual particles constituting the cluster if these particles are far from one another. This distinction is associated with the interference of the electromagnetic fields created by the particles of the cluster at distances that contribute significantly to ionization energy losses. Such a situation emerges, for example, when a high-energy electron–positron pair is produced in matter [1]. The point is that the characteristic angles of divergence of a high-energy pair produced in matter are very small. It follows that, over rather long a time interval, the transverse distance between the particles of the pair will be small in relation to the maximum impact-parameter values of $\rho_{\max} \sim v/\omega_p$, ($v$ is the particle velocity, and ω_p is the plasmon frequency) that contribute significantly to the ionization energy losses of the individual particles of the pair. The electromagnetic fields of the electron and the positron of the product pair compensate each other partly at distances of v/ω_p the from the pair in the transverse direction; therefore, the ionization energy losses of such a cluster are smaller than the ionization energy losses of the individual particles. By way of example, we indicate that, at photon energies of $\hbar\omega \sim 100$ GeV, characteristic angles of divergence of the particles forming the pair are estimated as $\theta_\pm \sim 4mc^2/\hbar\omega \sim 2 \times 10^{-5}$ rad, so that the reduction of the ionization energy losses of the product pair must manifest itself at longitudinal distances of $l \sim \rho_{\max}/\theta_\pm \sim 0.05$ cm from the pair-production vertex. This effect was observed in cosmic rays [1].

A similar effect occurs in the Coulomb explosion of fast molecules in a thin layer of matter [2]. The similarity of these two processes was noticed in [3].

For a long time, the electromagnetic field surrounding a high-energy charged particle (electron) produced in matter can differ significantly from the normal proper field of a similar particle that moves at a constant speed in the same direction [4, 5]. The effect is determined by the interference of the Coulomb field of the electron and the field of the electromagnetic wave that this electron emits. An ultrarelativistic electron emits waves predominantly at small angles with respect to its velocity, $\theta \sim mc^2/E$, where E is the electron energy. As a result, an electron, with its Coulomb field, and the emitted electromagnetic wave will be at small distances from each other for a long period of time; hence, the effect of interference between the two fields will be significant. In this sense, the electron and the electromagnetic wave emitted by it can be treated as a cluster formed by the Coulomb field of the electron and the emitted electromagnetic wave. Such clusters manifest themselves in many processes associated with radiation from ultrarelativistic electrons in matter, such as coherent radiation from relativistic electrons in oriented crystals and the Landau–Pomeranchuk effect, which consists in the suppression of bremsstrahlung from ultrahigh-energy electrons in amorphous media (see, for example, [5]). There naturally arises the question of whether such a cluster can manifest itself in ionization energy losses of a particle in a medium. This is the problem to be addressed in the present article.

2. We consider some special features of the evolution of the field of a particle following its production in matter and the ionization energy losses of the particle in this case. First, we analyze the evolution of the field of a high-energy particle produced in a medium, neglecting the dielectric properties of the medium.

We assume that a charged particle is instantaneously produced at the time instant $t = 0$ with a finite velocity $\mathbf{v}$.

The potentials of the particle field are determined by the equations ($c = 1$)

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\varphi(\mathbf{r}, t) = -4\pi e\delta(\mathbf{r} - \mathbf{v}t)\Theta(t), \quad (1)$$

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\mathbf{A}(\mathbf{r}, t) = -4\pi e\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t)\Theta(t), \quad (2)$$

where $\Theta(t)$ is the Heaviside step function. Solutions to Eqs. (1) and (2) can be represented in terms of the Fourier integrals

$$\varphi(\mathbf{r}, t) = \frac{e}{2\pi^2}\mathrm{Re}\int\frac{d^3k}{k(k - \mathbf{k}\cdot\mathbf{v})}e^{i\mathbf{k}\cdot\mathbf{r}}(e^{-i\mathbf{k}\cdot\mathbf{v}t} - e^{-ikt}), \quad (3)$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{e}{2\pi^2}\mathbf{v}\mathrm{Re}\int\frac{d^3k}{k(k - \mathbf{k}\cdot\mathbf{v})}e^{i\mathbf{k}\cdot\mathbf{v}t}(e^{-i\mathbf{k}\cdot\mathbf{v}t} - e^{-ikt}). \quad (4)$$

These solutions can also be recast into the form

$$\varphi(\mathbf{r}, t) = \Theta(t - r)\varphi_0(\mathbf{r}, t), \quad (5)$$

$$\mathbf{A}(\mathbf{r}, t) = \Theta(t - r)\mathbf{A}_0(\mathbf{r}, t), \quad (6)$$

where φ_0 and $\mathbf{A}_0$ determine the conventional Coulomb field of a charged particle that moves at a velocity $\mathbf{v}$,

$$\varphi_0(\mathbf{r}, t) = \frac{e}{[(z - vt)^2 + \rho^2\gamma^{-2}]^{1/2}}, \quad (7)$$

$$\mathbf{A}_0(\mathbf{r}, t) = \frac{e\mathbf{v}}{[(z - vt)^2 + \rho^2\gamma^{-2}]^{1/2}}. \quad (8)$$

Here, $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor, the z axis is aligned with $\mathbf{v}$, and $\boldsymbol{\rho}$ is a radius vector in the plane orthogonal to $\mathbf{v}$.

The first terms in (3) and (4) describe the potentials of the conventional Coulomb field of a particle that moves at a velocity $\mathbf{v}$. The second terms describe the field of the emitted wave for $t \longrightarrow \infty$. In each term in (3) and (4), the main contribution to the integrals with respect to $\mathbf{k}$ in every comes from $\mathbf{k}$ whose directions are close to the direction of $\mathbf{v}$—more specifically, from the region where the characteristic angle θ between $\mathbf{k}$ and $\mathbf{v}$ is $\theta \sim \gamma^{-1}$. For such $\mathbf{k}$, the relevant Fourier components of the surrounding field are suppressed over the time period from $t = 0$ to $t < (k - \mathbf{k}\cdot\mathbf{v})^{-1} \sim 2\gamma^2/k$ in relation to those in the region $t > 2\gamma^2/k$. This means that, over the period $\Delta t \sim 2\gamma^2/k$, the particle is in a "semibare" state deprived of its normal Coulomb field. Considering that the main contribution to the ionization energy losses of the particle comes from the region $k > \omega_p/v$, we can expect that the ionization energy losses are suppressed over the time interval $\Delta t \sim 2\gamma^2/k$. For electron energies of $E_e \sim 100$ GeV, we have $v\Delta t \sim 10^2$ cm.

Direct calculations reveal, however, that there is no such effect—that is, the ionization energy losses of the electron reach their normal value after a lapse of the time $\Delta t \sim \rho_{max}/v$. By using relations (5) and (6), we can indeed show (in accordance with the Bohr method for calculating the ionization energy losses [6]) that the energy losses of a particle at distances $z \gg \rho_{max}$ from its production vertex in matter are given by

$$T = T_0\left(1 - \frac{1}{\ln(\rho_{max}/\rho_{min})}\frac{\rho_{max}^2}{4z^2} + \ldots\right), \quad (9)$$

where

$$T_0 = \frac{\omega_p^2 e^2}{v}\ln\frac{\rho_{min}}{\rho_{max}} \quad (10)$$

and ρ_{min} is the minimum impact-parameter value that contributes significantly to the ionization energy losses. This value is determined on the basis of quantum considerations and is about p^{-1}, where p is the projectile momentum.

From Eq. (9), we can see that, at $z \gg \rho_{max}$, the correction to the conventional expression T_0 for the energy lost by the particle in matter owing to the prolonged existence of the electron in the "semibare" state is small. That the ionization energy losses increase within a time interval smaller than the time over which the normal Coulomb field of the electron is recovered can be explained by the fact that the splash Δ of the field strength at $t = r$ [see Eqs. (5) and (6)] compensates for the decrease in the ionization energy losses that is associated with the absence of the field for distances $r > t$.

3. Let us now take into account the dielectric properties of the medium. In this case, Eqs. (1) and (2) assume the form [7]

$$\hat{\varepsilon}\left(\nabla^2 - \hat{\varepsilon}\frac{\partial^2}{\partial t^2}\right)\varphi(\mathbf{r}, t) = -4\pi e\delta(\mathbf{r} - \mathbf{v}t)\Theta(t), \quad (11)$$

$$\left(\nabla^2 - \hat{\varepsilon}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}(\mathbf{r}, t) = -4\pi e\mathbf{v}\delta(\mathbf{r} - \mathbf{v}t)\Theta(t). \quad (12)$$

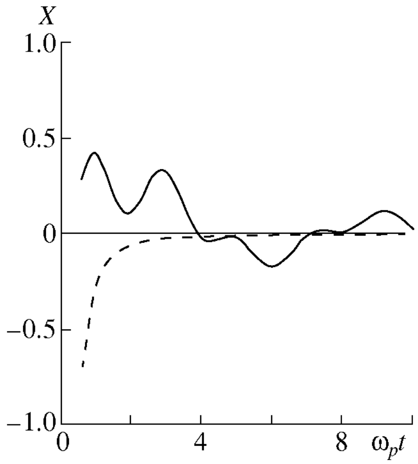
If the dielectric permittivity is given by

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{(\omega + i0)^2}, \quad (13)$$

solutions to these equations can be represented as

$$\varphi(\mathbf{r}, t) = \frac{e}{2\pi^2}\mathrm{Re}\int\frac{d^3k}{k^2}e^{i\mathbf{k}\cdot\mathbf{r}}\left\{-\frac{\omega_p e^{-i\mathbf{k}\cdot\mathbf{v}}}{\omega_p - \mathbf{k}\cdot\mathbf{v}} + \frac{\omega_p e^{-i\omega_p t}}{\omega_p - \mathbf{k}\cdot\mathbf{v}}\right. \quad (14)$$
$$\left. + \frac{\sqrt{k^2 + \omega_p^2}e^{-i\mathbf{k}\cdot\mathbf{v}t}}{\sqrt{k^2 + \omega_p^2} - \mathbf{k}\cdot\mathbf{v}} - \frac{\sqrt{k^2 + \omega_p^2}}{\sqrt{k^2 + \omega_p^2} - \mathbf{k}\cdot\mathbf{v}}e^{-i\sqrt{k^2 + \omega_p^2}t}\right\}\Theta(t),$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{e}{2\pi^2}v\mathrm{Re}\int d^3k e^{i\mathbf{k}\cdot\mathbf{r}}$$
$$\times\left\{\frac{e^{-i\mathbf{k}\cdot\mathbf{v}t}}{\sqrt{k^2 + \omega_p^2}(\sqrt{k^2 + \omega_p^2} - \mathbf{k}\cdot\mathbf{v})}\right. \quad (15)$$



Quantity X calculated according to the expressions (dashed curve) (9) and (solid curve) (16) versus $\omega_p t$.

$$- \left. \frac{e^{-i\sqrt{k^2+\omega_p^2}t}}{\sqrt{k^2+\omega_p^2}(\sqrt{k^2+\omega_p^2} - \mathbf{k}\cdot\mathbf{v})} \right\} \Theta(t).$$

It can easily be seen that, for $\omega_p \longrightarrow 0$, Eqs. (14) and (15) reduce to Eqs. (3) and (4). Calculating the ionization energy losses on the basis of the method described in [7], we obtain

$$T = T_0 + \frac{e^2\omega_p}{v}\frac{\sin(\omega_p t)}{t} - \frac{e^2\omega_p^2}{v} \tag{16}$$

$$\times[\mathrm{ci}(\omega_p t)\sin^2(\omega_p t) - \mathrm{si}(\omega_p t)\cos^2(\omega_p t)],$$

where

$$\mathrm{ci}(x) = -\int_x^\infty \frac{\cos t}{t}dt = C + \ln x + \int_0^x \frac{\cos t - 1}{t}dt,$$

$$\mathrm{si}(x) = -\int_x^\infty \frac{\sin t}{t}dt = -\frac{\pi}{2} + \int_0^x \frac{\sin t}{t}dt,$$

and $C = \ln\gamma = 0.577$ is the Euler constant.

The quantity $X = (v/\omega_p^2 e^2)(T - T_0)$, which determines the deviation of the ionization energy losses of the "semibare" electron from the normal energy losses in accordance with expressions (9) and (16), is displayed in the figure as a function of $\omega_p t$.

For electron energies of $E_e \sim 1$ GeV, $(v/\omega_p^2 e^2)T_0 \sim 17$; therefore, the difference of the T value computed according to (16) and T_0 at distances of a few ω_p^{-1} (that is, a few ρ_{max} for ultrarelativistic particles) is within 2%.

Thus, we see that the prolonged existence of an electron deprived of its normal Coulomb field has virtually no effect on the ionization energy loss of a particle in a medium. We emphasize, however, that the semibare-electron effect is significantly manifested in the radiation from relativistic particles [8].

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REFERENCES

1. A. E. Chudakov, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 651 (1955).
2. W. Brandt, A. Ratkowski, and H. Ritchie, Phys. Rev. Lett. **33**, 302, 1325 (1974).
3. N. F. Shul'ga and V. V. Syshchenko, Phys. Lett. A **165**, 175 (1992).
4. E. L. Feĭnberg, Zh. Éksp. Teor. Fiz. **50**, 202 (1966) [Sov. Phys. JETP **23**, 132 (1966)].
5. A. I. Akhiezer and N. F. Shul'ga, *Electrodynamics of High Energy in Matter* (Nauka, Moscow, 1993).
6. J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975; Inostrannaya Literatura, Moscow, 1965).
7. L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Nauka, Moscow, 1992; Pergamon, New York, 1984).
8. N. F. Shul'ga and S. P. Fomin, Phys. Lett. A **114**, 148 (1986).